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On New Types of Weakly Nano Open Functions

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Abstract:

In this paper, we must utilize the ideas of Nα-open and Nsα-open sets to characterize some new types of weakly nano open functions such as; Nα-open functions, Nα*-open functions, Nα**-open functions, Nsα-open functions, Nsα*-open functions and Nsα**-open functions. Also, we must explain the relationships between these types of weakly nano open functions and the concepts of nano open functions. Furthermore, we must prove some theorems, properties and remarks.

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Keywords: Nα-open sets, Nsα-open sets, Nα-open functions, Nα*-open functions, Nα**-open functions, Nsα-open functions, Nsα*-open functions and Nsα**-open functions.

1. Introduction

M. Lellis Thivagar and **C. Richard** [1,3] introduce nano topological space on a subset $\mathcal{C}$ of a universe which is characterized with respect to lower and upper approximations of $\mathcal{C}$. He studied about the weak forms of nano open sets. **Qays Hatem Imran** [4,5] introduced the idea of Nsα-open sets in nano topological spaces and also introduced new types of weakly nano continuity. The aim of this paper is to introduce new types of weakly nano open functions such as; Nα-open functions, Nα*-open functions, Nα**-open functions, Nsα-open functions, Nsα*-open functions and Nsα**-open functions. Also, we must explain the relationships between these types of weakly nano open functions and the concepts of nano open functions. Furthermore, we must prove some theorems, properties and remarks.

2. Preliminaries

Throughout this paper, $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C})),(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ and $(\mathcal{W},\rho_{\mathcal{R}}(\mathcal{E}))$ (or frugally $\mathcal{U},\mathcal{V}$ and $\mathcal{W}$) always mean nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a set $\mathcal{M}$ in a nano topological space $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))$, $Ncl(\mathcal{M})$, $Nint(\mathcal{M})$ and $\mathcal{M}^c=\mathcal{U}-\mathcal{M}$ denote the nano closure of $\mathcal{M}$, the nano interior of $\mathcal{M}$ and the nano complement of $\mathcal{M}$ respectively.

Definition 2.1: A subset $\mathcal{M}$ of a nano topological space $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))$ is said to be:
1) A nano α-open set (briefly Nα-open set) [3] if $\mathcal{M}\subseteq Nint(Ncl(Nint(\mathcal{M})))$. The family of all Nα-open sets of $\mathcal{U}$ is denoted by $\alpha\tau_{\mathcal{R}}(\mathcal{C})$.
2) A nano semi-α-open set (briefly Nsα-open set) [4] if there exists a Nα-open set $\mathcal{S}$ in $\mathcal{U}$ such that $\mathcal{S}\subseteq\mathcal{M}\subseteq Ncl(\mathcal{S})$ or equivalently if $\mathcal{M}\subseteq Ncl(Nint(Ncl(Nint(\mathcal{M}))))$. The family of all Nsα-open sets of $\mathcal{U}$ is denoted by $s\alpha\tau_{\mathcal{R}}(\mathcal{C})$.

Remark 2.2:[4] In a nano topological space $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))$, the following statements hold and the reverse of each statement is not true:
1) Every N-open set is a Nα-open and Nsα-open.
2) Every Nα-open set is a Nsα-open.

Example 2.3: Let $\mathcal{U}=\{p_1,p_2,p_3,p_4\}$ with $\mathcal{U}/\mathcal{R}=\{\{p_1\},\{p_3\},\{p_2,p_4\}\}$ and $\mathcal{C}=\{p_1,p_2\}$. Then $\tau_{\mathcal{R}}(\mathcal{C})=\{\phi,\{p_1\},\{p_2,p_4\},\{p_1,p_2,p_4\},\mathcal{U}\}$ is a nano topological space. The family of all Nα-open sets of $\mathcal{U}$ is: $\alpha\tau_{\mathcal{R}}(\mathcal{C})=\{\phi,\{p_1\},\{p_2,p_4\},\{p_1,p_2,p_4\},\mathcal{U}\}$. The family of all Nsα-open sets of $\mathcal{U}$ is: $s\alpha\tau_{\mathcal{R}}(\mathcal{C})=\alpha\tau_{\mathcal{R}}(\mathcal{C})\cup\{\{p_1,p_3\},\{p_2,p_3,p_4\}\}$.

