Effect of magnetic field on peristaltic flow of Williamson fluid through a porous medium in an inclined tapered asymmetric channel.

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Abstract

Effects of magnetic field on peristaltic flow of Williamson fluid through a porous medium as well as effects of non-slip and heat transfer are considered in an inclined tapered asymmetric channel. The problem is studied under long wave length and low Reynolds number assumption, the perturbation technique is used to solve the problem as the equations are non linear. The stream function, the temperature distribution and the pressure rise are calculated. The effect of various parameter on the pumping characteristic and on the temperature profiles as well as stream functions and velocity profiles are discussed with the help of graphs.

Keywords: Williamson fluid, Peristaltic transport, magnetic field, porous medium , non-slip effects, heat transfer, an inclined tapered asymmetric channel.

Introduction

Peristaltic is well known mechanism for pumping biological and industrial fluids. Even though it is observed in living systems for many centuries, the mathematical modeling of peristaltic transport has begun with important works by fung and Yih[2] using laboratory frame of reference. Many of the contributors to the area of peristaltic pumping have either followed Shapiro or fung. Most of the studies on peristaltic flow deal with Newtonian fluids. The complex rheology of biological fluids has motivated investigations involving different non Newtonian fluids. Peristaltic flow of non-Newtonian fluids. Peristaltic flow of non-Newtonian fluids in a tube was first studies by Raju and Devanathan [8]. Ravi Kumar et al. [11] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Y. V. K. Ravi Kumar et. al. [10] studied the peristaltic pumping of a magneto hydrodynamic casson fluid in an inclined channel. Ravi Kumar et.al. [9] studied the peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel. Mekheimer[6].studied the peristaltic transport of MHD flow in an inclined planner channel. Hayat et.al. [3] extended the idea of elshehawey et. al. [1] for partial slip condition. Srinivas et al. [12] studied the peristaltic transport in an asymmetric channel with heat transfer. Srinivas et al. [13] studied the non-linear peristaltic transport in an inclined asymmetric channel. Vajravelu et al.[15] analyzed peristaltic transport of a casson fluid in contact with a Newtonian fluid in circular tube with permeable wall. Nadeem and Akram [7] discussed peristaltic flow of a Williamson fluid in an asymmetric channel. It is observed that most of the physiological fluids for example, blood can not be described by Newtonian model. Hence, several non-Newtonian models are being proposed by various researchers to investigate the flow behavior in Physiological system of a living body. Among them Williamson model is expected to explain most of the features of a physiological fluid. Moreover, this model is nonlinear and Newtonian fluid model can be deduced as a special case from this model. In this paper, we present the peristaltic motion of MHD flow and heat transfer of Williamson fluid in an inclined tapered asymmetric channel through porous medium with the effects of non-slip conditions. By using the perturbation technique for small values of weissenberg number, the non linear governing equations are solved.

2- Mathematical model

Let us consider the MHD flow and heat transfer of Williamson fluid through a porous medium of two – dimensional inclined tapered a symmetric channel. We assume that infinite wave train traveling with velocity c along the non –uniform walls. We choose a rectangular coordinate system for the channel with $\overline{X}$ along the direction of wave propagation and parallel to the centre line and $\overline{Y}$ transverse to it. The wall of the tapered a symmetric channel are given in fig.(1) by the equations:

$$\overline{H_1}(\overline{x},\overline{t}) = -d - m'\overline{x} - a_1 \sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t}) + \phi] ......lower \text{ wall}$$

$$\overline{H_2}(\overline{x},\overline{t}) = d + m'\overline{x} + a_2 \sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t})] ......upper \text{ wall} \qquad ......(1)$$

Where a_1, a_2 are the amplitudes of the waves, λ is the wave length, 2d is the width of the channel at the inlet, $m'(m' << 1)$ is the non-uniform parameters, the phase difference ϕ varies in the range $0 \le \phi \le \pi, \phi = 0$ represents to symmetric channel in which waves are out of phase and when $\phi = \pi$ the waves are in phase, and further a_1, a_2, d and ϕ satisfies the condition:

$$a_1^2 + a_2^2 + 2a_1a_2 \cos\phi \le (2d)^2 \qquad ....(2)$$

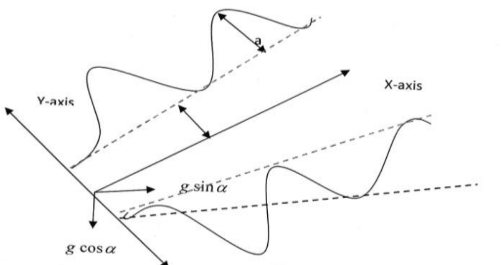


Fig.(1) geometry of problem

The constitutive equations for an incompressible Williamson fluid are:

$$S = -pI + \tau \qquad ....(3)$$

$$\tau = -[\mu_\infty + (\mu_0 + \mu_\infty)(1 - \Gamma\overline{\dot{\gamma}})^{-1}\overline{\dot{\gamma}} \qquad .....(4)$$

In which $-PI$ is the spherical part of the stress due to constraint of incompressibility, τ is the extra stress tensor, μ_∞ is the infinite shear rate viscosity, μ_0 is the zero shear rate viscosity, Γ is the time constant and $\overline{\dot{\gamma}}$ is defined as :

$$\overline{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \overline{\dot{\gamma}}_{ij} \overline{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi} \qquad ...(5)$$

Here $\prod$ is the second invariant strain tensor, which $\prod = Tr(A_1^2)$ where
$A_1 = \nabla\overline{V} + (\nabla\overline{V})^T$

We consider the constitutive equation (4), the case for which $\mu_\infty = 0$ and $\Gamma\overline{\dot{y}} < 1$. The component of extra stress tensor therefore can be written as

$$\tau = -\mu_0[(1-\Gamma\overline{\dot{y}})^{-1}\overline{\dot{y}} = -\mu_0[(1+\Gamma\overline{\dot{y}})^{-1}\overline{\dot{y}} \qquad ....(6)$$

In laboratory frame, the equations governing two dimensional motion of an incompressible MHD flow and heat transfer of Williamson fluid through a porous medium :

$$\frac{\partial\overline{U}}{\partial\overline{X}} + \frac{\partial\overline{V}}{\partial\overline{Y}} = 0 \qquad .....(7)$$

$$\rho\left(\frac{\partial\overline{U}}{\partial\overline{t}} + \overline{U}\frac{\partial\overline{U}}{\partial\overline{X}} + \overline{V}\frac{\partial\overline{U}}{\partial\overline{Y}}\right) = -\frac{\partial\overline{P}}{\partial\overline{X}} - \frac{\partial\overline{t}_{\overline{XX}}}{\partial\overline{X}} - \frac{\partial\overline{t}_{\overline{XY}}}{\partial\overline{Y}})$$
$$-\sigma B_0^2\overline{U} - \frac{\mu_0}{k_0}\overline{U} - \rho g\sin\alpha \qquad .....(8)$$

