Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.7, No.5, 2017

QUEUING CHARACTERISTICS OF THE DENTAL DEPARTMENT AT ESSIKADO HOSPITAL

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Abstract

In this paper, the queuing characteristics at the dental department of Essikado Hospital in the Sekondi-Takoradi Metropolis was analysed using a Single-server queuing Model. The data was collected over a period of one week, from 8:30am-11:30 am each day.

By using the queuing rule First-come, First-served as practiced by the case study and M/M/s queuing model, the performance measures were calculated. The average/mean arrival and mean service times were found to be 1.6 patients/hour and 4.4477 patients/hour respectively. The average number of patients in the system and in the queue was 0.5619 patients and 0.2021 patients respectively. Also, the average time spent in the system and the average time it takes for service to start was 0.3512 hours and 0.1263 hours respectively.

The paper recommends the need to obtain a central tray setup system for instruments required for the different dental procedures so as to reduce the time spent in sorting them up thereby reducing the amount of time spent in queue. It also respectfully submits a suggestion on the need to increase the number of dentist from one to two.

Keywords: Essikado Hospital; Queuing characteristics; First-come, First- served; Single-server Model; Utilization Factor; M/M/s Queuing Model; Mean Arrival Rate; and mean Service Rates.

1. INTRODUCTION

Queuing theory deals with the study of queues which abound in practical situations and arise so long as arrival rate of any system is faster than the system can handle. Following Nkeiruka et al (2013), queuing theory is applicable to any situation in general life ranging from customers arriving at a bank, customers at a supermarket waiting to be attended to by a cashier and in health care settings. Queuing theory is the mathematical approach to the analysis of waiting lines in any setting where arrival rate of subjects is faster than the system can handle. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted (Sundarapandian, 2009). In situations where facilities are limited and cannot satisfy the demand made upon them, bottlenecks occur which manifest as queue but patients are not interested in waiting in queues. When patients are not interested in waiting in queues, there is danger that waiting time will become excessive leading to the loss of some patients to competitors. Customer satisfaction has become a serious concern in the service sector.

In the healthcare industry, a number of initiatives have been introduced to enhance customer satisfaction. The healthcare industry providers globally are experiencing increasing pressure to concurrently reduce cost and improve the access and quality of care they deliver. Dentistry is a healthcare specialism that is often forgotten about or given less importance than other areas. However as diets change to include more sugary snacks it has fewer fruits and vegetables due mainly to globalisation and a wider variety of goods, there is an increasing need for people to have regular dental check-ups and sometimes even actual dental treatment. Unfortunately Ghana, like most developing countries, is not blessed with numerous supply of dentist. There are only a few dentists in Ghana to meet the needs of the Ghanaian population especially in the rural areas. A dental clinic is an integral part of a social and medical organization, the function of which is to provide for the population dental care and whose out-patient services reach out to the family and its home environment.

Nowadays, there are a lot of dental clinics in the Sekondi-Takoradi Metropolis. Some of the services they provide include pain relief (tooth extraction, temporary incision and drainage) and dental restoration (simple amalgam filling, temporary dressing). The Essikado Hospital is one of the hospitals in the Sekondi-Takoradi metropolis that has a dental department. The hospital dental department was started in 1991. The dental department offers a full range of all dental health services which include all types of fillings (amalgams, composite, and Root cal treatment), extractions, orthodontic treatment, and prosthetic treatment.

The dental department is confronted with long waiting times, delays and queues of patients. Long waiting time in any hospital is considered as an indicator of poor quality and needs improvement. Managing waiting lines create a great dilemma for managers seeking to improve the return on investment of their operations. Customers also dislike waiting for long time. This paper is designed to help the management of Essikado Hospital on the employee adequacy and also help to reduce patient-waiting time for services at the dental department. The main purpose of this study is to examine the queuing characteristics of patients at the dental department of the Essikado Hospital. Specifically, this paper seeks to find the average number of arrivals entering the dental department of the hospital; the average service time of customers at the dental department and the average time a patient spends in the queue at the hospital.

The findings of the paper will assist the hospital administrators and management to improve on customer service, and maximize the utilization of its resources (doctors, nurses, hospital beds, etc.). The findings could also be used for appropriate staffing and facilities design.

2 MATERIALS AND METHODS

The chapter looks at and gives report on the population from which the sample was drawn, instruments and procedures used for the collection and analysis of the data. The overall model structure is explained and the methodology used to achieve the objectives under study is clearly stated.

2.1 Data Source and Collection Techniques

The data for this study were collected from the dental department of the Essikado hospital. The data is purely from a primary source. Patients coming for dental treatment were not made to join the queue at the out-patient department (OPD). They were quickly attended to and made to join the queue at the dental section. This is because the dentist closes by 1.00 p.m. The data of most fundamental importance were arrival time, service time and department time of patients. These data were collected within a three-hour interval. Data was collected over a period of one week from 8:30 a.m. to 11:30 a.m. each day. The research goal during the collection of data was to get time between patient consecutive arrivals and service length. More interest was taken in periods where each of the following events occurs: a patient arrival, a beginning of service and an end of service in the dental clinic.