Theorem 2.4:[4] For any subset $\mathcal{M}$ of a nano topological space $(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))$, $\mathcal{M}\in\alpha\tau_{\mathcal{R}}(\mathcal{C})$ iff there exists a N-open set $\mathcal{P}$ such that $\mathcal{P}\subseteq\mathcal{M}\subseteq Nint(Ncl(\mathcal{P}))$.

Definition 2.5: Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a function, then ℓ is said to be:

1) Nano open (briefly N-open) [2] iff for each $\mathcal{M}$ N-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a N-open set in $\mathcal{V}$.
2) Nano α-open (briefly Nα-open) [6] iff for each $\mathcal{M}$ N-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$.

Theorem 2.6:[2] A function $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ is N-open iff $\ell(Nint(\mathcal{M}))\subseteq Nint(\ell(\mathcal{M}))$, for every $\mathcal{M}\subseteq\mathcal{U}$.

Definition 2.7:[2] Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a function, then ℓ is said to be nano continuous (briefly N-continuous) iff for each $\mathcal{M}$ N-open set in $\mathcal{V}$, then $\ell^{-1}(\mathcal{M})$ is a N-open set in $\mathcal{U}$.

Theorem 2.8:[2] A function $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ is N-continuous iff $\ell(Ncl(\mathcal{M}))\subseteq Ncl(\ell(\mathcal{M}))$, for every $\mathcal{M}\subseteq\mathcal{U}$.

3. Weakly Nano Open Functions

Definition 3.1: Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a function, then ℓ is said to be:
1) Nano α*-open (briefly Nα*-open) iff for each $\mathcal{M}$ Nα-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$.
2) Nano α**-open (briefly Nα**-open) iff for each $\mathcal{M}$ Nα-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a N-open set in $\mathcal{V}$.

Definition 3.2: Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a function, then ℓ is said to be:
1) Nano semi-α-open (briefly Nsα-open) iff for each $\mathcal{M}$ N-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a Nsα-open set in $\mathcal{V}$.
2) Nano semi-α*-open (briefly Nsα*-open) iff for each $\mathcal{M}$ Nsα-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a Nsα-open set in $\mathcal{V}$.
3) Nano semi-α**-open (briefly Nsα**-open) iff for each $\mathcal{M}$ Nsα-open set in $\mathcal{U}$, then $\ell(\mathcal{M})$ is a N-open set in $\mathcal{V}$.

Theorem 3.3:
1) Every N-open function is a Nα-open, so it is Nsα-open, but the reverse is not true in general.
2) Every Nα-open function is a Nsα-open, but the reverse is not true in general.

Proof:
1) Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a N-open function and $\mathcal{M}$ be a N-open set in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since any N-open set is Nα-open (Nsα-open), $\ell(\mathcal{M})$ is a Nα-open (Nsα-open) set in $\mathcal{V}$. Hence ℓ is a Nα-open (Nsα-open) function.
2) Let $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ be a Nα-open function and $\mathcal{M}$ be a N-open set in $\mathcal{U}$. Then $\ell(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since any Nα-open set is Nsα-open, $\ell(\mathcal{M})$ is a Nsα-open set in $\mathcal{V}$. Hence ℓ is a Nsα-open function.

Example 3.4: Let $\mathcal{U}=\{1,2,3,4\}$ with $\mathcal{U}/\mathcal{R}=\{\{2\},\{4\},\{1,3\}\}$ and $\mathcal{C}=\{1,2\}$. Then $\tau_{\mathcal{R}}(\mathcal{C})=\{\phi,\{3\},\{1,3\},\{1,2,3\},\mathcal{U}\}$ is a nano topological space. The family of all Nα-open (Nsα-open) sets of $\mathcal{U}$ is: $\alpha\tau_{\mathcal{R}}(\mathcal{C})=\mathrm{s}\alpha\tau_{\mathcal{R}}(\mathcal{C})=\tau_{\mathcal{R}}(\mathcal{C})\cup\{\{2,3\},\{3,4\},\{1,3,4\},\{2,3,4\}\}$. Define a function $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))$ as $\ell(1)=1,\ell(2)=4,\ell(3)=3$ and $\ell(4)=2$. Then ℓ is a Nα-open, so it is Nsα-open but not N-open function.

