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On M-Compact Ring

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Abstract: *In the present paper, we have introduced some new definitions On M-cover ring, M-compact ring, weakly M-compact ring, weakly M-compact c. ring, M-compact c. ring, M-compact locally ring and M-compact strong locally ring, we obtain some examples and results related to M-cover ring, M-compact ring, weakly M-compact ring, weakly M-compact c. ring, M-compact c. ring, M-compact locally ring and M-compact strong locally ring.*

Keywords: *rings, M-cover ring, M-compact ring, ring homomorphism, ring isomorphism.*

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1- Introduction:

The notion of groupoid was introduced by H. Brandt [Math. Ann., 96(1926), 360- 366; MR 1512323]. A groupoid $(G,*)$ is a set on which is defined a non associative binary operation which is closed on G, the groupoid $(G,*)$ is a semigroup if the binary operation $*$ is associative [4].

Milne [2], introduced details on a ring. We call $(R,*,\circ)$ to be a ring if the following conditions are satisfied.

1) $(R, *)$ is a group.
2) $(R,\circ)$ is a semigroup.
3) (i) a × (b + c) = a × b + a × c
   (ii) (a + b) × c = a × c + b × c

for all a ,b ,c ∈ R, a nonempty subset P of R is said to be a subring of R if P is a ring under the operations of R.

We investigated on *M-cover ring, M-compact ring, weakly M-compact ring, weakly M-compact c. ring, M-compact c. ring, M-compact locally ring and M-compact strong locally ring*, we obtain some good examples and results related to these concepts above.

2- Definitions:

<u>Definition (1)</u>: Let $(R,*,\circ)$ be a ring , and I be an indexed set.

Let $M = \{ R_i \, ; \, R_i \subset R \, , (R_i,*,\circ)$ is a proper subring of $(R,*,\circ), \forall i \in I \}$ be a family of proper subrings of $(R,*,\circ)$, (I is a finite or an infinite set), we say that M is a *M-cover ring* of $(R,*,\circ)$ if $R = \bigcup_{i\in I} R_i$.

Definition (2): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is a *weakly M-compact ring* if there is a finite sub-*M-cover ring* of $(R,*,\circ)$.

Definition (3): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is *M-compact ring* if for every *M-cover ring* of $(R,*,\circ)$ there exists a finite *sub-M-cover ring* of $(R,*,\circ)$.

Definition (4): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is *weakly M-compact c. ring* if there is a countable *M-cover ring* of $(R,*,\circ)$.

Definition (5): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is *M-compact c. ring* if for every *M-cover ring* of $(R,*,\circ)$ there exists a countable *sub-M-cover ring* of $(R,*,\circ)$.

Definition (6): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is a *M-compact locally ring* if for every element *x* of R there is a subring (proper) of R include *x* .

Definition (7): Let $(R,*,\circ)$ be a ring, we say that $(R,*,\circ)$ is a *M-compact strong locally ring* if for every element *x* of R (except the unite element) there is a unique subring (proper) of R include *x*.

Definition (8): Let $(R,*,\circ)$ be a ring, the subring $(H,*,\circ)$ of the ring $(R,*,\circ)$ is called a *M-compact subring* (*weakly M-compact subring* , *weakly M-compact c. subring*, *M-compact c. subring*, *M-compact locally subring*, *M-compact strong locally subring*), if $(H,*,\circ)$ is a *M-compact ring* (*weakly M-compact ring*, *weakly M-compact c. ring*, *M-compact c. ring*, *M-compact locally ring*, *M-compact strong locally ring*), respectively.

Definition (9) [3]: Let $(R,*,\circ)$ and $(\bar{R} \, ,\bar{*},\bar{\circ})$ are two rings, we say that

1- $f: (R,*,\circ) \rightarrow (\bar{R} \, ,\bar{*},\bar{\circ})$ is a *homomorphism* if $f(x * y)= f(x) \mathbin{\bar{*}} f(y)$ and $f(x \circ y) = f(x) \mathbin{\bar{\circ}} f(y), \forall x, y \in G$.

2- $f: (R,*,\circ) \rightarrow (\bar{R} \, ,\bar{*},\bar{\circ})$ is an *isomorphism* if f is a *bijective homomorphism*.

