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On Semi Alpha Syndetic Sets

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Abstract: In this peper we present new class of syndetic sets called Semi-α-Synditic sets and study their basic properties in topological groups

Keywords:- Semi-α-open set, Semi-α-closed set, Semi-α-compact and Semi-α-syndetic set.

1. Introduction:

In 2000, Semi-α-open set was presented by Navalagi [7]. Gottschalk and Hedlund [4] presented the notions of left (right) syndetic set in topological group. He defined a subset A of topological group G, is called left (right) syndetic if there exists a compact subset M of G such that $AM = G$.Al-Kutaibi [2] presented the concept of Semi-syndetic sets and feebly syndetic sets in topological groups. In 2016, Al-Khafaji [3] presented paper α-syndetic sets.

The purpose of this paper is to present the concept of Semi-α-syndetic Sets ans study their basic properties in topological groups.

2. Preliminaries:

Throughout this paper, (G, T) (or simply G) always mean topological space on which no separation axioms are assumed unless otherwise mentioned, For a set A in a topological Space (G, T), $Cl(A)$, $int(A)$ and $A^c = G - A$ denote the closure of A, the interior of A and the Complement of A respectively.

Definition 2.1[6]:- A subset A of a topological space (G, T) is Called α-open set iff $A \subseteq int\left(cl\left(int(A)\right)\right)$. The family of all α-open sets of G is denoted by T_α.

Definition 2.2[7]:- A subset A of atopological Space (G, T) is called Semi-α-open set iff there exists an α-open set U in G Such that $U \subseteq A \subseteq Cl(U)$. The family of all Semi-α-open sets of G is denoted by $S_\alpha O(G)$. The complement of Semi-α-open set is called Semi-α-closed set. The family of all Semi-α-closed sets is denoted by $S_\alpha C(G)$.

Proposition 2.3[6]:- Let (G, T) be a topological space, $A \subseteq G$. Then A is Semi-α-open set iff $A \subseteq Cl\left(int\left(Cl(int(A))\right)\right)$.

Remark 2.4 [6]:

i. Every open set is α-open, so it is Semi-α-open set, but the converse is not true in general.
ii. Every α-open set is Semi-α-open set, but the converse is not true in general.

Example 2.5: Let $G = \{a, b, c, d\}$, $T_G = \{G, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, then

$T_\alpha = T_G \cup \{\{a, b, c\}\}$,

$S_\alpha O(G) = T_\alpha \cup \{\{b, c, d\}, \{a, c, d\}, \{b, c\}, \{b, d\}, \{a, b\}, \{a, c\}\}$

i. Let $A = \{a, b, d\}$, A is α-open set but it is not an open. Let $B = \{b, c\}$, B is Semi-α-open set but it is not an open set
ii. Let $A = \{b, d\}$, A is Semi-α-open set but it is not an α-open set.

Definition 2.6[6]:- Let A be a subset of a topological space (G, T). The intersection of all Semi-α-closed sets containing A is called Semi-α-Closure of A. The Semi-α-Closure of A is denoted by $S_\alpha - Cl(A)$.

Definition 2.7[1]:- Let (G, T) be a topological space, $A \subseteq G$, of a family W of subsets of G is said to be a Semi-α-open cover of A iff W cover A and W is a subfamily of $S_\alpha O(G)$.

Definition 2.8[1]:- A topological space (G, T) is said to be semi-α-compact iff every Semi-α-open cover of G has a finite sub cover.

Remark 2.9[1]:- Every Semi-α-compact space is compact.

Proposition 2.10[1]:- The union of two Semi-α-compact subsets of G is Semi-α-compact.

Definition 2.11[5]:- A topological group is a set G which carries group Stricture and a topology and satisfied the two axioms:

i. The map $(a, b) \to ab$ of $G \times G$ into G is continuous. (That is, the operation of G is continuous).
ii. The map $a \to a^{-1}$ (The inversion map) of G into G is continuous.

3. Semi-α-Syndetic Sets:

Definition 3.1:- Let A be a subset of a topological group G, then A is called left (right) Semi-α-Synditic if there exists a Semi-α-compact subset M of G such that $AM = G$ $(MA = G)$.

Remark 3.2:- In the following results we will prove the cace of left Semi-α-syndetic and the case of right Semi-α-syndetic will be similar.

Proposition 3.3:- If M is Semi-α-compact set in a topological group G, then M^{-1} is Semi-α-compact set.