2.2 Queuing Model and Kendall's Notation

The basic queuing model is shown below. It can be used to model for example, machine or operators processing orders or communication equipment processing information. Among others, a queuing model is characterized by:

a. The arrival process of customers: Usually we assume that the inter-arrival times are independent and have a common distribution. In many practical situations customers arrive according to a Passion stream (that is, exponential inter-arrival times). Customers may arrive one by one, or in batches. An example of batch arrivals is the customs office at the border where travel documents of bus passengers have to be checked.
b. The behaviour of customers: Customers may be patient and willing to wait (for a long time), or customers may be impatient and leave after a while. For example, in call centres, customers will hand up when they have to wait too long before an operator is available, and they possibly try again after a while.
c. The service times: Usually we assume that he service times are independent and identically distributed, and that they are independent of the inter arrival times. For example, the service times can be deterministic or exponentially distributed. It is also possible that service times are dependent of the queue length. For example, the processing rates of the machines in a production system can be increased once the number of jobs waiting to be processed becomes too large.
d. The service discipline: Customers can be served one by one in batches. We have many possibilities for the order in which they enter service:
    i. first come first served, i.e. in order of arrival;
    ii. random order;
    iii. last come first served (e.g. in a computer stack or a shunt buffer in a production line);
    iv. priorities (e.g. rush orders first, shortest processing time first); processor sharing (in computers that equally divide their processing power over all jobs in the system).
e. The service capacity: There may be a single server or a group of servers helping the customers.
f. The waiting room: There can be limitations with respect to the number of customers in the system. For example, in a data communication network, only finitely many cells can be buffered in a switch. The determination of good buffer sizes is an important issue in the design of these networks.

Kendall introduced a shorthand notation to characterize a range of these queuing models. It is a three-part code a/b/c. The first letter specifies the inter-arrival time, the second one the service time distribution and the third signifies the number of servers. For example, a/b/1 is a queuing model with a single server.
In the basic model, customers arrive one by one and they are always allowed to enter the system, there is always room, there are no priority rules and customers are served in order of arrival. It will be explicitly indicated (e.g. by additional letters) when one of these assumptions does not hold.

2.3 Occupation Rate

In a single-server system G/G/1 with arrival rate, λ and mean service time $E(B)$ the amount of work arriving per unit time equals $\lambda E(B)$. The server can handle 1 unit work per unit time. To avoid that the queue eventually grows to infinity, it requires that $\lambda E(B) < 1$. It is common to use the notation

$$\rho = \lambda E(B).$$

If $\rho < 1$, then ρ is called the occupation rate or server utilisation, because it is the fraction of time the server is working.

In a multi-server system G/G/c it requires that $\lambda E(B) < c$. Here the occupation rate per server is $\rho = \lambda \frac{E(B)}{c}$.

2.4 Performance Measure

Relevant performance measures in the analysis of queuing models are:

- The distribution of the waiting time and the sojourn time of a customer. The sojourn time is the waiting time plus the service time.
- The distribution of the number of customers in the system (including or excluding the one or those in service)
- The distribution of the amount of work in the system. That is the sum of service times of the waiting customers and the residual service time of the customer in service.
- The distribution of the busy period of the server. This is a period of time during which the server is working continuously.

The aim of investigations in queuing theory is to get the main performance measures of the system which are the probabilistic properties (distribution function, density function, mean, variance) of the following random variables: number of customers in the system, number of waiting customers, utilization of the server/s, response time of a customer, waiting time of a customer, idle time of the server, busy time of a server. Of course, the answers heavily depend on the assumptions concerning the distribution of inter-arrival times, service times, number of servers, capacity and service discipline. It is quite rare, except for elementary or Markovian systems, that the distributions can be computed. Usually their mean or transforms can be calculated.

2.5 Formulation of Methods

Under this section, we discuss the methods of computing the various parameters used in queuing models.

2.5.1 The mean arrival rate

Let λ be the mean arrival rate and let *n* be the number of patient that entered the system between 8:30 a.m. – 11:30 a.m. Also, let *h* be the number of hours between 8:30 a.m. – 11:30 a.m. Then, the mean arrival rate is given by the formula

$$\lambda = \frac{n}{h} \text{ arrival per hour} \tag{1}$$

2.5.2 The mean service rate

The mean service rate at the dental section

Let $S_1, S_2, \ldots, S_n$ be the observed service times when patients arrived at the hospital. Let n be the number of patients that are attended to by the dentist and let b be the start service time for each patient and let e be the finished service time for each patient. From these defined letters.

$$S_{1} = \frac{e_1 - b_1}{n_1}$$

$$S_{2=}\frac{e_{2-}b_2}{n_2}$$

$$\vdots$$

$$S_{n=}\frac{e_n-b_n}{n_n}$$

Little's Formula

The Little's formula is given by:

$$L \ = \lambda\, W \tag{2}$$

This formula is used to determine the amount of time that a patient would spend in the system. From this formula, L is defined as the average number of patient present in the queuing system and $\lambda$is defined as the mean arrival rate. W is also defined as the expected time a patient spends in the queuing system.