Definition (10) [3]: Let $(R,*,\circ)$ and $(\bar{R} \, ,\bar{*},\bar{\circ})$ are two rings ,we say that $(R,*,\circ)$ is an isomorphic to $(\bar{R} \, ,\bar{*},\bar{\circ})$, denoted that $(R,*,\circ) \cong (\bar{R} \, ,\bar{*},\bar{\circ})$, if there is an *isomorphism* $f : (R,*,\circ) \rightarrow (\bar{R} \, ,\bar{*},\bar{\circ})$.

3- Examples:

Example (1): The ring $(\mathbb{Z}_2, +_2, \cdot_2)$, has no *M-cover ring*. The ring $(\mathbb{Z}_2, +_2, \cdot_2)$ is not *M-compact ring*, while the ring $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus, \otimes)$ has *M-cover ring* {(0,0),(1,1)},{(0,0),(1,0)},{(0,0),(0,1)}. Such that $(a,b)\oplus(c,d) = (a +_2 c, b +_2 d), (a,b)\otimes(c,d) = (a \cdot_2 c, b \cdot_2 d)$.

Example (2): Let $R = \{0 ,1 ,2 , . . . \}$, defined a binary operator $\prec$ as follows;

$a \prec b = \begin{cases} \max\{a,b\} & a \neq b \\ 0 & a = b \end{cases}, \forall \, a, b \in R$. It is easy to show that $(R, \prec)$ is a group. The ring $(R, \prec, *)$ is a *M-compact C. ring* (* define by $a * b = 0 \, for \, all \, a, b \in R$).

The ring $(R, \prec, *)$ is not *M-compact ring*, since the family of subrings $\{ (\{0, a, b\}, \prec, *); \, a, b \in \mathbb{N} \}$ is a *M-cover ring* of $(R, \prec, *)$ has no finite *sub-M-cover ring* of $(R, \prec, *)$.

Example (3): Let $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus, \otimes)$ be a ring. Then $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus, \otimes)$ is a *M-compact strong locally ring*(also *M-compact locally ring)*, since there are only three subrings of $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus, \otimes)$ which are $(M_1, \oplus, \otimes)$, $(M_2, \oplus, \otimes)$ and $(M_3, \oplus, \otimes)$, (except the trivial subrings) where

$M_1 = \{(0,0), (0,1)\}$, $M_2 = \{(0,0), (1,0)\}$, $M_3 = \{(0,0), (1,1)\}$.

Example (4): Let $R = \{-n, \ldots, -2, -1, 0\}$, defined a binary operator $\succ$ as follows;

$a \succ b = \begin{cases} \min\{a,b\} & a \neq b \\ 0 & a = b \end{cases}$, $\forall\, a, b \in R$. It is easy to show that $(R, \succ)$ is a group.

The ring $(R, \succ, *)$ ($*$ define by $a * b = 0 \; for \; all \; a, b \in R$) is *M-compact ring*.

Example (5): Let $X = \{0\} \cup \mathbb{R}^+$, defined a binary operator $\prec$ as follows;
$a \prec b = \begin{cases} \max\{a,b\} & a \neq b \\ 0 & a = b \end{cases}$, $\forall\, a, b \in X$. It is easy to show that $(X, \prec)$ is a group.
The ring $(X, \prec, *)$ ($*$ define by $a * b = 0 \; for \; all \; a, b \in X$) is *M-compact locally ring*.
The ring $(X, \prec, *)$ is not *M-compact c. ring*, since the family of subrings $\{ (\{0, a, b\}, \prec, *);\ a, b \in \mathbb{R}^+ \}$ is a *M-cover ring* of $(X, \prec, *)$ has no countable *sub-M-cover ring* of $(X, \prec, *)$.

4- Main Results:

The prove of all the following lemmas are direct from definitions.

Lemma (1): If $(R, *, \circ)$ is a *M-compact ring*, then $(R, *, \circ)$ is a *M-compact c. ring*.

Lemma (2): If $(R, *, \circ)$ is a *M-compact ring*, then $(R, *, \circ)$ is a *weakly M-compact ring*.

Lemma (3): If $(R, *, \circ)$ is a *M-compact c. ring*, then $(R, *, \circ)$ is a *weakly M-compact c. ring*.

Lemma (4): If $(R, *, \circ)$ is a *Weakly M-compact ring*, then $(R, *, \circ)$ is a *weakly M-compact c. ring*.

Lemma (5): If $(R, *, \circ)$ is a *M-compact strong locally ring*, then $(R, *, \circ)$ is a *M-compact locally ring*.