$$\rho\left(\frac{\partial\overline{V}}{\partial\overline{t}} + \overline{U}\frac{\partial\overline{V}}{\partial\overline{X}} + \overline{V}\frac{\partial\overline{V}}{\partial\overline{Y}}\right) = -\frac{\partial\overline{P}}{\partial\overline{Y}} - \frac{\partial\overline{t}_{\overline{XY}}}{\partial\overline{X}} - \frac{\partial\overline{t}_{\overline{YY}}}{\partial\overline{Y}}$$
$$-\frac{\mu_0}{k_0}\overline{V} + \rho g\cos\alpha. \qquad ......(9)$$

$$\rho C_p\left(\frac{\partial T}{\partial\overline{t}} + \overline{U}\frac{\partial T}{\partial\overline{X}} + \overline{V}\frac{\partial T}{\partial\overline{Y}}\right) = k_1\left[\frac{\partial^2 T}{\partial(\overline{X})^2} + \frac{\partial^2 T}{\partial(\overline{Y})^2}\right] +$$
$$\mu_0\left[2\left(\frac{\partial\overline{U}}{\partial\overline{X}}\right)^2 + 2\left(\frac{\partial\overline{V}}{\partial\overline{Y}}\right)^2 + \left(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}\right)^2 \qquad .....(10)$$

$$\overline{\dot{y}} = \sqrt{2\left[\left(\frac{\partial\overline{U}}{\partial\overline{X}}\right)^2 + \left(\frac{\partial\overline{V}}{\partial\overline{Y}}\right)^2\right] + \left(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}\right)^2} \qquad ....(11)$$

$$\overline{\tau}_{\overline{XX}} = -\mu_0[(1+\Gamma\overline{\dot{y}}).2\frac{\partial\overline{U}}{\partial\overline{X}} = -2\mu_0[(1+\Gamma\overline{\dot{y}})\frac{\partial\overline{U}}{\partial\overline{X}} \qquad .....(12)$$

$$\overline{\tau}_{\overline{XY}} = -\mu_0[(1+\Gamma\overline{\dot{y}}).\left(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}\right) \qquad .....(13)$$

$$\overline{\tau}_{\overline{YY}} = -\mu_0[(1+\Gamma\overline{\dot{y}}).2\frac{\partial\overline{V}}{\partial\overline{Y}} = -2\mu_0[(1+\Gamma\overline{\dot{y}})\frac{\partial\overline{V}}{\partial\overline{Y}} \qquad .....(14)$$

In which ρ is the density of fluid, P is the pressure, $(\overline{U},\overline{V})$ are velocity component in the direction of the laboratory frame $(\overline{X},\overline{Y}),\sigma$ is the electrically conductivity, B_0 is the applied magnetic field strength, g is acceleration due to gravity, α is the inclination of the channel with the horizontal, K_1 is the thermal conductivity, C_p is specific heat at constant pressure. We employ the following dimensionless variables into governing equations of motion:

$$x=\frac{\overline{X}}{\lambda},\ y=\frac{\overline{Y}}{d},\ t=\frac{c\bar{t}}{\lambda},\ u=\frac{\overline{U}}{c},\ v=\frac{\overline{V}}{\delta c},\ h_1=\frac{\overline{H_1}}{d},\ h_2=\frac{\overline{H_2}}{d},\ p=\frac{d^2\overline{P}}{c\lambda\mu_0},\ a=\frac{a_1}{d},\ b=\frac{a_2}{d},$$

$$m=\frac{m'\lambda}{d},\ \theta=\frac{T-T_0}{T_1-T_0},\ \delta=\frac{d}{\lambda},\ \mathrm{Re}=\frac{\rho c d}{\mu_0},\ M=\sqrt{\frac{\sigma}{\mu_0}}B_0 d,\ k^2=\frac{d^2}{k_0},\ \mathrm{Pr}=\frac{\mu_0 C_p}{k_1},$$

$$Ec=\frac{c^2}{C_p(T_1-T_0)},\ t_{xx}=\frac{\lambda}{\mu_0 c}\bar{t}_{\overline{XX}},\ t_{xy}=\frac{d}{\mu_0 c}\bar{t}_{\overline{XY}},\ t_{yy}=\frac{d}{\mu_0 c}\bar{t}_{\overline{YY}},\ \eta=\frac{-d\rho g}{c\mu_0},\ \dot{y}=\frac{d}{c}\overline{\dot{y}} \qquad \text{.....(15)}$$

In which c is wave speed, λ is wave length, θ is non-dimensional temperature, T_1 is the temperature of the upper wall, T_0 is the temperature of the lower wall. In the laboratory frame $(\overline{X},\overline{Y})$ the flow is un steady, However if treated it as steady flow in the wave frame (x, y).

Invoking Eq.(15) into Eqs.(7)-(14) we have :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{......(16)}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\delta^2\frac{\partial}{\partial x}t_{xx}-\frac{\partial}{\partial y}t_{xy}$$
$$-(M^2+k^2)u+\eta\sin\alpha \qquad \text{......(17)}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\delta^2\frac{\partial t_{xy}}{\partial x}-\delta\frac{\partial t_{yy}}{\partial y}-k^2\delta^2 v-\eta\delta\cos\alpha \qquad \text{.........(18)}$$

$$\delta\,\mathrm{Re}\,\mathrm{Pr}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+2\delta^2 Ec.$$
$$\mathrm{Pr}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\mathrm{Ec.Pr}\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2 \qquad \text{..........(19)}$$

$$\overline{\dot{y}}=\sqrt{2\delta^2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)^2} \qquad \text{........(20)}$$

$$t_{xx}=-2[1+we\,\dot{y}]\frac{\partial u}{\partial x} \qquad \text{.........(21)}$$

$$t_{xy}=-[1+we\,\dot{y}]\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right) \qquad \text{.........(22)}$$

$$t_{yy}=-2\delta[1+we\,\dot{y}]\frac{\partial v}{\partial y} \qquad \text{.........(23)}$$

Where $\delta,\mathrm{Re},M,K^2,\mathrm{Pr},we$ are non-dimensional parameters called respectively the wave number, the Reynolds number, Hartmann number, the inverse of Darcy number, prandtle number, weissenberg number.

Introducting the stream function $u = \frac{\partial \psi}{\partial y}, v = \frac{-\partial \psi}{\partial x}$

In the eqs.(16)-(23) and under the assumption of long wave length $(\delta << 1)$ and low Reynolds number, we get the following resulting:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}[1 + we\frac{\partial^2 \psi}{\partial y^2}]\frac{\partial^2 \psi}{\partial y^2} - N_1\frac{\partial \psi}{\partial y} + \eta \sin \alpha \qquad ....(24)$$

$$\frac{\partial p}{\partial y} = 0 \qquad ....(25)$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec(\frac{\partial^2 \psi}{\partial y^2})^2 = 0 \qquad ....(26)$$

$$\dot{y} = \frac{\partial^2 \psi}{\partial y^2} \qquad ....(27)$$

$$t_{xx} = -2[1 + we\frac{\partial^2 \psi}{\partial y^2}]\frac{\partial^2 \psi}{\partial y^2} \qquad ....(28)$$

$$t_{xy} = -[1 + we\frac{\partial^2 \psi}{\partial y^2}]\frac{\partial^2 \psi}{\partial y^2} \qquad ....(29)$$

$$t_{yy} = 0 \qquad ....(30)$$

Where $N_1 = M^2 + K^2$; it is noted from eq.(25), P does not depend on y. The appropriate boundary conditions in dimensionless form are :

$$\left.\begin{array}{l} \psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} = 0 \text{ and } \theta = 1 \ at\ (y = h_2) \\ \psi = \frac{-F}{2}, \frac{\partial \psi}{\partial y} = 0 \text{ and } \frac{\partial \theta}{\partial y} = 0 \ at\ (y = h_1) \end{array}\right] \{\text{non slip conditions}\} \qquad .....(31)$$