We could say from Little's formula that if equation (3.2) is Little's second flow of equation. Then

$$W_q = \frac{L_q}{W} \tag{3}$$

Equation (3.5) which follows directly from Little's second flow equation was used for the single-channel and the multiple-channel waiting line models. The general expression that applies to waiting line models is that the average time in the system W, is equal to the average time, W_q plus the average service time. A system with a mean service rate μ, the average or mean service time is $\frac{1}{\mu}$. Hence, we have the following general relationships.

$$L_q = \lambda\, W_q \tag{4}$$

and

$$L_s = \lambda\, W_s \tag{5}$$

Where equations (3.5) and (3.6) are defined as the average number of patients in the waiting line and the average number of patient in the system respectively where

W_q is expected waiting time in queue

W_s is expected time a customer spends in the system

W_q is expected number of customers in the queue

W_s is expected number of customers in the system.

2.6 The Operating Characteristics of M/M/I Model (Single Server – Infinite Population)

$\rho = \frac{1}{\mu}$ is defined as the average utilization of the system i.e. the probability that the system is.

$$\rho(\text{n customers during period T}) = \frac{e^{-\lambda t}(\lambda\, t)^n}{n!} \tag{6}$$

When the time taken to serve different customers is independent then;

$$\rho(\text{not more than T time period needed to serve a customer}) = 1 - e^{\mu T} \tag{7}$$

1. Probability that the system is idle i.e. the probability that there are no customers in the system;

$$P_o = 1 - P = 1 - \frac{\lambda}{\mu} \tag{8}$$

2. The probability of having exactly n customers in the system

$$P_n = P^n\, P_0 = P^n\left(1 - \lambda\frac{1}{\mu}\right) = P^n(1-P) \tag{9}$$

3. Expected number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} \tag{10}$$

Or

$$L_s = \frac{P}{1-P} \tag{11}$$

4. Expected number of customers in the queue.

$$L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\mu - \lambda} - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\lambda - \mu)} = \frac{P^2}{1-P} \tag{12}$$

Equation (3.13) can also be noted as the average length of all queues including empty queues.

5. The average length of non-empty queues (i.e. those which contain at least one customer)

$$L_n = \frac{\mu}{\mu - \lambda} = \frac{1}{1 - P} \tag{13}$$

6. Expected waiting time in the queue

$$W_q = \frac{1}{\lambda} L_q \tag{14}$$

Substituting equation (3.11) into (3.15), we have

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{P}{\mu - \lambda} \tag{15}$$

7. Expected time a customer spends in the system

$$W_s = \frac{1}{\lambda} \text{ X } L_s \tag{16}$$

Putting equation (3.11) into equation (3.17), we have

$$W_q = \frac{1}{\mu - \lambda} \tag{17}$$

Since the mean service rate is?, the average (expected) time for completing the service is $\frac{1}{\mu}$.

Therefore, the expected time a customer would spend in the system would be equal to the expected waiting time in the queue plus the average servicing time. Thus,

$$W_s = W_q + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)} + \frac{1}{\mu} = \frac{1}{\mu - \lambda} \tag{18}$$

Which is the same as equation (3.18), as shown earlier.

8. The probability that a customer spends more than *t* units of time in the system

$$W_s(t) = e^{\frac{-t}{s}} \tag{19}$$

9. The probability that a customer spends more than *t* units of time in the queue

$$W_q = Pe^{-\frac{t}{W_s}} \tag{20}$$

2.7 Single Server Model – Finite Population

This model is based on similar assumptions as that of M/M/1 model except that the input population is finite. For this model, the system structure is such that we have a total of M customers, a customer is either in the system (consisting of a queue and a single server) or outside the system and in a sense, arriving. When a customer is the arriving condition, then the time it takes him to arrive is a random variable having an exponential distribution with mean equal to $\frac{1}{\lambda}$. When there are *n* customers in the system, then there is M? *N* customers in the arriving state. From this, the total average rate of arrivals in the system is $\lambda\,(M-n)$.

The single server model with finite population is self-regulating. This means that when the system gets busy, with many customers in the queue, then the rate at which additional customers arrive is reduced thus lowering the congestion in the system. In this model, there is a depending relationship between arrivals. For the reason of dependency relationships between arrivals, the Poisson probability distribution law cannot be strictly applied when the input population is finite. Instead of the arrivals' statement as an average for the population, we classify them as an average of a unite time.

Hence, the exponential distribution with mean $\frac{1}{\lambda}$.

2.8 The Operating Characteristics of a single Server – Finite Population Model.

The number of customers in the source population = M

Average inter-arrival between successive arrivals $= \frac{1}{\lambda}$.

Service rate $= \lambda$.