Example 3.5: Let $\mathcal{U}=\{p_1,p_2,p_3,p_4\}$ with $\mathcal{U}/\mathcal{R}=\{\{p_1\},\{p_3\},\{p_2,p_4\}\}$ and $\mathcal{C}=\{p_1,p_2\}$. Then $\tau_{\mathcal{R}}(\mathcal{C})=\{\phi,\{p_1\},\{p_2,p_4\},\{p_1,p_2,p_4\},\mathcal{U}\}$ is a nano topological space.
Let $\mathcal{V}=\{q_1,q_2,q_3,q_4\}$ with $\mathcal{V}/\mathcal{R}=\{\{q_2\},\{q_4\},\{q_1,q_3\}\}$ and $\mathcal{D}=\{q_1,q_2\}$. Then $\sigma_{\mathcal{R}}(\mathcal{D})=\{\phi,\{q_2\},\{q_1,q_3\},\{q_1,q_2,q_3\},\mathcal{V}\}$ is a nano topological space.
Define a function $\ell:(\mathcal{U},\tau_{\mathcal{R}}(\mathcal{C}))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(\mathcal{D}))$ as $\ell(p_1)=\ell(p_2)=q_2,\ell(p_3)=\ell(p_4)=q_4$. Then ℓ is a Nsα-open function but it is not Nα-open function.

Remark 3.6: The concepts of N-open function and Nα*-open function are independent, as the following examples show:

Example 3.7: In example (3.4), the function ℓ is a Nα*-open but it is not N-open.

Example 3.8: Let $\mathcal{U}=\{p_1,p_2,p_3,p_4\}$ with $\mathcal{U}/\mathcal{R}=\{\{p_1\},\{p_4\},\{p_2,p_3\}\}$ and $\mathcal{C}=\{p_1,p_4\}$. Then $\tau_{\mathcal{R}}(\mathcal{C})=\{\phi,\{p_1,p_4\},\mathcal{U}\}$ is a nano topological space.

Let $\mathcal{V} = \{q_1, q_2, q_3, q_4\}$ with $\mathcal{V}/\mathcal{R} = \{\{q_1\}, \{q_3\}, \{q_2, q_4\}\}$ and $\mathcal{D} = \{q_1, q_2\}$. Then $\sigma_{\mathcal{R}}(\mathcal{D}) = \{\phi, \{q_1\}, \{q_2, q_4\}, \{q_1, q_2, q_4\}, \mathcal{V}\}$ is a nano topological space.
Define a function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ as $\ell(p_2) = q_1, \ell(p_1) = q_2, \ell(p_3) = q_3$ and $\ell(p_4) = q_4$. Then ℓ is a N-open function but it is not Nα*-open.

Proposition 3.9:
1) If $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ is a N-open, N-continuous function, then ℓ is a Nα*-open function.
2) $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ is a Nα*-open function iff $\ell: (\mathcal{U}, \alpha\tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \alpha\sigma_{\mathcal{R}}(\mathcal{D}))$ is a N-open.

Proof:
1) Let $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ is a N-open, N-continuous function. To prove ℓ is a Nα*-open function. Let $\mathcal{M} \in \alpha\tau_{\mathcal{R}}(\mathcal{C})$, then there exists a N-open set $\mathcal{N}$ such that $\mathcal{N} \subseteq \mathcal{M} \subseteq Nint(Ncl(\mathcal{N}))$ (by theorem (2.4)). Hence $\ell(\mathcal{N}) \subseteq \ell(\mathcal{M}) \subseteq \ell(Nint(Ncl(\mathcal{N})))$ but $\ell(Nint(Ncl(\mathcal{N}))) \subseteq Nint(\ell(Ncl(\mathcal{N})))$ (since ℓ is a N-open function).
Then $\ell(\mathcal{N}) \subseteq \ell(\mathcal{M}) \subseteq \ell(Nint(Ncl(\mathcal{N})))) \subseteq Nint(\ell(Ncl(\mathcal{N})))$.
But $Nint(\ell(Ncl(\mathcal{N}))) \subseteq Nint(Ncl(\ell(\mathcal{N})))$ (since ℓ is a N-continuous function). Therefore we get $\ell(\mathcal{N}) \subseteq \ell(\mathcal{M}) \subseteq Nint(Ncl(\ell(\mathcal{N})))$. But $\ell(\mathcal{N})$ is a N-open set in $\mathcal{V}$ (since ℓ is a N-open function). Hence $\ell(\mathcal{M}) \in \alpha\sigma_{\mathcal{R}}(\mathcal{D})$ (by theorem (2.4)). Thus is a Nα*-open function.
2) The proof of a part (2) is easily.