$h_1(x, t)$ and $h_2(x ,t)$ represent the dimension less form of the surfaces of peristaltic walls:

$$h_1 = -1 - mx - a\sin(2\pi(x - t) + \phi)$$
$$h_2 = 1 + mx + b\sin(2\pi(x - t)) \qquad ....(32)$$

3 perturbation solution

The Eq.(24) is non- linear and hence its exact solution is not possible . we use the perturbation technique to find the solution. For perturbation solution, we expand ψ , F, and P and θ as follow :

$$\psi = \psi_0 + we\psi_1 + ...$$
$$F = F_0 + weF_1 + ...$$
$$p = p_0 + wep_1 + ...$$
$$\theta = \theta_0 + \text{we}\,\theta_1 + ... \qquad .....(33)$$

Substituting the above expressions in equations (24), (26) and equations (31),we get the following system of equations:

(3.1) system of order (we$^{(0)}$)

$$\frac{\partial p_0}{\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial^2 \psi_0}{\partial y^2} - N_1 \frac{\partial \psi_0}{\partial y}\right) + \eta \sin\alpha \qquad ....(34)$$

Now diff. eq.(34) by y we have :

$$0 = \frac{\partial^4 \psi_0}{\partial y^4} - N_1 \frac{\partial^2 \psi_0}{\partial y^2} \qquad ....(35)$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta_0}{\partial y^2} + Ec\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 = 0 \qquad ....(36)$$

Along with the corresponding boundary conditions:

$$\psi_0 = \frac{F_0}{2},\ \frac{\partial \psi_0}{\partial y} = 0,\ \frac{\partial \theta_0}{\partial y} = 0 \quad at\left(y = h_2\right)$$
$$\psi_0 = \frac{-F_0}{2},\ \frac{\partial \psi_0}{\partial y} = 0,\ \frac{\partial \theta_0}{\partial y} = 0 \quad at\left(y = h_1\right) \qquad ....(37)$$

(3.2) System of order (we$^{(1)}$)

$$\frac{\partial p_1}{\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial^2 \psi_1}{\partial y^2} + \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2\right) - N_1 \frac{\partial \psi_1}{\partial y}) + \eta \sin\alpha \qquad ....(38)$$

Now diff. eq.(38) by y we have :

$$0 = \frac{\partial^4 \psi_1}{\partial y^4} + \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 - N_1 \frac{\partial^2 \psi_1}{\partial y^2} \qquad ....(39)$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta_1}{\partial y^2} + 2Ec\left(\frac{\partial^2 \psi_0}{\partial y^2}.\frac{\partial^2 \psi_1}{\partial y^2}\right) = 0 \qquad ....(40)$$

Along with the corresponding boundary conditions:

$$\psi_1=\frac{F_1}{2},\quad \frac{\partial \psi_1}{\partial y}=0,\quad \theta_1=0,\quad at\,(y=h_2)$$

$$\psi_1=\frac{-F_1}{2},\ \frac{\partial \psi_1}{\partial y}=0,\ \frac{\partial \theta_1}{\partial y}=0,\ at\,(y=h_1) \qquad \text{....(41)}$$

In order to discuss the results quantitatively we assume the instantaneous volume rate of the flow F(x, t),

periodic in (x-t), (Kothandapani and prakash[4], kothandapani et al.[5], srivastawa et al. [14]) as:

$$F(x,t)=\theta'+b\sin(2\pi(x-t)+\phi)+a\sin(2\pi(x-t)) \qquad \text{.....(42)}$$

In which θ' is the time- average of the flow over one period of the wave and

$$F=\int_{h_1}^{h_2} u\,dy \qquad \text{.....(43)}$$

Using $F_0=F-o(\mathrm{we}F_1)$ and then neglecting the terms greater than $o(\mathrm{we})$

4.1 Solution for the zeroth order system $(\alpha^{(0)})$

We solve Eq. (35) and we can find the solution of the zeroth order system which is :

$$\psi_0=a_3+a_2e^{-n_1y}\,n_2+a_1e^{n_1y}\,n_2+a_4y\,\text{.........} \qquad \text{.....(44)}$$

$$where\,(n_1=\sqrt{N_1};n_2=\frac{1}{N_1});$$

are constants can be obtained by using the boundary conditions in Eq.(37). a_1,a_2,a_3,a_4

If We substitute the expression for ψ_0 into Eq.(36) and solve the resulting equation we can find the following equation:

$$\theta_0=-\frac{1}{4}a_2{}^2e^{-2n_1y}Ecn_1^2n_2^2\,\mathrm{Pr}-\frac{1}{4}a_1{}^2e^{2n_1y}Ecn_1^2n_2^2\,\mathrm{Pr}-\mathrm{a_1a_2}Ecn_1^4n_2^2\,\mathrm{Pry}^2+\mathrm{c}_1+\mathrm{yc}_2 \text{......(45)}$$

are constant can be obtained by using the boundary condition in eq.(c_1,c_2

4.2 Solution of the first order system (($\delta^{(1)}$)

We substitute the expression for ψ_0 into Eq.(39) and solve the resulting equation we can find the solution of the first order system in the form:

$$\psi_1=\frac{-e^{-2n_1y}\,(a_2^2n_1^4n_2^2+a_1^2e^{4n_1y}\,n_1^4n_2^2-3e^{n_1y}b_1+b_2}{3n_1^2}+b_3+yb_4 \qquad \text{.....(46)}$$

b_1,b_2,b_3,b_4 are constants can be determinates by using the boundary conditions in Eq.(41).

If we substitute the expression for ψ_0 and ψ_1 into eq.(40) and solve the resulting equation we can find the following equation:

$$\theta_1 = -\frac{1}{54} a_2{}^2 e^{-2n_1 y} Ecn_2 \Pr - (16a_2{}^3 e^{-3n_1 y} n_1^4 n_2^2 \text{ -}144 \mathrm{a}_1 \mathrm{a}_2^2 e^{-n_1 y} n_1^4 n_2^2 - 16a_1{}^3 e^{3n_1 y} n_1^4 n_2^2$$
$$+9\,\mathrm{a}_2(3\mathrm{b}_2 e^{-2n_1 y} - 16a_1{}^2 e^{n_1 y} n_1^4 n_2^2 + 6b_1 n_1^2\, \mathrm{y}^2) + 27\,\mathrm{a}_1(b_1 e^{2n_1 y} + 2\,\mathrm{b}_2\, n_1^2\, \mathrm{y}^2)) + \mathrm{c}_3 + \mathrm{yc}_4 ......(47)$$

c_3 , c_4 are constant can be obtained by using the boundary conditions in eq.(41).