Probability that the system would be idle is given by

$$P_0 = \left[\sum_{i=0}^{m} \frac{M!}{(M-1)!}\left(\frac{\lambda}{\mu}\right)^t\right]^{-1} \tag{21}$$

Probability of n customers in the system can be represented as

$$P_n = \left[P_0\left(\frac{\lambda}{\mu}\right)^n \sum_{i=0}^{m} \frac{M!}{(M-1)!}\right]; 0 < \int n \leq M \; or \; n > M \qquad (22)$$

Expected length of the queue is specified by

$$L_q = M - \frac{\lambda + \mu}{\mu}(1 - P_0) \qquad (23)$$

Expected number of customers in the system can be represented by the formula

$$L_s = L_q + (1 - P_0) = M - \frac{\lambda + \mu}{\mu}(1 - P_0) \qquad (24)$$

Expected waiting time of a customer in the queue can be expressed as follows:

$$W_q = \frac{L_q}{\mu(1 - P_0)} = \frac{1}{\omega}\left(\frac{M}{1 - P_0} - \frac{\lambda + \mu}{\lambda}\right) \qquad (25)$$

Expected time a customer spends in the system is given by

$$W_s = W_q + \frac{1}{\lambda} = \frac{1}{\mu}\left(\frac{M}{1 - P_0} - \frac{\lambda + \mu}{\lambda}\right) + 1 \qquad (26)$$

2.9The operating Characteristics of a Multiple Server Model (M/M/K/∞)

The model is based on the following assumptions;

(i) The arrival of customers follows Poisson probability distribution

(ii) The service time has an exponential distribution

(iii) There are *k* servers, each of which provides identical services.

(iv) A single waiting line is formed

(v) The input population is infinite

(vi) The service is on a first-come-first-served basis

(vii) The arrival rate is smaller than the combined service rate of all *k* service facilities.

2.10 Operating Characteristics of a single Server – Finite Population Model.

We define the operating characteristics of a single server as follows:

Average rate of arrival $= \lambda$

Numbers of servers = k

Mean combine rate of all servers $= k\mu$

Utilization factor of the entire system $P = \frac{\lambda}{k\mu}$

Probability that the system shall be idle is given by.

$$P_0 = \left[\sum_{i=0}^{k-1} \frac{\left(\frac{\lambda}{\mu}\right)^t}{i!} + \frac{\left(\frac{\lambda}{\mu}\right)}{K!\,(1-P)}\right]^{-1} \quad (27)$$

The probability that there would be exactly n customers in the system is.

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}; n \leq k \quad (28)$$

And

$$P_n = P_0 \left[\frac{\left(\frac{\lambda}{\mu}\right)^n}{K!\,K^{n-1}}\right]; n > k \quad (29)$$

The expected number of customers in the waiting line is given by

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k P}{K!\,(1-P)^2} P_0 \quad (30)$$

The expected number of customers in the system is given by

$$L_s = L_q + \frac{\lambda}{\mu} \quad (31)$$

The expected waiting time in the queue can be expressed as

$$W_q = \frac{L_q}{\lambda} \quad (32)$$

The expected time a customer spends in the system

$$W_s = W_q + \frac{1}{\lambda} \tag{33}$$

2.11 The Erlang Distribution

The Erlang distribution is used to model situation where the inter-arrival times do not appear to be exponential. It is a continuous random variable (T) whose density function is specialized by two parameters; a scale parameter, μ, and a shape parameter k (where k is a positive integer).

A shape parameter and a scale parameter are kinds of a numerical parameter of a parametric family of probability distributions. The larger the scale factor, the more spread out the distribution. The scale parameter is useful in modelling applications since they are flexible enough to model a variety of data sets. A shape parameter allows a distribution to take on a variety of shapes, depending on the value of the shape parameter.

The Erlang distribution $E_k(\mu)$ is given by

$$F(t) = \int \frac{\mu(\mu t)^{k-1}}{(k-1)} e^{-\mu t}\, dt;\ t > 0 \tag{34}$$

The distribution function is represented by

$$F(t) = 1 - \sum_{j=0}^{k-1} \frac{\mu(\mu t)^{k-1}}{j!} e^{-\mu t}\,; t > 0 \tag{35}$$

Applying integration by parts, the mean, variance and squared coefficients of variation are given as follows:

$$E(T) = \frac{k}{\mu} \tag{36}$$

$$\sigma^2 = \frac{k}{\mu} \tag{37}$$

$$C^2(T) = \frac{1}{K} \tag{38}$$

2.12 Poisson Process

The Poisson process id an extremely useful process for modelling purposes in many practical applications such as, e.g.; to model arrival process for queuing models or demand processes for inventory systems. It could be used to approximate stochastic processes. In the Poison probability distribution, the observer records the number of events in a time interval. The Poisson is a continuous-time process. Let $N(t)$ be the number of arrivals in $[0,t]$ for a Poisson process with rate, λ, i.e. the time between successive arrivals is exponentially distributed with parameter, λ, and independent of the past. Then, the Poisson distribution with parameter, λt for $N(t)$ can be calculated as;

$$P[N(t) = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}; kk = 0,1,2, \dots \quad (39)$$

Applying integration by parts gives

$$E[N(t)] = \lambda t \quad (40)$$

And

$$\sigma^2[N(t)] = \lambda t \quad (41)$$

The mean of the Poisson Process is the same as the variable (Wayne, 1991).

2.13 Pure Birth Model and Death Models

Most pure birth-death process is used to study the number of customers in a queue. It is a special case of continuous stochastic process where the state transitions are of only two types: λ_i births which increase the state variable by one and μ_i deaths which decrease the state by one.