Remark 3.10: Every Nα*-open function is a Nα-open and Nsα-open but the reverse is not true in general as the following example show:

Example 3.11: Let $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$ with $\mathcal{U}/\mathcal{R} = \{\{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}\}$ and $\mathcal{C} = \{p_1, p_4\}$.
Then $\tau_{\mathcal{R}}(\mathcal{C}) = \{\phi, \{p_1, p_4\}, \mathcal{U}\}$ is a nano topological space.
Let $\mathcal{V} = \{q_1, q_2, q_3, q_4\}$ with $\mathcal{V}/\mathcal{R} = \{\{q_2\}, \{q_3\}, \{q_1, q_4\}\}$ and $\mathcal{D} = \{q_1, q_3\}$. Then $\sigma_{\mathcal{R}}(\mathcal{D}) = \{\phi, \{q_3\}, \{q_1, q_4\}, \{q_1, q_3, q_4\}, \mathcal{V}\}$ is a nano topological space.
Define a function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ as $\ell(p_1) = q_1, \ell(p_2) = q_2, \ell(p_3) = q_3$ and $\ell(p_4) = q_4$. Then ℓ is a Nα-open function and Nsα-open function but not Nα*-open.

Remark 3.12: The concepts of N-open function and Nsα*-open function are independent, for examples:

Example 3.13: In example (3.5), the function ℓ is a Nsα*-open but it is not N-open.

Example 3.14: Let $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$ with $\mathcal{U}/\mathcal{R} = \{\{p_1\}, \{p_3\}, \{p_2, p_4\}\}$ and $\mathcal{C} = \{p_2, p_4\}$. Then $\tau_{\mathcal{R}}(\mathcal{C}) = \{\phi, \{p_2, p_4\}, \mathcal{U}\}$ is a nano topological space.
Let $\mathcal{V} = \{q_1, q_2, q_3, q_4\}$ with $\mathcal{V}/\mathcal{R} = \{\{q_1\}, \{q_3\}, \{q_2, q_4\}\}$ and $\mathcal{D} = \{q_1, q_2\}$. Then $\sigma_{\mathcal{R}}(\mathcal{D}) = \{\phi, \{q_1\}, \{q_2, q_4\}, \{q_1, q_2, q_4\}, \mathcal{V}\}$ is a nano topological space.
Define a function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ as $\ell(p_1) = q_2, \ell(p_2) = q_1, \ell(p_3) = q_4$ and $\ell(p_4) = q_1$. Then ℓ is a N-open function but it is not Nsα*-open.

Proposition 3.15: A function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ is a Nsα*-open iff $\ell: (\mathcal{U}, s\alpha\tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, s\alpha\sigma_{\mathcal{R}}(\mathcal{D}))$ is a N-open function.

Proof: Obvious.

Remark 3.16: The concepts of Nα*-open function and Nsα*-open function are independent as the following examples show:

Example 3.17: In example (3.11), the function ℓ is a Nsα*-open but it is not Nα*-open.

Example 3.18: Let $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$ with $\mathcal{U}/\mathcal{R} = \{\{p_1\}, \{p_3\}, \{p_2, p_4\}\}$ and $\mathcal{C} = \{p_1, p_2\}$. Then $\tau_{\mathcal{R}}(\mathcal{C}) = \{\phi, \{p_1\}, \{p_2, p_4\}, \{p_1, p_2, p_4\}, \mathcal{U}\}$ is a nano topological space.
Let $\mathcal{V} = \{q_1, q_2, q_3, q_4\}$ with $\mathcal{V}/\mathcal{R} = \{\{q_1\}, \{q_3\}, \{q_2, q_4\}\}$ and $\mathcal{D} = \{q_1, q_2\}$. Then $\sigma_{\mathcal{R}}(\mathcal{D}) = \{\phi, \{q_1\}, \{q_2, q_4\}, \{q_1, q_2, q_4\}, \mathcal{V}\}$ is a nano topological space.