The non- dimensional expression for the average rise in pressure Δp is given as follows:

$$\Delta p = \int_0^1 \int_0^1 \frac{\partial p}{\partial x} dx dt \qquad .....(48)$$

5.1 Pumping characteristic

We plot the expression for Δp in Eq.(48) against θ' for various values of parameters of interest in Fig.(2-9). Numerical calculations for several values of various parameters. The effect of these parameters on Δp have been evaluated numerically using mathematica and the results are presented graphically. In fig.(2), The effects of non-uniform parameter m on Δp are seen, observed that in the pumping $\Delta p > 0$ and Co-pumping $\Delta p < 0$ for the fluid, an increase in m causes increasing in the pumping $\Delta p > 0$ and Co-pumping $\Delta p < 0$ and decreasing in the free pumping ($\Delta p = 0$). It is noticed the pumping curves intersect at a point (0.3758, -11.22) in the fourth quadrant at fixed $\theta' \in [0.3, 0.5]$. In fig.(3), the effect of parameter η on Δp are seen, observed that an increasing in η causes increasing in $(\Delta p > 0 \text{ and } \Delta \mathrm{p} < 0)$ and decreasing in the free pumping $\Delta p = 0$ which the same behavior of effect m on Δp, and it is examined that the pumping curves are parallel at all the region of pumping. The effect of inclination of the channel α on Δp is showed in Fig.(4), it observed that an increase in α causes increasing on $(\Delta p > 0 \text{ and } \Delta \mathrm{p} < 0)$ and slightly decreasing on $\Delta p = 0$ and the pumping curves are parallel. Fig.(5), showed the effect of Hartmann number M on Δp, it noticed that an increasing in M lead to slightly decreasing on $\Delta p > 0$ clearly decreasing on ($\Delta p = 0$) and the curves of pumping are intersected at a point (2.762, 20.08) in the first quadrant at $\theta \in [2.6, 3]$. The effects of amplitude lower wall of channel a on Δp is seen in Fig.(6), it observed that an increase in a lead to clearly increasing on $\Delta p > 0$,clearly decreasing on $\Delta p < 0$ and slightly decreasing on $\Delta p = 0$,and the curves of pumping intersected at point (1.212, -0.08389) of the central line. Fig.(7) showed the effect of phase difference ϕ on Δp, it noticed that an increase in ϕ causes decreasing on $\Delta p < 0$ and the curves intersected at the point (1.202, -0.08389) of the central line. Fig.(8) showed the effect of parameter (K) on Δp it observed that an increase in k cases slightly increasing on $\Delta p > 0$,clearly increasing on $\Delta p = 0$ and clearly decreasing on $\Delta p < 0$ and the curves intersect at a point (2.649, 18.32) in the first quadrant at $\theta \in [2.5, 2.8]$. The effect of amplitudes of the upper wall of the channel b on Δp are showed in fig.(9). It noticed that an increase in b causes clearly increasing on $\Delta p > 0$ and clearly decreasing on $\Delta p < 0$ and the curves intersect at a point (1.231, -0.2861) of the central line.

5.2 Velocity distribution

Influences of different parameters on the velocity distribution have been illustrated in Fig.(10-15) these figures are scratched at the fixed values of x=0.3, t=0.5 . the effect of parameter m on the axial velocity u is shown in fig.(10), it can be found that the axial velocity u decrease at the centre of channel and on region of [0, 0.6] [-0.6, 0] but at the region of [0.7, 1], [-1, ,-0.7] the velocity u increasing, Fig.(11) shows that the influence of ϕ on the axial velocity u, it observed that an increase in ϕ causes increasing in u at the lower part of channel. Fig.(12) displays the effect of a on the axial velocity u, it examined that an increase in a causes decreasing in u at all part of channel. .Fig.(13) showed that an increase in b causes decreasing in u at the centre and upper part of channel. The influence of M and k on the axial velocity u is shown in Fig.(14) and Fig.(15) which is the same behavior of effect m on u. the influence of θ' on u is displays in Fig. (16), it is observed that an increase in θ' causes clearly increasing at all part of channel. In all figures of velocity profiles the curve of velocity is parabolic.

5.3 trapping phenomenon

The trapping for different values of m, ϕ, a, b, M, K and θ' are shown in Figs.(17-23) at fixed values of(t=0.1). The stream lines for different parameters of problem are closed. in Fig.(17), the influence of parameter (m) is shown, it has been noticed that the size and number of bolus increase in the lower and upper of the tapered channel. The stream lines for different values of ϕ are shown in fig.(18), it is examined that the size and number of bolus decrease which are at beginning they are seems adherent and then they take a divergent way with an increase value of ϕ .Fig.(19), showed the effect of parameter (a) on trapping it is noticed that the size and number of bolus increase with an increasing of a. The effects of parameter b on the trapping is plotted in Fig.(20), it is bounded that the of trapping bolus decreasing slightly in the lower and upper wall with an increasing of previous parameters. Figs.(21) display the effect of parameters M on the trapping, it is examined that the size and number of trapping bolus decreasing and then they are disappear with an increase of M. The effect of (k) and θ' are shown in Figs. (22) and (23), which is noticed there is clear decrease in the size and number of bolus in the upper and lower part of channel with an increase of previous parameters.

5.4 Temperature characteristics

The expressions for temperature are given by Eqs.(45) and.(47) the effects of various parameters on temperature for fixed values of (x=0.3, t=0.5) are shown, the eqs.(45) and (47) has been numerically evaluated and the results are presented in fig. (24-32). From fig. (24), it can be found that the temperature profile decreasing in the centre and lower part of channel and at the region [0,0.6] in the upper part of channel but at the region [0.6,1.2] then temperature profile increasing with an increasing of the parameter m and the curves of temperature intersected at the point (0.5,1.2) in the first quadrant. The influence of parameter (ϕ) on temperature profile is plotted in fig. (25), it is noticed that there is no difference in temperature profile with an increase of ϕ and the curves of temperature are applicable to each other. Fig. (26) showed the influence of parameter a on temperature profile and it is founded that increasing in a lead to some behavior of effect m on temperature profile and the curves are investigated at the point (0.8,1.1) in the first quadrant. The effect of parameter Pr on temperature is noticed in fig.(27), it observed that an increasing in Pr causes increasing on temperature profile and its curves intersected at the point (0.9248,1.045) in the first quadrant. The effect of parameter Ec on temperature is shown in fig. (28), it founded that an increasing in Ec lead to similar behavior of effect Pr on temperature and its curves intersected at the point (0.9537,1.016) in the first quadrant Fig.(29) displays the influence of parameter b, it is examined that an increase in b causes decreasing in temperature profile and the curves are parallel. Figs. (30) and (31) shows the effects of M and k respectively it is observed that an increase in previous parameters lead to slightly increasing on temperature profile on the lower part of channel and the curves are applicable to each other in the upper part of channel. The influence of parameter θ'

on temperature profile is shown in Fig. (32), it is found that an increase in θ' causes an increasing on temperature and the curves intersected at the point (0.9609,0.9928).