When a birth occurs, the process goes from state *n* to *n* + 1. When a death occurs, the process goes from *n* to state *n* – 1. The process is specialized by birth rates.

$$(\lambda_i), \quad i = 0, 1, 2, 3, \dots \infty$$

and death rates

$$(\mu_i), \quad i = 1, 2, 3, \dots \infty$$

Poisson process is a pure birth process where $\lambda_i = \lambda$ for all $i \geq 0$
Birth process is the same as arrival process and death process is the departure process.
The birth-death process describes probabilistically how the number of customers *N*(*t*) in the queuing system changes as time (*i*) increases.
The birth-death process is governed by these assumptions:

(i) Given $N(t) = n$, the current probability distribution of the remaining time until the next birth (arrival) is exponential with parameter λ_n.

(ii) Given $N(t) = n$, the current probability distribution of the remaining time until the next death (service completion) is exponential with parameter μ_n.

(iii) The random variable of assumptions one and the random variable of assumption two are mutually independent. The next transition in the state of the process is either $n \to (n+1)$or $n \to (n-1)$ depending on whether the former or the latter random variable is smaller. (Wayne, 1991).

2.14 The Traffic Intensity (ρ)

An important parameter in any queuing system is the traffic intensity also called the load or the utilization, defined as the ratio of the mean service time $E(X) = \frac{1}{\mu}$ over the mean inter-arrival time $(\mathrm{T}) = \frac{1}{\mu}$.

$$\rho = \frac{E(x)}{E(\mathrm{T})} = \frac{\lambda}{\mu} \tag{42}$$

When $\lambda$and μ are the mean inter-arrival and service rate, respectively.
Clearly, if $\rho > 1$ or $E(x) > E[\mathrm{T}]$, which means that the mean service time is longer than the mean inter-arrival time, then the queue will grow indefinitely longer for large t, because packets are arriving faster on average than they could be served. In this case $(\rho > 1)$, the queuing system is unstable or will never reach a steady-state. The case where ρ = 1 is critical. In practice, therefore, mostly situations where $\rho < 1$ are of interest.
If $\rho < 1$ a steady state can be reached. The considerations are a direct consequence of the law of conservation of packets in the system

3.15 Queuing Model

2.15.1 Kendalls' notation

In the year 1953, Kendall introduced a notation that is commonly used to described and classify the type of a queuing model that a queuing system corresponds to. The general syntax is A/B/n/K/m, where A specifies the inter-arrival process, B the service process, n the number of servers, K the number of positions in the queues and m restricts the number of allowed arrivals in the queuing system. Examples for both the inter-arrival distribution A and the service distribution B are M (memoryless or Markovian) for the exponential distribution, G for a general distribution and D for a deterministic distribution.

When other letters are used besides these three common assignments, the meaning will be defined. For example, M/G/1 stands for a queuing system with exponentially distributed Inter-arrival times, a general service distribution and 1 server. If one of the two last identifiers K and m is not written, they should be interpreted as infinity. Hence, M/G/1 has an infinitely long queue and no restriction on the number of allowed arrivals. A queue is therefore, described in a shorthand notation by A/B/C/K/N/D or the more concise A/B/C in the concise version, assumed that $K = \infty$, $N = \infty$ and D = FCFS (first come, first served). For Kendall's notation, the letters A and B can also be described by:

Table 3.1 Description of Kendall's notation

Letter	Description
A	The arrival process
B	The service time distribution
C	The number of servers
K	The number of channels in the system
N	The calling population
D	The queue discipline

Table 3.2 Examples of queuing models

Letter	Description
M	Exponential distribution
D	Deterministic inter-arrival times
E^K	Erlang distribution (k=shape parameter)
G	General distribution

2.15.2 Examples of queuing models

(i)

M/M/1 Model

In this model, random arrivals and exponentially distributed service times are assumed. Poisson distribution is used to define the random arrivals. The M/M/1 Model has only one server serving customers on first come, first served bases. The population here is infinite, so arriving customers are unaffected by the queue size. The parameters given for M/M/1 model are λ (the average arrival rate), μ (the average service rate) which may be calculated from the average service time.

(ii) **M/M/M/∞**

This describes a queuing system with M number of servers and infinite number of waiting lines.

(iii) **M/M/1/∞/K**

This describes a queuing system with a single server, infinite number of waiting lines and finite customer population k.

(iv) **M/E^2/2/K**

This describes a queuing system with Poisson arrivals, Erlangian of order 2 service time distribution, 2 servers, and maximum number of k in a queue.

(v) **M/D/5/40/200/FCFS**

Where M is the exponential distributed times, D is the deterministic service times, 5 is the number of servers, 40 is the number of buffers (35 for waiting), a total population of 200 customers and FCFS is the service discipline (first come, first served).

2.16 Deterministic Queuing Models

Queuing models can be categorized as being deterministic if customers arrive at known intervals and the service time is known with certainty. Suppose that customers come to bank's teller counter every 5 minutes. Thus the interval between the arrivals of any two successive customers in exactly 5 minutes, suppose that the banker takes exactly 5 minutes to serve each customer. Here the arrival and the service rates are each equal to 12 customers per hour. That is, 60 minutes/5 minutes = 12 customers per every 5 minutes. In this situation, there shall be never a queue and the banker shall always be busy with work.