Define a function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ as $\ell(p_1) = q_1, \ell(p_2) = q_2, \ell(p_3) = q_2$ and $\ell(p_4) = q_4$. Then ℓ is a Nα*-open function but it is not Nsα*-open.

Theorem 3.19: If a function $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ is Nα*-open and N-continuous, then it is Nsα*-open.

Proof: Let $\ell: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ be a Nα*-open and N-continuous function. Let $\mathcal{M}$ be a Nsα-open set in $\mathcal{U}$. Then there exists a Nα-open set say $\mathcal{S}$ such that $\mathcal{S} \subseteq \mathcal{M} \subseteq Ncl(\mathcal{S})$. Therefore $\ell(\mathcal{S}) \subseteq \ell(\mathcal{M}) \subseteq \ell(Ncl(\mathcal{S})) \subseteq Ncl(\ell(\mathcal{S}))$ (since ℓ is a N-continuous), but $\ell(\mathcal{S}) \in \alpha\tau_{\mathcal{R}}(\mathcal{C})$ (since ℓ is a Nα*-open function). Hence $\ell(\mathcal{S}) \subseteq \ell(\mathcal{M}) \subseteq Ncl(\ell(\mathcal{S}))$. Thus, $\ell(\mathcal{M}) \in s\alpha\tau_{\mathcal{R}}(\mathcal{C})$. Therefore, ℓ is a Nsα*-open function.

Remark 3.20: The following diagram explains the relationship between weakly nano open functions.

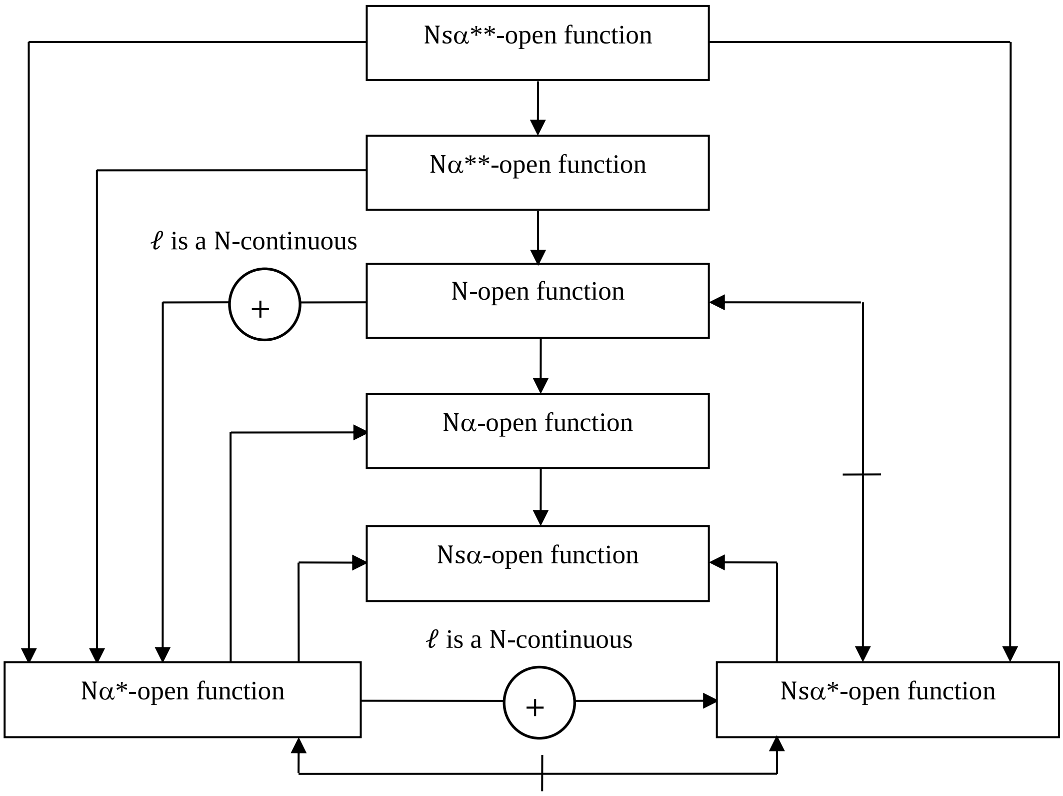

Diagram (3.1)