The following theorems are direct from definitions,

Theorem (1): Let $(\mathrm{R}, *, \circ)$ be ring, if R weakly *M-compact* ring, then it is *M-compact* locally ring.

Proof: Let $x \in R$ and R weakly *M-compact ring*, then there is cover ring of R $\Longrightarrow R = \bigcup_{i \in I} R_i$ $(R_i \; is \; proper \; sub \; ring \; of \; R \; \vee \; i \in I)$
$x \in \bigcup_{i \in I} R_i \Longrightarrow x \in R_i \; for \; some \; i$. R is *M-compact locally ring*.

Corollary (1): Let $(\mathrm{R}, *, \circ)$ be a ring. Then
1) *M-compact ring* $\Longrightarrow$ *M-compact locally ring*.
2) *M-compact c. ring* $\Longrightarrow$ *M-compact locally ring*.
3) *weakly M-compact c. ring* $\Longrightarrow$ *M-compact locally ring*.

Theorem (2): Any finite non cyclic ring of order a nonprime number, is a *M-compact locally ring*.

Proof: Let $(R,*,\circ)$ is a finite ring, for every element x of G the subring $(\langle x\rangle,*,\circ)$ of $(R,*,\circ)$ include x, it is clear that $R \neq \langle x\rangle$, since $(R,*,\circ)$ is not cyclic ring, and therefore $(R,*,\circ)$ is a *M-compact locally ring*.

Theorem (3): If $(R,*,\circ)$is a finite ring. then the following are equivalent;
1) $(R,*,\circ)$ is a *M-compact ring*.
2) $(R,*,\circ)$ is a *M-compact c. ring*.
3) $(R,*,\circ)$ is a *weakly M-compact ring*.
4) $(R,*,\circ)$ is a *weakly M-compact c. ring*.

Theorem (4): Any finite ring has a prime order is not *M-compact locally ring*.

Proof: Let $(R,*,\circ)$ is a ring with $|R| = p$, p prime number, by "Lagrange theorem" the order of every subgroup of R divides p, but p is prime, so there is no proper subgroup of R except the unit element and hence is no proper subring of R. Then $(R,*,\circ)$ is not *M-compact locally ring*.

Corollary (2):

1) Any finite ring has prime order is not *M-compact ring*.
2) Any finite ring has prime order is not *M-compact c. ring*.
3) Any finite ring has prime order is not *weakly M-compact ring*.
4) Any finite ring has prime order is not *weakly M-compact c. ring*.

Corollary (3): Any finite ring has prime order $(R,*,\circ)$ is not *M-compact strong locally ring*.

Theorem (5): If $(R,*,\circ)$ ring, then $(R,*,\circ)$ is a *M-compact ring* is not simple ring.

Proof: If $(R,*,\circ)$ is a *M-compact ring*, then there is *M-cover ring* and hence there is proper subring of $(R,*,\circ)$ $\Longrightarrow$ $(R,*,\circ)$ is not simple ring.

Theorem (6): Any simple ring $(R,*,\circ)$ is not *weakly M-compact ring*.

Proof: Clear, any simple ring has no proper sub ring and has no *M-cover*.
i.e $(R,*,\circ)$ is not *weakly M-compact ring* .

Corollary (4): Any $(R,*,\circ)$ simple ring is not *M-compact ring*.

Theorem (7): Any $(R,*,\circ)$ simple ring is not *M-compact c. ring*.

Proof: Clear, any simple ring has no proper sub ring and has no *M-cover*.
i.e $(R,*,\circ)$ is not *M-compact ring*.

Theorem (8): Any cyclic ring is not *M-compact locally ring*.

Proof: Assume $(R,*,\circ)$ is a cyclic ring which is a *M-compact locally ring* so for every element x of R there is a subring of R include x, but R is a cyclic so there is an element say g such that $\langle g\rangle = R$ (R generated by g) and hence any subring contains g must be equal to R, that is there is no proper subring contains g.

Corollary (5):

1) Any cyclic ring is not *M-compact ring*.
2) Any cyclic ring is not *M-compact c. ring*.

3) Any cyclic ring is not *weakly M-compact ring*.
4) Any cyclic ring is not *weakly M-compact c. ring*.
5) Any cyclic ring is not *M-compact strong locally ring*.

Theorem (9): If $(G,*,\circ) \cong (\bar{G},\bar{*},\bar{\circ})$. Then

$(G,*,\circ)$ is a *M-compact ring* $\Leftrightarrow$ $(\bar{G},\bar{*},\bar{\circ})$ is a *M-compact ring*.