Now, suppose that he banker can serve 15 customers per hour, the consequence of this higher service rate would be that the banker would be busy 4/5th of the time and idle in 1/5th of the time. The teller shall take 4 minutes to serve a customer and wait for the next for the next customer to come. Here also, there shall never be a queue as before.

If on the other hand the banker can serve only 10 customers per hour, then the result would be that he would be always busy and the queue length will increase continuously without limit with the passage of time. It is easy to see when the service rate is less than the arrival rate, the service facility cannot cope with all the arrivals and eventually the system leads to explosive situations. This problem can be resolved by providing additional service station(s).

Symbolically let λ be the arrival rate of customers per unit time and μ be the service rate per unit.

If $\lambda > \mu$, the waiting line shall be formed which will increase indefinitely; the service facility would always be busy; and the service system will eventually fail. If $\lambda \geq \mu$, there shall be no waiting time; the proportion of time the service facility would be idle is 1 - λ/μ. Where $\lambda/\mu = P$ is called the average utilization or the traffic ratio, or the clearing ratio. This indicates the proportion of time or the probability that the service station is busy.

From this model,

If $P > 1$, the arrivals come at a faster rate than the server can accommodate.

The expected queue increases without limit and a steady state does not occur.

This would make the system to fail ultimately.

If $P \geq 1$, the system would work and P is the utilization factor of the system, that is the proportion of time the system busy. Also, if $P < 1$, then steady-state probabilities would occur.

The deterministic queuing model may exist when we are dealing with, for example movements of items for processing in a highly automated plant. However, generally and more particularly when human beings are involved, the arrivals and servicing time are variable and uncertain. Thus, variable arrival rates and servicing times are the more realistic assumptions. The probabilistic queuing models are based on these assumptions.

3. RESULTS AND DISCUSSION

This section is devoted to the analysis and the modelling of the data collected from the dental department at Essikado Hospital, Essikado. The mean arrival rates and the mean service rates would be calculated and the results would be used to measure the performance of the entire system.

Tables 4.4.1 to 4.4.5 present the number of patients that arrived at the dental department of the Hospital from Monday to Friday. In each table, the first column shows the patient number while the second and third columns show the arrival time and the service start time respectively. The fourth and fifth columns show the service end time and the service times respectively. Observe from Table 4.4.1 that six patients reported for dental service on Monday. From Table 4.4.2, five patients reported on Tuesday. Five patients were recorded on Wednesday as illustrated in Table 4.4.3. Also, on Thursday, we recorded four patients (see Table 4.4.4). See further from Table 4.4.5 that four patients were recorded on Friday. The first and fifth patients recorded the highest service time.

We determine the arrival rates for the week, that is, Monday to Friday. The number of patients that arrived per unit of time is referred to as mean arrival rate. The counting starts from 8:30 am and ends at 11:30 am. Let the mean arrival rate (λ) be given by

$$\lambda = \frac{number\ of\ patients\ in\ the\ system\ between\ the\ time\ interval}{number\ of\ hours\ between\ 8{:}30am - 11{:}30am} \text{patients/hr}$$

Then the mean arrival rate for Monday, for instance, is given by

$$\lambda = \frac{6\ patient}{3\ hours} = 2 \text{ patients/hour}$$

The mean arrival rates are displayed in Table 4.5.1. The first column shows the days while the second column shows the arrival rates for each day of the week. Monday recorded the highest mean arrival rate of 2 patients/hour. The lowest mean arrival rate was 1.3333patients/hour which was recorded on Thursday as well as Friday. Tuesday and Wednesday also recorded the same value for the mean arrival rate which is 1.6667patients/hour.

Our next attempt is directed towards the determination of the mean service rates in the week. The mean service rate can be defined as the number of patients that are served per unit of time. The mean service rate (μ) is given by,

$$\mu = \frac{number\ of\ patients}{total\ number\ of\ hours\ spent} \text{patients/hour}$$

Mean service rate for Monday can be calculated as follows:

$$Total\ service\ time = 6 + 19 + 18 + 24 + 12 + 5 = 84 minutes = 1.4 \text{ hours}$$

$$\mu = \frac{6}{1.4} = 4.2857 \text{ patients/hour}$$

Table 4.5.2 shows the mean service rates for the week. The first column shows the days while the second column shows the mean service rates for the entire week. The highest mean service rate was recorded on Thursday with a value of 6.1538patients/hour. Whiles the least value of 3.3333patients/hour was recorded on Friday. Monday and Wednesday recorded mean service rates of 4.2857patients/hour and 4.7619patients/hour respectively.

Furthermore, the utilization factor was determined. The utilization factor (ρ) is the probability that the system is busy when the system is in equilibrium. This is given by

$$\rho = \frac{\lambda}{\mu}$$

We compute the utilization factor for Monday as illustrated below.

$$\rho = \frac{2}{4.2857} = 0.4667$$

The results of the utilization factor for each day of the week is as displayed in Table 4.5.3. The first column shows the days while the second column shows the utilization factor for each day of the week. Perceive that the busiest day of the server was on Monday with 46.67% and followed by Tuesday with a utilization factor of 45%. The least of the utilization factors is 21.67% which occurred on Thursday.