Proof:- Let $f: G \to G$ be the inverse map, that is, $f(a) = a^{-1}$ for all a in G, let H be Semi-α-open set cover of M^{-1}, then $f(H)$ is Semi-α-open set cover of $f(M^{-1}) = (M^{-1})^{-1} = M$, but M is Semi-α-compact, which implies $f(H)$ has a finite Sub cover H^*, then $f(H^*)$ covers $f(M) = M^{-1}$. Hence, M^{-1} is Semi-α-compact set.

Proposition 3.4:- Let G be a topological group and let $A \subseteq G$, then A is left (right) Semi-α-Syndetic set in G iff there exists a Semi-α-compact subset M of G such that every left (right) translation of M intersects A.

Proof:- Suppose that A is a left Semi-α- Syndetic set, then there exists a Semi-α-compact subset M of G such that $AM = G$, let $g \in G$, then there exists $x \in A, m \in M$ such that $g = xm$ which implies $x = gm^{-1}$ and then $x \in gm^{-1}$ but m^{-1} is Semi-α-compact. Hence $gm^{-1} \cap A \neq \emptyset$, (i.e m^{-1} is the Semi-α-compact set we need).

Conversely, let $g \in G$, there exists Semi-α-compact subset M of G such that, $gM \cap A \neq \emptyset$, for each g in G, there exists $x \in A$, $m \in M$ such that $gm = x$, so $g = xm^{-1}$, which implies $G = AM^{-1}$, and since M^{-1} is Semi-α-compact then A is a left Semi-α-Syndetic.

Proposition 3.5:- Let A be a subset of a topological group G, then A is left (right) Semi-α-Syndetic in G iff A^{-1} is right (left) Semi-α-Syndetic.

Proof:- Let A be a left Semi-α-Syndetic, then there exists Semi-α-compact subset M of G such that $AM = G$. Since $G = G^{-1} = (AM)^{-1} = M^{-1}A^{-1}$ and Since M^{-1} is Semi-α-compact, then A^{-1} is right Semi-α-syndetic.

Proposition 3.6:- Let G be a topological group, let A, B be two subset of G Such that $A \subseteq B$. If A is left (right) Semi-α-Syndetic set, then so is B.

Proof:- Let A be a left Semi-α-syndetic set, then there exists a Semi-α-compact subset M of G such that $AM = G$, since $A \subseteq B$, then $BM = G$, which implies that B is left Semi-α-Syndetic.

Theorem 3.7:- Let G be a topological group, then

i. If A is a left Semi-α-Syndetic set in G, then $Cl(A)$ and Semi-α-$Cl(A)$ are left Semi-α-Syndetic.
ii. The union of any family of left (right) Semi-α-Syndetic sets is left (right) Semi-α-Syndetic.

Proof:- i and ii directly from proposition (3.6)

Theorem 3.8:- Let G be a topological group, let A, B be two left (right) Semi-α-Syndetic Subset og G, then $A \cap B$ is a left (right) Semi-α-Syndetic.

Proof:- Let A and B be two left Semi-α-Syndetic, then there exist Semi-α-compact sets M and N such that $AM = G$ and $BN = G$ and then $(A \cap B)(M \cup N) = A(M \cup N) \cap B(M \cup N) = G \cap G = G$ and since $(M \cup N)$ is Semi-α-compact, then $(A \cap B)$ is left Semi-α-Syndetic.

Theorem 3.9:- Let A be a subset of topological group G. If A is a Subgroup of G or if G is an abelian group, then A is left Semi-α-Syndetic in G iff A is a right Semi-α-Syndetic in G.

Proof:- Let A be a left Semi-α-Syndetic subgroup of G, then $G = AM$, that means $G^{-1} = M^{-1}A^{-1}$ and hence $G = M^{-1}A$, but M^{-1} is Semi-α-compact, which implies $A$is a right Semi-α-Syndetic in G.

Theorem 3.10:- Let A be a Semi-α-Syndetic subgroup of a topological group G, then the quotient space G/A is compact.

Proof:- Let A be a Semi-α-Syndetic subgroup of a topological group G, then there exists a Semi-α-compact set M such that $MA = G$. Let $f: G \to G/A$ be the quotient map. Clear that $f(M) \subseteq G/A$, Let $gA \in G/A$, then $gA \subseteq G$ which implies $gA \subseteq MA$, that is $gA \in f(M)$ and then, $G/A \subseteq f(M)$. Hence $G/M = f(M)$. Since f is continuous and M is Semi-α-compact, then $f(M) = G/A$ is compact set.

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