Proof: ($\Longrightarrow$) Let $(\bar{G_\iota},\bar{*}_\iota,\bar{\circ}_\iota)$ be any *M-cover ring* of $(\bar{G},\bar{*},\bar{\circ})$ $\Longrightarrow$ $\bar{G} = \cup_{i\in I} \bar{G_\iota}$, but f is an isomorphism $\Longrightarrow f(G) = \bar{G} = \cup_{i\in I} \bar{G_\iota}$

$\Longrightarrow G = f^{-1}(\cup_{i\in I} \bar{G_\iota}) = \cup_{i\in I} f^{-1}(\bar{G_\iota})$, and $f^{-1}(\bar{G_\iota})$ is a ring $\forall i \in I$,

but $(G,*,\circ)$ is a *M-compact ring* so there is a finite set J such that

$G = \cup_{j\in J} f^{-1}(\bar{G_j}) = f^{-1}(\cup_{j\in J} \bar{G_j}) \Longrightarrow \bar{G} = f(G) = f\left(f^{-1}(\cup_{j\in J} \bar{G_j})\right) = \cup_{j\in J} \bar{G_J}$ and $(\bar{G_J},\bar{*}_J)$ is a ring $\forall j \in J$

$\Longrightarrow (\bar{G},\bar{*},\bar{\circ})$ is a *M-compact ring* .

$\Longleftarrow$ Let $(G_i,*_i,\circ_i)$ be any *M-cover ring* of $(G,*,\circ)$ $\Longrightarrow G = \cup_{i\in I} G_i$, but f is an isomorphism $\Longrightarrow \bar{G} = f(G) = f(\cup_{i\in I} G_i) = \cup_{i\in I} f(G_i)$,and is a ring $\forall i \in I$, but $(\bar{G},\bar{*})$ is a *M-compact ring* so there is a finite set J such that $\bar{G} = \cup_{j\in J} f(G_j) = f(\cup_{j\in J} G_j)$
$\Longrightarrow G = f^{-1}(\bar{G}) = f^{-1}\left(f(\cup_{j\in J} G_j)\right) = \cup_{j\in J} G_j$

and $(G_j,*_j,\circ_j)$ is a ring $\forall j \in J$ $\Longrightarrow$ $(G,*,\circ)$ is a *M-compact ring*.

Theorem (10): If $f : (G,*,\circ) \longrightarrow (\bar{G},\bar{*},\bar{\circ})$ is an *isomorphism* and $(H,*,\circ)$ is a *M-compact subring* of $(G,*,\circ)$, then $f(H)$ is a *M-compact subring* of $(\bar{G},\bar{*},\bar{\circ})$.

Proof: Let $\{W_i : i \in I\}$ is *M-cover* of $f(H)$ $\Longrightarrow$ $\cup_{i\in I} W_i = f(H)$Since f is *isomorphism*, $f^{-1}(f(H)) = f^{-1}(\cup_{i\in I} W_i) = f(H)$

$H = \cup_{i\in I} f^{-1}(W_i)$ [$f^{-1}(W_i)$ is sub rings of $(G,*,\circ)$ for all $i \in I$]

$\Longrightarrow$ $\exists\, finite\ set\ J \subseteq I \ni H = \cup_{i\in J} f^{-1}(W_i)$ [since $(H,*,\circ)$ is a *M-compact subring* of $(G,*,\circ)$]

$\Longrightarrow f(H) = f(\cup_{i\in J} f^{-1}(W_i)) = \cup_{i\in J} f(f^{-1}(W_i)) = \cup_{i\in J} W_i$

$\Longrightarrow$ $f(H)$ is a *M-compact subring* of $(\bar{G},\bar{*},\bar{\circ})$.

Theorem (11): If $f : (G,*,\circ) \longrightarrow (\bar{G},\bar{*},\bar{\circ})$ is an *isomorphism* and $(S,\bar{*},\bar{\circ})$ is a *M-compact subring* of $(\bar{G},\bar{*},\bar{\circ})$. then $f^{-1}(S)$ is a *M-compact subring* of $(G,*,\circ)$.
Proof: Let $\{W_i : i \in I\}$ is *M-cover* of $f^{-1}(S) \Longrightarrow \cup_{i\in I} W_i = f^{-1}(S)$.