Operating Characteristics

We define the operating characteristics of patients as follows:

a. The average number of patients in the system, denoted by L_s

b. The average number of patients in the queue, L_q

c. The average time spent by a patient from arrival until fully served, denoted by W_s

d. The average time it takes a patient to start being served, denoted by W_q

For instance, we compute the operating characteristics for Monday as follow:

$$W_q = \frac{\rho}{\mu - \lambda}$$

$$W_q = \frac{0.4667}{4.2857 - 2} = 0.2042 \text{ hours}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = 0.2042 + \frac{1}{4.2857} = 0.4375 \text{ hours}$$

$$L_s = \lambda\text{Ws} = 2(0.4375) = 0.875 \text{ patients}$$

$$L_q = \lambda\text{Wq} = 2(0.2042) = 0.4084 \text{ patients}$$

Thus, on Monday, the average number of patients in the system is 0.875 patients, the average number of patients in the queue is 0.4084 patients, the average time spent by a patient from arrival until fully served is 0.4373 hours and the average time it takes a patient to start being served is 0.2024 hours. Results of the operating characteristics for the week is displayed in Table 4.5.4. The first column represents the days while the second column shows the operating characteristics for each day of the week. The highest of the average number of patients in the system was 0.875patients on Monday and the least was 0.2766 patients on Thursday. Also, the least average number of patients in queue was 0.0599patients which occurred on Thursday and a highest of 0.4084patients on Monday. The longest waiting time in queue and system are 0.2209hours and 0.4909hours respectively and occurred on Tuesday. The shortest waiting time in queue and system are 0.0449hours and 0.2074 hours respectively and occurred on Thursday.

We can see from Table 4.1.6 that the busiest of all the days was Monday since it recorded the highest number of patients and the percentage of time the server is being utilized by a patient (server utilization) was 46.67%. This was followed by Tuesday. Thursday recorded the least of 21.67%. It can also be seen that more patients wait in the queue on Mondays than any other day. The average time spent by a patient from arrival until fully served on Monday is 0.4375 hours, which is the highest compared to all the other days. On Thursday, the average time

spent by a patient until fully served is 0.2074 hours which is the lowest compared to all the other days.
Also, Thursday recorded the lowest average time it takes a patient to start being served whiles Monday recorded the highest followed by Tuesday, Friday and Wednesday respectively.
In order to make any meaningful decision as regard the operating characteristics of patients at the dental department of the hospital, we consider it imperative to compare the operating characteristics of patients for different number of servers using average arrival rate and average service rate. An average of the arrival rate at the dental department is computed as follows:

$$\lambda = \frac{2 + 1.6667 + 1.6667 + 1.3333 + 1.3333}{5} = 1.6 \text{ patients/hour}$$

Similarly, the average of the service rate during the five days data collection is given by,

$$\mu = \frac{4.2857+ 3.7037+4.7619+6.1538+3.3333}{5} = 4.4477 \text{ patients/hour}$$

Now, using the average of the arrival and service rates, we display the operating characteristics of the system for different number of servers as in Table 4.5.5. Observe that there is a reduction in the value of the operating characteristics as the number of service renders increases. An increase in the number of service renders from one to two indicates that the servers will be
17.99% busy at the dental department. The average number of patients in the system reduced from 0.5619 patients to 0.3718 patients. The average number of patients in the queue decreased from 0.2021 patients to 0.012 patients. The average time spent by a patient from arrival until fully served decreased from 0.3512 hours to 0.2324 hours and the average time it takes a patient to start receiving dental service reduced from 0.1263 hours to 0.0075 hours. Finally, increasing the number of servers from two to three also shows a decrease in the operating characteristics. The server utilisation factor reduced from 17.99% to 11.99%. The average number of patients in the system reduced from 0.3718 patients to 0.3606 patients. The average number of patients in the queue decreased from 0.012 patients to 0.0008 patients. Also, the average time spent by a patient from arrival until fully served reduced from 0.2324 hours to 0.2254 hours and the average time it takes a patient to start to receive service reduced from 0.0075 hours to 0.0005 hours.

4. CONCLUSIONS AND RECOMMENDATION

Patients' satisfaction is very important to hospital management because the patients are the people who sell the good image of the hospital to others which help to increase the revenue of the hospital. The objective of every health facility is to help reduce patients' waiting time, increase revenue and improve upon customer services and care.
The study looked at the queuing system at the dental department of Essikado Hospital in the
Sekondi Takoradi Metropolis. It looked at patients' arrival rates, service rates and the average time it takes a patient to start being served. These three parameters were then used to measure the waiting time of patients in the queue and in the entire system. They were also used to find the number of patients in the queue and in the whole system.
Our results show that Monday is the busiest day at the dental Department because it recorded the highest number of patients. Friday recorded only four patients. This may be due to the fact that the dental department is closed on weekends so people with dental issues over the weekend have to wait till Monday. During the data collection period, it was observed that most of the patients who spent longer time in the dental treatment room

were those who had major dental issues like the extraction of tooth. Patients with minor issues like gum bleeding are those who spent less time at the dental treatment room.

From the analysis, the values of the operating characteristics of the single server model were very high compared to that of the two-server model and three-server model.