Theorem 3.21: Let $\ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D}))$ and $\ell_2: (\mathcal{V}, \sigma_{\mathcal{R}}(\mathcal{D})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ be two functions, then:

1) If ℓ_1 is N-open function and ℓ_2 is Nα-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα-open function.
2) If ℓ_1 is Nα-open function and ℓ_2 is Nα*-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα-open function.
3) If ℓ_1 and ℓ_2 are Nα*-open functions, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
4) If ℓ_1 and ℓ_2 are Nsα*-open functions, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nsα*-open function.
5) If ℓ_1 and ℓ_2 are Nα**-open functions, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα**-open function.
6) If ℓ_1 and ℓ_2 are Nsα**-open functions, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nsα**-open function.
7) If ℓ_1 is Nα**-open function and ℓ_2 is Nα*-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
8) If ℓ_1 is Nα-open function and ℓ_2 is Nα**-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a N-open function.

9) If ℓ_1 is Nα**-open function and ℓ_2 is Nα-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
10) If ℓ_1 is Nα**-open function and ℓ_2 is N-open function, then $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα**-open function.

Proof:

1) Let $\mathcal{M}$ be a N-open set in $\mathcal{U}$. Since ℓ_1 is a N-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since ℓ_2 is a Nα-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα-open function.
2) Let $\mathcal{M}$ be a N-open set in $\mathcal{U}$. Since ℓ_1 is a Nα-open function, $\ell_1(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since ℓ_2 is a Nα*-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα-open function.
3) Let $\mathcal{M}$ be a Nα-open set in $\mathcal{U}$. Since ℓ_1 is a Nα*-open function, $\ell_1(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since ℓ_2 is a Nα*-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
4) Let $\mathcal{M}$ be a Nsα-open set in $\mathcal{U}$. Since ℓ_1 is a Nsα*-open function, $\ell_1(\mathcal{M})$ is a Nsα-open set in $\mathcal{V}$. Since ℓ_2 is a Nsα*-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nsα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nsα*-open function.
5) Let $\mathcal{M}$ be a Nα-open set in $\mathcal{U}$. Since ℓ_1 is a Nα**-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since any N-open set is Nα-open, $\ell_1(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since ℓ_2 is a Nα**-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a N-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα**-open function.
6) Let $\mathcal{M}$ be a Nsα-open set in $\mathcal{U}$. Since ℓ_1 is a Nsα**-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since any N-open set is Nsα-open, $\ell_1(\mathcal{M})$ is a Nsα-open set in $\mathcal{V}$. Since ℓ_2 is a Nsα**-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a N-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nsα**-open function.
7) Let $\mathcal{M}$ be a Nα-open set in $\mathcal{U}$. Since ℓ_1 is a Nα**-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since any N-open set is Nα-open, $\ell_1(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since ℓ_2 is a Nα*-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
8) Let $\mathcal{M}$ be a N-open set in $\mathcal{U}$. Since ℓ_1 is a Nα-open function, $\ell_1(\mathcal{M})$ is a Nα-open set in $\mathcal{V}$. Since ℓ_2 is a Nα**-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a N-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a N-open function.
9) Let $\mathcal{M}$ be a Nα-open set in $\mathcal{U}$. Since ℓ_1 is a Nα**-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since ℓ_2 is a Nα-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a Nα-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα*-open function.
10) Let $\mathcal{M}$ be a Nα-open set in $\mathcal{U}$. Since ℓ_1 is a Nα**-open function, $\ell_1(\mathcal{M})$ is a N-open set in $\mathcal{V}$. Since ℓ_2 is a N-open function, $\ell_2 \circ \ell_1(\mathcal{M}) = \ell_2(\ell_1(\mathcal{M}))$ is a N-open set in $\mathcal{W}$. Thus, $\ell_2 \circ \ell_1: (\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{C})) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(\mathcal{E}))$ is a Nα**-open function.

4. Conclusion

We must utilize the ideas of Nα-open and Nsα-open sets to characterize some new types of weakly nano open functions such as; Nα-open, Nα*-open, Nα**-open, Nsα-open, Nsα*-open and Nsα**-open functions. The Nα-open and Nsα-open sets can be used to derive some nano compactness, and nano connectedness.

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