Since f is *isomorphism*, $f(f^{-1}(S)) = f(\bigcup_{i\in I} W_i) = S \Longrightarrow S = \bigcup_{i\in I} f(W_i)$ [$f(W_i)$ is sub rings of $(\bar{G},\bar{*},\bar{\circ})$ for all $i \in I$]

$\Longrightarrow \exists\, finite\ set\ J \subseteq I \ni S = \bigcup_{i\in J} f(W_i)$ [since $(S,*,\circ)$ is a *M-compact subring* of $(\bar{G},\bar{*},\bar{\circ})$]

$\Longrightarrow f^{-1}(S) = f^{-1}\left(\bigcup_{i\in J} f(W_i)\right) = \bigcup_{i\in J} f^{-1}\left(f(W_i)\right) = \bigcup_{i\in J} W_i$

$\Longrightarrow$ S is a *M-compact subring* of $(G,*,\circ)$.

Theorem (12): If $(A,\circledast,\circ)$ is a ring and $(G,*,\bar{\circ})$ is a *M-compact ring*, then $(A\times G,\oplus,\otimes)$ is a *M-compact ring*. Where

$(a_1,g_1)\oplus(a_2,g_2) = (a_1 \circledast a_2, g_1 * g_2)$, $(a_1,g_1)\otimes(a_2,g_2) = (a_1{}^{\circ}\, a_2, g_1{}^{\bar{\circ}}g_2)$,

$\forall\, (a_1,g_1),(a_2,g_2) \in A\times G$.

Proof: Let $\{(A\times G_i,\oplus,\otimes);\ G_i \subset G, (A\times G_i,\oplus,\otimes)$ is a subring of $(A\times G,\otimes), \forall i \in I\}$ be any *M-cover ring* of $(A\times G,\oplus,\otimes)$, it is clear that $(G_i,*,\circ)$ is a subrings of $(G,*,\circ)$, such that $A\times G = \bigcup_{i\in I} A\times G_i = A\times(\bigcup_{i\in I} G_i)$ $\Longrightarrow G = \bigcup_{i\in I} G_i$, but $(G,*,\circ)$ is a *M-compact ring*, so there is a finite set J such that $G = \bigcup_{j\in J} G_j$, and hence

$A\times G = A\times\left(\bigcup_{j\in J} G_j\right) = \bigcup_{j\in J} A\times G_j \Longrightarrow (A\times G,\oplus,\otimes)$ is a *M-compact ring*.

Theorem (13): If $(G,*,\circ)$ and $(\bar{G},\bar{*},\bar{\circ})$ are *M-compact strong locally rings*, then

$(G\times\bar{G},\oplus,\otimes)$ is a *M-compact strong locally ring*.

Proof: Let $(x,y)\in G\times\bar{G} \Longrightarrow x\in G$ and $y\in\bar{G}$, but $(G,*,\circ)$ and $(\bar{G},\bar{*},\bar{\circ})$ are *M-compact strong locally rings* $\Longrightarrow \exists!\, G_x$ and $\exists!\, \bar{G}_y$ subrings of G and $\bar{G}$, respectively, such that $x\in G_x$ and $y\in\bar{G}_y$ $\Longrightarrow (x,y)\in G_x\times\bar{G}_y$ and $G_x\times\bar{G}_y$ is a unique subrings of $G\times\bar{G} \Longrightarrow (G\times\bar{G},\oplus,\otimes)$ is a *M-compact strong locally ring.*

Corollary (6): If $(G,*,\circ)$ and $(\bar{G},\bar{*},\bar{\circ})$ are *M-compact locally rings*, then $(G\times\bar{G},\oplus,\otimes)$ is a *M-compact locally ring*.

Theorem (14): If $(G,*,\circ)$ and $(\bar{G},\bar{*},\bar{\circ})$ are *M-compact rings*, then $(G\times\bar{G},\oplus,\otimes)$ is a *M-compact ring* .

Proof: Let $(G,*,\circ)$ and $(\bar{G},\bar{*},\bar{\circ})$ are *M-compact rings* $\Longrightarrow$ there exists a *M-cover ring* of $(G,*,\circ)$ say $\{G_a\}_{a\in A}$ and a *M-cover ring* of $(\bar{G},\bar{*},\bar{\circ})$ say $\{\bar{G}_b\}_{b\in B} \Longrightarrow G\times\bar{G} = (\bigcup_{a\in A} G_a)\times(\bigcup_{b\in B}\bar{G}_b) = \bigcup_{a\in A,b\in B}(G_a\times\bar{G}_b) \Longrightarrow \{G_a\times\bar{G}_b\}_{a\in A,b\in B}$ is a *M-cover ring* of

$(G\times\bar{G},\oplus,\otimes)$.