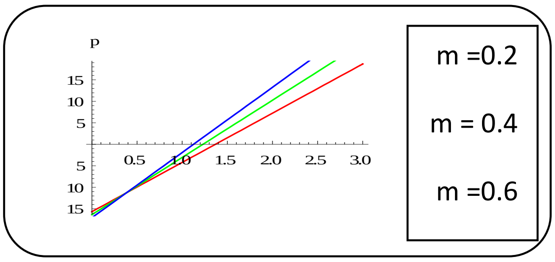


Fig.(2) Effect of non-uniform parameter m on Δp

$$t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \alpha = \pi/3,$$
$$M = 2, K = 1, \eta = 1, x = 0.5$$

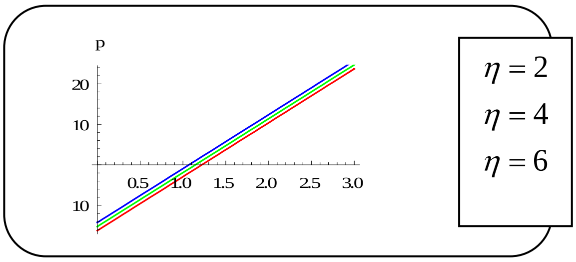


Fig.(3) Effect of η on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$$
$$\alpha = \pi/6, M = 2, K = 1, x = 0.5$$

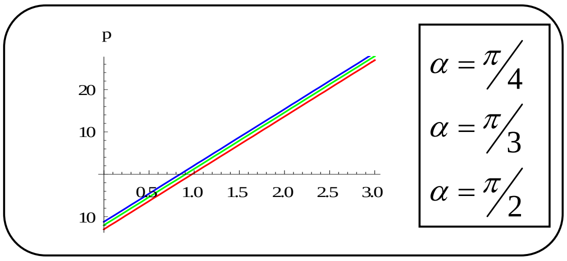


Fig.(4) effect of inclination of the channel on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$$
$$M = 2, K = 1, \eta = 6, x = 0.5$$

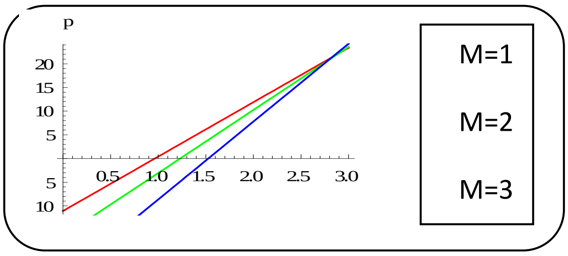


Fig.(5) Effect of Hartmann number M on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$$
$$\alpha = \pi/3, K = 1, \eta = 1, x = 0.5$$

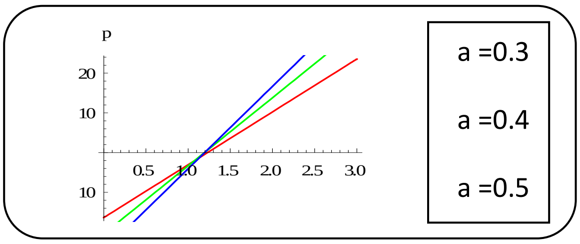


Fig.(6) Effect of the amplitude of lower wall of channel on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.2, \alpha = \pi/3$$
$$M = 2, K = 1, \eta = 1, x = 0.5$$

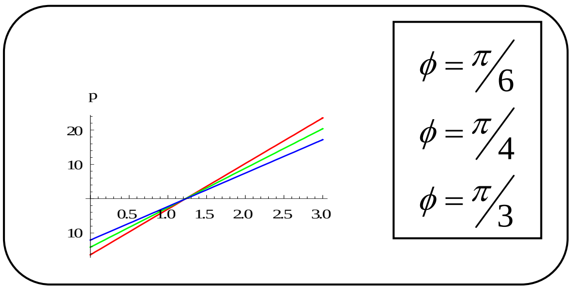


Fig.(7) Effect of phase difference on Δp

$$m = 0.4, t = 0.5, a = 0.3, b = 0.2, M = 2$$
$$\alpha = \pi/3, K = 1, \eta = 1, x = 0.5$$

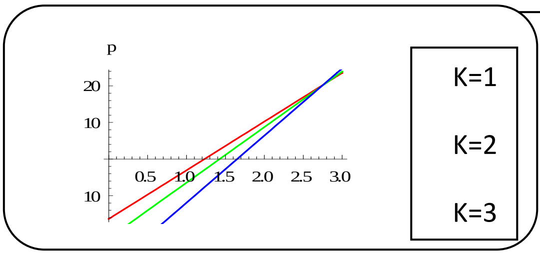


Fig.(8) Effect of the inverse of Decry number K on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$$
$$\alpha = \pi/3, M = 2, \eta = 1, x = 0.5$$

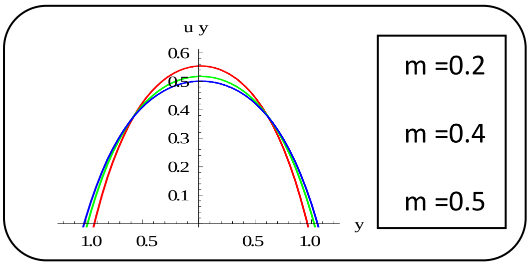
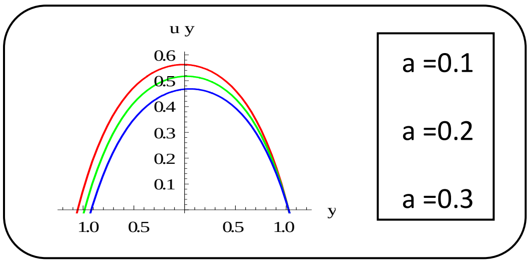


Fig.(10) Effect of non –uniform parameter m on u(y)

$$t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, M = 2,$$
$$we = 0.0001, K = 1, \theta' = 1, x = 0.3$$



Fig(12) Effect of amplitude of lower wall of channel on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.1, M = 2,$$
$$we = 0.0001, K = 1, \theta' = 1, x = 0.3$$

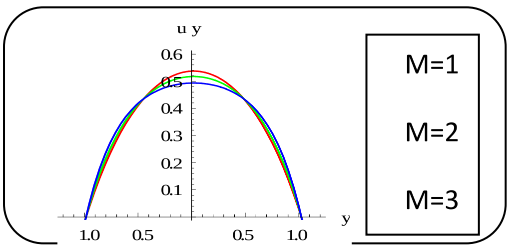


Fig.(14) Effect of Hartmann number M on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.1, M = 2,$$
$$we = 0.0001, K = 1, \theta' = 1, x = 0.3$$

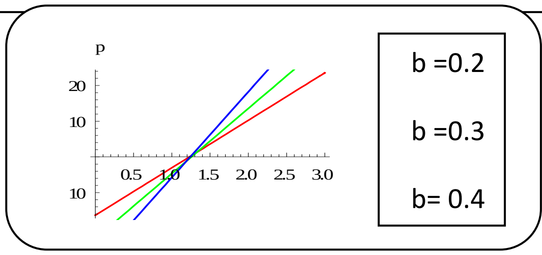


Fig.(9) Effect of amplitude of upper wall of channel b on Δp

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, \alpha = \pi/3$$
$$M = 2, K = 1, \eta = 1, x = 0.5$$