Based on the findings of this study, we make the following recommendations to help management improve upon patients' satisfaction and also help reduce their waiting times for health care. Patients are uncomfortable with toothache which is very painful and as such will wish to be attended to by a dentist without waiting for long hours. In the first stance we respectfully recommended that number of dentist should be increased from one to two in order to reduce the amount of time spent in the queue. Also, management should obtain a central tray setup system for instruments required for the different dental procedures done in the dental department so as to reduce time spent in sorting them up thereby reducing the service time. It is important to note that the model's waiting time predictions pertain only to waiting time due to server unavailability. The service-end time was recorded as the time patients leave the dental department. It is therefore our candid suggestion to future researchers to work on the total time a patient spends taking into account the time the patient spends at the pharmacy department to take his/her drugs. Last but not least, we recommend that a simulation model should be developed to confirm the results of the analytical model that was developed in this work.

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4.4.1 Presentation of data collected on Monday

Patient number	Arrival Time	Service Start Time	Service End Time	Service Time
1	08:38	10:53	10:59	00:06
2	08:49	10:55	11:14	00:19
3	09:02	10:58	11:16	00:18
4	09:31	11:01	11:25	00:24
5	09:32	11:20	11:32	00:12
6	09:47	11:30	11:35	00:05

4.4.2 Presentation of data collected on Tuesday

Patient number	Arrival Time	Service Start Time	Service End Time	Service Time
1	08:32	10:10	10:30	00:20
2	08:36	10:16	10:35	00:19
3	08:40	10:36	10:40	00:04
4	09:00	10:41	10:59	00:18
5	09:10	11:05	11:25	00:20

4.4.3 Presentation of data collected on Wednesday

Patient number	Arrival Time	Service Start Time	Service End Time	Service Time
1	08:40	10:46	11:00	00:14
2	08:52	10:49	11:05	00:16
3	09:39	10:50	11:13	00:23
4	10:14	10:54	10:59	00:05
5	10:17	11:16	11:21	00:05

4.4.4 Presentation of data collected on Thursday

Patient number	Arrival Time	Service Start Time	Service End Time	Service Time
1	08:39	09:56	10:11	00:15
2	08:48	09:59	10:13	00:14
3	08:55	10:02	10:08	00:06
4	10:15	10:22	10:26	00:04

4.4.5 Presentation of data collected on Friday

Patient number	Arrival Time	Service Start Time	Service End Time	Service Time
1	08:33	09:44	10:11	00:20
2	08:37	09:47	10:13	00:18
3	08:45	09:50	10:09	00:19
4	09:17	09:57	10:12	00:15

4.5 ANALYSIS OF DATA COLLECTED

Table 4.5.1 Summary of data collected on Monday

Patient number	Arrival Time	Service time (minutes)	Time spent in queue(minutes)
1	08:38	00:06	135
2	08:49	00:19	126
3	09:02	00:18	116
4	09:31	00:24	90
5	09:32	00:12	108
6	09:47	00:05	95

Table 4.5.1 Mean arrival rates for the week

Day	Mean arrival rate(patients/hour)
Monday	2
Tuesday	1.6667
Wednesday	1.6667
Thursday	1.3333
Friday	1.3333

4.5.2 Service rate

Table 4.5.2 Mean service rates for the week

Day	Mean service rate(patients/hour)
Monday	4.2857
Tuesday	3.7037
Wednesday	4.7619
Thursday	6.1538
Friday	3.3333

Table 4.5.3 Utilization factor for the week

Day	Utilization factor
Monday	0.4667
Tuesday	0.45
Wednesday	0.35
Thursday	0.2167
Friday	0.4

Table 4.5.4 Operating characteristics

Day	**Operating characteristics**			
	L_s	L_q	W_s	W_q
Monday	**0.875**	0.4084	**0.4375**	**0.2042**
Tuesday	**0.8182**	**0.3682**	**0.4909**	**0.2209**
Wednesday	**0.5385**	**0.1885**	**0.3231**	**0.1131**
Thursday	**0.2766**	**0.0599**	**0.2074**	**0.0449**
Friday	**0.6667**	**0.2667**	**0.5**	**0.2**

Table 4.1.6 SUMMARY OF THE OPERATING CHARACTERISTICS OF ALL THE DAYS

OPERATING CHARACTERISTICS	DAYS				
	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
μ(patients/hour)	4.2857	3.7037	4.7619	6.1538	3.3333
λ(patients/hour)	2	1.6667	1.6667	1.3333	1.3333
L_s(patients)	0.875	0.8182	0.5385	0.2766	0.6667
L_q(patients)	0.4084	0.3682	0.1885	0.0599	0.2667
W_s (hours)	0.4375	0.4909	0.3231	0.2074	0.5
W_q (hours)	0.2042	0.2209	0.1131	0.0449	0.2
P	0.4667	0.45	0.35	0.2167	0.4

Table 4.3 Operating Characteristics

OPERATING CHARACTERISTICS	QUEUE MODEL		
	m/m/1	m/m/2	m/m/3
L_s	0.5619	0.3718	0.3606
L_q	0.2021	0.012	0.0008
W_s	0.3512	0.2324	0.2254
W_q	0.1263	0.0075	0.0005
P	0.3597	0.1799	0.1199