Now, Let $\{\mathcal{W}_i\}_{i\in I}$ be any *M-cover ring* of $(G\times\bar{G},\oplus,\otimes)$

$\Longrightarrow G\times\bar{G} = \bigcup_{i\in I}\mathcal{W}_i$, such that $\mathcal{W}_i = \mathcal{U}_i\times\mathcal{V}_i$, where $\{\mathcal{U}_i\}_{i\in I}$ are subrings of $(G,*,\circ)$ and $\{\mathcal{V}_i\}_{i\in I}$are subrings of $(\bar{G},\bar{*},\bar{\circ})$. But $(G,*,\circ)$ is a *M-compact ring*, so there is a *M-cover ring* of

$(G,*)$ contains $\{\mathcal{U}_i\}_{i\in I}$ which have a finite *sub-M-cover ring* (*i.e.* there is a finite set J) such that $G = \bigcup_{j\in J} \mathcal{U}_j$, let $\mathcal{U}_{j_1} \in \{\mathcal{U}_j\}_{j\in J} \Rightarrow \{\mathcal{U}_{j_1} \times \mathcal{V}_i\}_{i\in I}$ is a *M-cover ring* of the *M-compact ring* $(\mathcal{U}_{j_1} \times \bar{G}, \oplus, \otimes)$ (from **Theorem 10** since $(\mathcal{U}_{j_1}, *, \circ)$ is a ring and $(\bar{G}, \bar{*}, \bar{\circ})$ is a *M-compact ring*), so there is a finite set S such that

$$\mathcal{U}_{j_1} \times \bar{G} = \bigcup_{s\in S}(\mathcal{U}_{j_1} \times \mathcal{V}_s) = \mathcal{U}_{j_1} \times (\bigcup_{s\in S} \mathcal{V}_s)$$

$$\Rightarrow \bigcup_{j\in J}(\mathcal{U}_j \times (\bigcup_{s\in S} \mathcal{V}_s)) = (\bigcup_{j\in J} \mathcal{U}_j) \times (\bigcup_{s\in S} \mathcal{V}_s) = G \times \bar{G}$$

$\Rightarrow \quad G \times \bar{G} = (\bigcup_{j\in J} \mathcal{U}_j) \times (\bigcup_{s\in S} \mathcal{V}_s) = \bigcup_{j\in J, s\in S} (\mathcal{U}_j \times \mathcal{V}_s)$, where $\mathcal{U}_j \times \mathcal{V}_s$ are subrings of $G \times \bar{G}$. And therefore $G \times \bar{G}$ is a *M-compact ring*.

Corollary (7): If $(G,*,\circ)$ is a *M-compact ring* (*M-compact strong locally ring*, *M-compact locally ring*, *weakly M-compact ring*, *weakly M-compact c. ring*), then $(G^2, \oplus, \otimes)$ is a *M-compact ring* (*M-compact strong locally ring, M-compact locally ring, weakly M-compact ring, weakly M-compact c. ring*), respectively.

Theorem (15): If $(G,*,\circ)$ is a *M-compact ring* (*M-compact strong locally ring*, *M-compact locally ring*, *weakly M-compact ring*, *weakly M-compact c. ring*), then $(G^n, \oplus, \otimes)$ is a *M-compact ring* (*M-compact strong locally ring*, *M-compact locally ring*, *weakly M-compact ring*, *weakly M-compact c. ring*), respectively, for each $n \in \mathbb{N}$.

Theorem (16): The product of any finite collection of *M-compact rings* (*M-compact strong locally rings*, *M-compact locally rings*, *weakly M-compact rings*, *weakly M-compact c. rings*), is a *M-compact ring* (*M-compact strong locally ring*, *M-compact locally ring* , *weakly M-compact ring*, *weakly M-compact c. rings*).

References:

[1] T. W. Körner, Topological rings, Part III, Spring 2008.
[2] J. S. Milne, Ring Theory, September 24, 2010, Version 3.10.
[3] P. Schapira, General Topology, Course at Paris VI University, 2010-2011 (v.3).
[4] W. B. Vasantha Kandasamy, Groupoids and Smarandache Groupoids, 2012.
[5] W. B. Vasantha Kandasamy , Smarandache Special Definite Algebraic Structures, InfoLearn Quest, Ann Arbor, 2009.