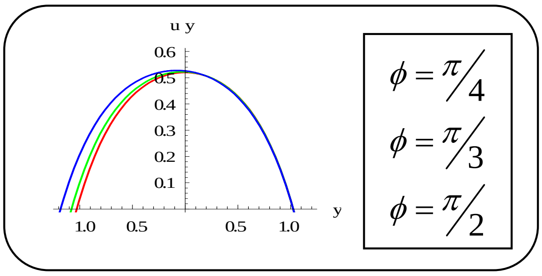


Fig.(11) Effect of phase difference on the axial velocity on u(y)

$$m = 0.4, t = 0.5, a = 0.2, b = 0.1, M = 2,$$
$$we = 0.0001, K = 1, \theta' = 1, x = 0.3$$

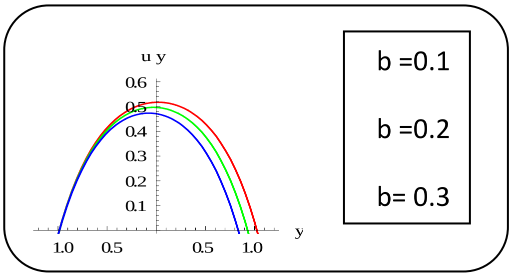
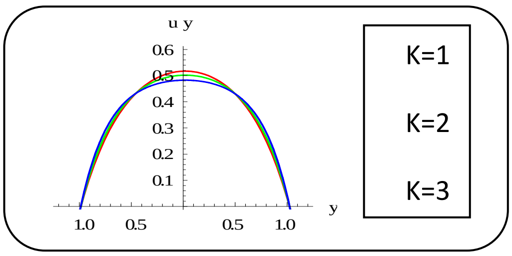


Fig.(13) Effect of amplitudes of upper wall channel on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, M = 2,$$
$$we = 0.0001, K = 1, \theta' = 1, x = 0.3$$



Fig(15) Effect of the inverse of the Dercy number K on axial velocity u(y).

$m = 0.4, t = 0.5, \phi = \pi/6, \mathrm{a} = 0.2, b = 0.1,$
$M = 2, we = 0.0001, \theta' = 1, x = 0.3$

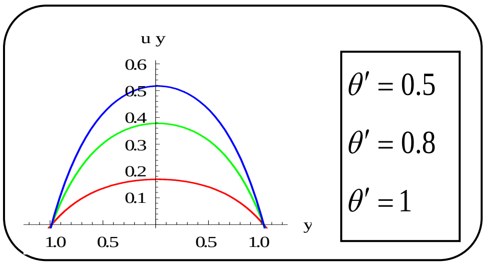


Fig.(16) Effect of the time average θ' on axial velocity u(y)

$m = 0.4, t = 0.5, \phi = \pi/6, \mathrm{a} = 0.2, b = 0.1,$
$M = 2, we = 0.0001, \mathrm{k} = 1, x = 0.3$

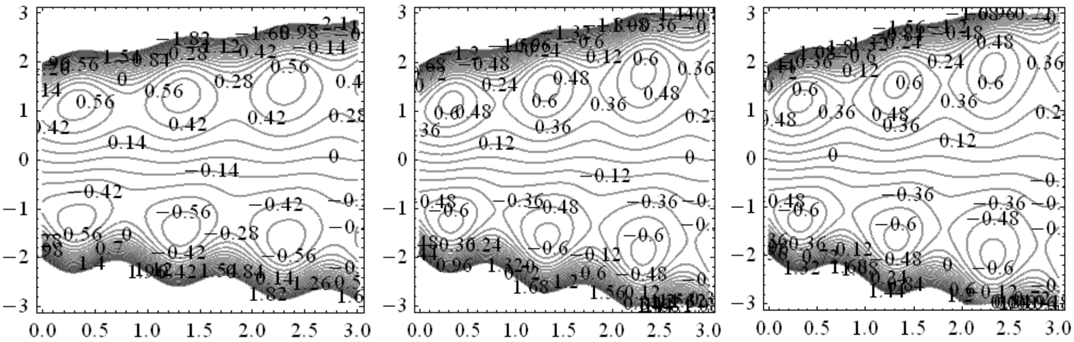


Fig.(17) Stream lines for:

$t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, M = 2, \mathrm{we} = 0.0001, K = 1, \theta' = 1$

$(a) m = 0.2, (b) m = 0.3, (c) m = 0.35$

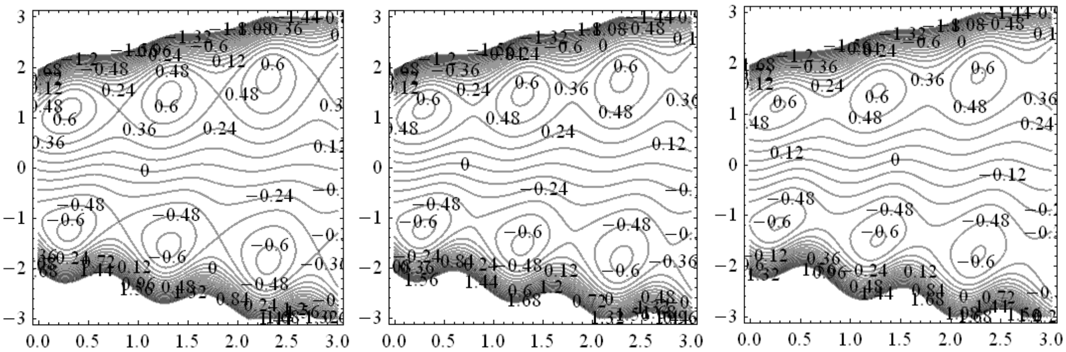


Fig.(18) Stream lines for:

$m = 0.3, t = 0.1, a = 0.2, b = 0.1, M = 2, \mathrm{we} = 0.0001, K = 1, \theta' = 1$

$(a) \phi = \pi/4, (b) \phi = \pi/3, (c) \phi = \pi/2$

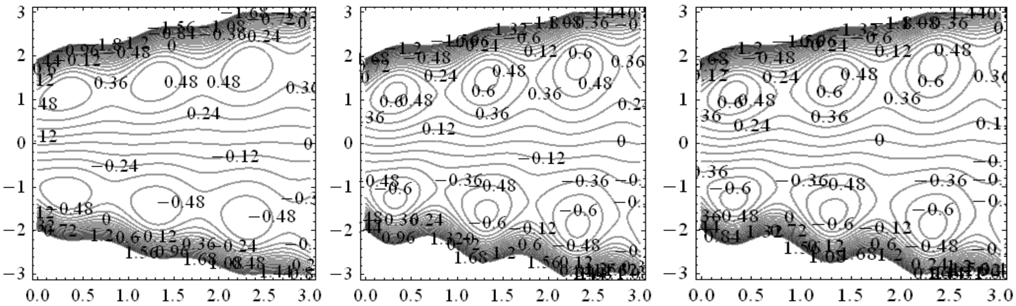


Fig.(19) Stream lines for

$$m = 0.3, t = 0.1, \phi = \pi/6, b = 0.1, M = 2, \text{we} = 0.0001, K = 1, \theta' = 1$$

$(a) a = 0.1, (b) a = 0.2, (c) m = 0.22$

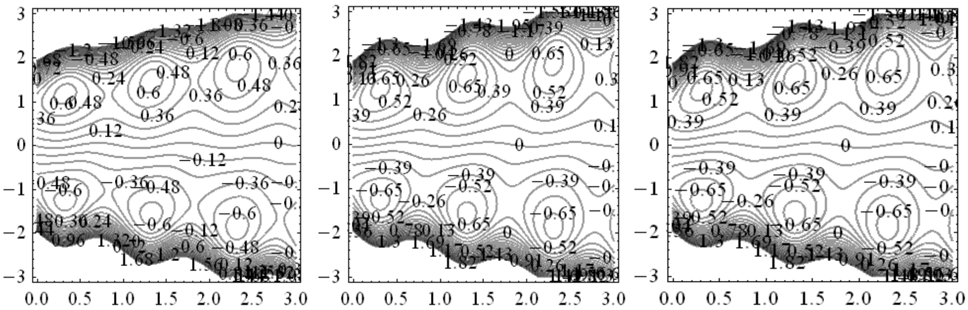


Fig.(20) Stream lines for

$$m = 0.3, t = 0.1, \phi = \pi/6, a = 0.2, M = 2, \text{we} = 0.0001, K = 1, \theta' = 1$$

$(a) b = 0.1, (b) b = 0.2, (c) b = 0.22$

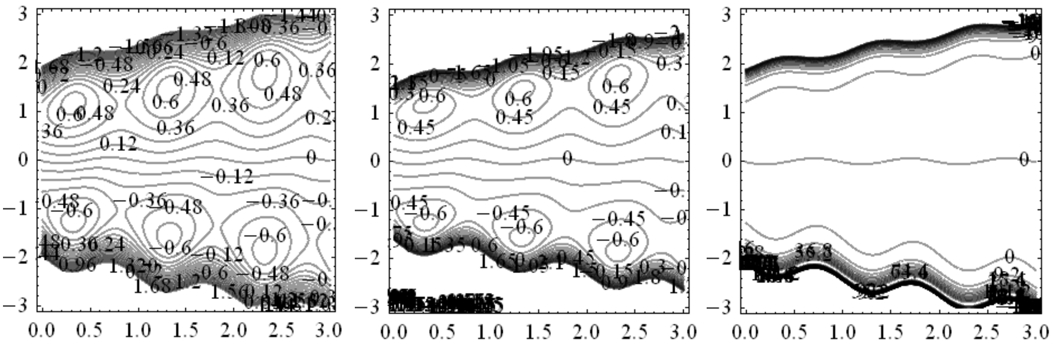


Fig.(21) Stream lines for

$$m = 0.3, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, \text{we} = 0.0001, \text{k} = 1, \theta' = 1$$

$(a)\ \text{M} = 2 (b)\ \text{M} = 5, (c)\ \text{M} = 8,$

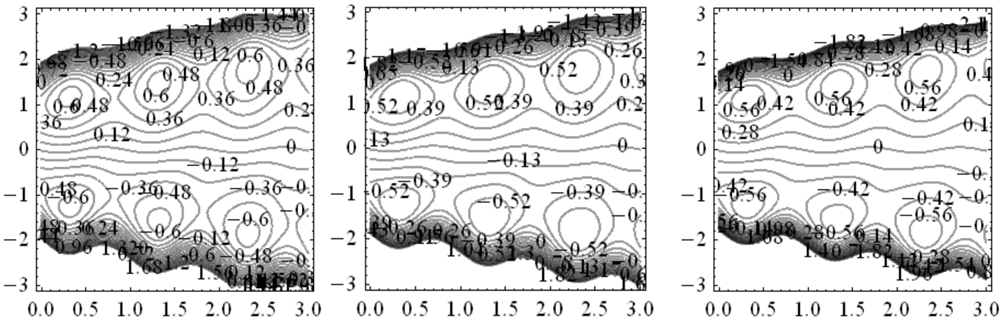


Fig.(22) Stream lines for

$m = 0.3, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, M = 2, \text{we} = 0.0001, \theta' = 1$

$(a) K = 1 (b) K = 2, (c) K = 3,$

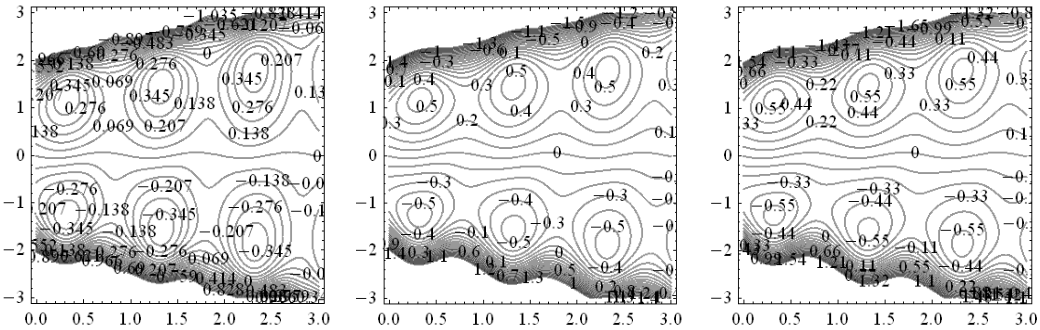


Fig.(23) Stream lines for

$m = 0.3, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, M = 2, \text{we} = 0.0001, K = 1,$

$(a)\theta' = 0.5, (b)\theta' = 0.8, (c)\theta' = 0.88,$

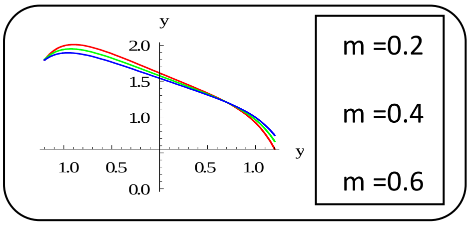


Fig.(24) Effect of non-uniform parameter m on temperature

$t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, \mathrm{M} = 2$

$\text{we} = 0.0001, \mathrm{k} = 1, \mathrm{Pr} = 1, \mathrm{Ec} = 1, \theta' = 1, x = 0.3$

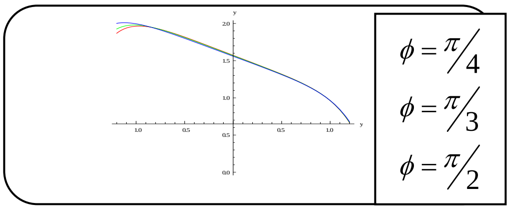


Fig.(25) Effect of phase difference parameter on temperature

$m = 0.2, t = 0.5, \mathrm{a} = 0.2, b = 0.1, \mathrm{M} = 2$

$\text{we} = 0.0001, \mathrm{k} = 1, \mathrm{Pr} = 1, \mathrm{Ec} = 1, \theta' = 1, x = 0.3$

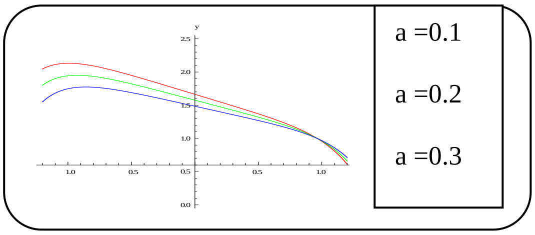


Fig.(26) Effect of amplitude of lower wall of channel on temperature

$$m=0.2, t=0.5, \phi=\pi/6, b=0.1, \mathrm{M}=2$$
$$\mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Pr}=1, \mathrm{Ec}=1, \theta'=1, x=0.3$$

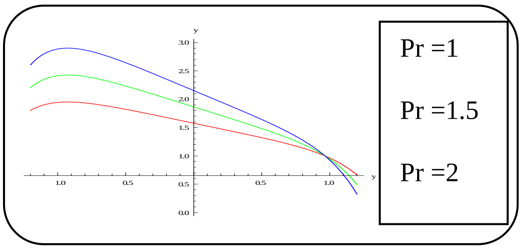


Fig.(27) Effect of Pr on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, b=0.1,$$
$$\mathrm{M}=2, \mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Ec}=1, \theta'=1, x=0.3$$

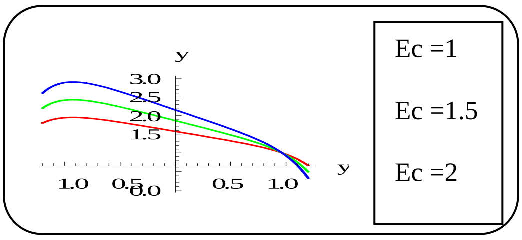


Fig.(28) Effect of Ec on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, b=0.1, \mathrm{M}=2$$
$$\mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Pr}=1, \theta'=1, x=0.3$$

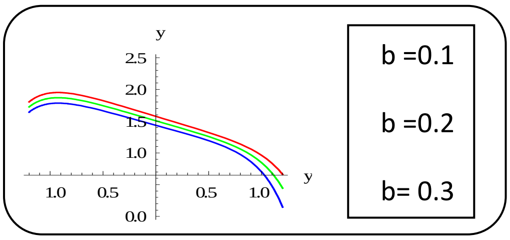


Fig.(29) Effect of amplitude of upper wall of channel on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, \mathrm{M}=2$$
$$\mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Pr}=1, \mathrm{Ec}=1, \theta'=1, x=0.3$$

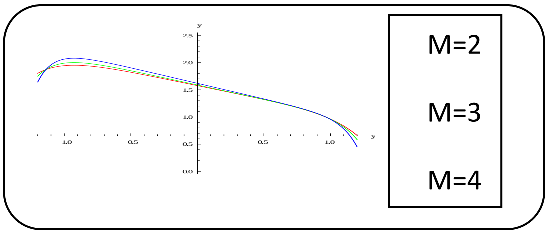
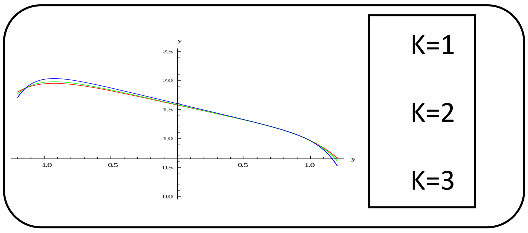


Fig.(30) Effect of Hartmann number M on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, b=0.1,$$
$$\mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Pr}=1, \mathrm{Ec}=1, \theta'=1, x=0.3$$



Fig(31) Effect of the inverse of the Dercy number K on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, b=0.1,$$
$$\mathrm{M}=2, \mathrm{we}=0.0001, \mathrm{Pr}=1, \mathrm{Ec}=1, \theta'=1, x=0.3$$

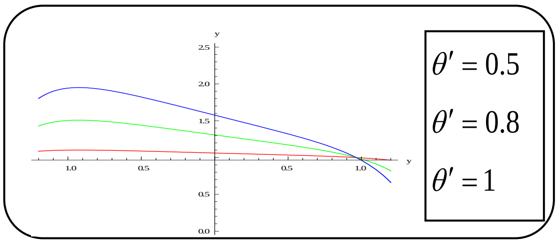


Fig.(32) Effect of θ' on temperature

$$m=0.2, t=0.5, \phi=\pi/6, \mathrm{a}=0.2, b=0.1, \mathrm{M}=2$$
$$\mathrm{we}=0.0001, \mathrm{k}=1, \mathrm{Pr}=1, \mathrm{Ec}=1, x=0.3$$

6 Concluding remarks

In this paper, we presented the peristaltic transport of Williamson fluid under the effect of magnetic fields and heat transfer through porous medium in an inclined tapered a symmetric channel. The effects of Hartmann number (M), the inverse of Darcy number (K), wave amplitudes (a& b), channel non-uniform parameter (m), phase difference ϕ on the pressure rise, axial velocity, stream lines and temperature profile are also studied in details, it noticed that :

1- The pressure rise over a wave length Δp increase in the pumping $\Delta p > 0$ with an increase of m, a, b, η, α .
2- The pressure rise over a wave length Δp increase in the pumping $\Delta p = 0$ with an increase of M, K.
3- The pressure rise over a wave length Δp increase in the pumping $\Delta p < 0$ with an increase of m. η, α
4- The pressure rise over a wave length Δp decrease in the pumping $\Delta p > 0$ with an increase in M, ϕ.
5- The pressure rise over a wave length Δp decrease in the pumping $\Delta p = 0$ with an increase of m, a, η, α
6- The pressure rise over a wave length Δp decrease in the pumping $\Delta p < 0$ with an increase of M, a, K, b.
7- The pressure rise over a wave length Δp decrease in the pumping $\Delta p < 0$ by wobbling case by increase of ϕ
8- It is observed that the pumping curves intersected at different points by increasing of parameters m, M, a, ϕ, K, b and parallel by increasing η, α .
9- At the centre of channel, the axial velocity u increase with an increase of θ' while the increase of m, a, b, M, K the velocity axial decrease.
10- At the upper part of channel, the axial velocity u increase with an increase of θ' while the axial velocity decrease with an increase of a, b .
11- At the lower part of channel, the axial velocity u increase with an increase of θ', ϕ and the axial velocity decrease with an increase of a .
12- There is wobbling behavior in the axial velocity in the centre, lower and upper part of channel with an increase of m, M,K.
13- The curves of velocity profiles at all figures are parabolic.
14- The temperature profile increase with an increase of Pr.Ec,M,K, θ' while the temperature decrease with an increase of b.
15- There is wobbling behavior in the temperature profile with an increase of m,a.
16- The curves of temperature profiles are intersected at different points with an increase of m, a, Pr, Ec, θ'
17- The curves of temperature profiles are applicable to each other by increasing of ϕ ,M, K.
18- The size and number of the trapped bolus increase with an increase of m, a in the lower and upper part of channel.
19- There is slightly decreasing in the size of bolus with an increase of b.
20- The size and the number of the trapped bolus in the trapped channel decreasing with an increase of M, θ', ϕ, k .

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