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Modelling Rates of Inflation in Kenya: An Application of Garch and Egarch Models

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Abstract

The purpose of this study was to determine an effective Arch-type model for forecasting Kenya's inflation. Using Kenya monthly inflation data from January 1990 to December 2015, the performance of GARCH and EGARCH type models was analyzed to come up with the best model for forecasting Kenyan inflation data. Since the inflation series is non-stationary, the Consumer Price Index (CPI) was first transformed to return series by logarithmic transformation. Afterwards, the data was tested for the presence of ARCH effects and serial correlation using both Ljung Box Pierce Q test and Engle Arch test. The test showed presence of heteroscedasticity and correlation in the inflation return series which is a key feature of a financial time series data. The project adopted AIC and BIC in selecting the the best model. From the fitted models EGARCH (1,1) had the smallest AIC and BIC values followed by the GARCH(1,1) model. Model diagnostic test was conducted on the selected model EGARCH (1,1) model to determine its adequacy and goodness of fit. QQ plot was fitted to the residuals of the model and fairly straight line was produced looking roughly linear. Furthermore weighted Ljung Box Test on standard squared residuals showed the absence of correlation in the model. In conclusion, EGARCH(1,1) model is the best model for forecasting Kenyan inflation data.

Keywords: AIC, BIC, Model adequacy and heteroscedasticity

1. Introduction

A great deal of data in business, economics, engineering and natural sciences occur in the form of the time series where observations close together in time domain are more correlated than observations further apart. Chatfield (2000) defines time series as a series or sequence x_t of data points measured typically at successive times. The data points are commonly spaced in time. Time series comprise methods that attempt to understand underlying generation process of the data points and construct a mathematical model to represent the process (Mbeah, 2013). Accordingly time series involves explaining past observations in order to try and predict those in the future (Ahiati, 2007). In the same manner Chatfield (2000) defines time series as measurements made continuously through time or taken at a discrete set of time points. A significant macroeconomic issue that has posed a great challenge to most Nations and states monetary authorities today is tracking and predicting reliably the movement in the general price level. Kenya like most developing countries has had significant challenges between policy formulation, policy implementation and policy targets. In most cases it is difficult to attain the targets due to insufficient models that can be used to forecast inflation and its real determinants. Furthermore, inflation is a major monetary policy performance indicator and is useful in informing the investors, the general public and government about the trends in movement of the currency. The investor would use the forecasted inflation rates to make better investments decisions. As a result, a clear understanding of inflation forecasting technique is crucial for the success of monetary policy in tracking the movement of macroeconomic aggregates and in maintaining stable and sustainable economic growth. When the general level of price is relatively stable the uncertainties of time-related activities such as investment diminish. The consequence is the promotion of full employment and robust economic growth. Understandably government top priority is to ensure a healthy economy which promotes the well-being of its citizens. Government through its ability to tax, spend and control money supply, attempts to promote full employment, price stability and economic growth (Ezekiel et al., 2015).Time series models have many forms, and represents different stochastic process which could be linear and non-linear. Among the linear models include autoregressive (AR) model of order (p), moving average (MA) of order(q) and autoregressive moving average (ARMA) model of order (p,q). A combination of the above model produces the autoregressive moving average (ARIMA). The non-linear time series model represent or reflect the changes of variance along with time known as heteroscedasticity. With these models, changes in variability are related to and/or predicted by recent past values of the observed series. The wide variety of non-linear models include the symmetric models such as Autoregressive Conditional Heteroscedastic (ARCH) model with order (p) and Generalized ARCH (GARCH) model with order(p,q). Other asymmetric models are the Power ARCH (PARCH), Threshold GARCH (TGARCH), Exponential GARCH (EGACRH), Integrated GARCH (IGARCH), etc. All these asymmetric models have order (p,q). The above mentioned non-linear models form part of a large family of the ARCH type models. Autoregressive (AR), Moving Average (MA) and ARMA models are often very useful in modeling general time series. However, they all have the assumption of homoskedacity (or equal variance) for the errors. This may not be appropriate when dealing with financial markets variables such as stock price indices or inflation rate. Heteroscedacity affects the accuracy of forecast confidence limits and has to be handled properly by constructing appropriate non-constant variance models (Amos, 2009). Financial markets variables typically have the following three characteristics which general time series models have failed to consider: The distribution of a financial series X_t , has heavier tails than normal; values of X_t do not have much correlation but values of X_t are highly correlated. Finally the changes in X_t tend to cluster. Large (small) changes in X_t tends to be followed by large (small) changes, as documented by Mandelbrot (1963). In order to avoid the long lag structure of the ARCH model and solve negative coefficient problem, a Generalized ARCH (GARCH) model was developed by Bollersleve (1986). The GARCH model has been modified to accommodate the possibility of serial correlation in volatility. It contains a linear combination of lags of the squared residuals from the conditional variance (Goudarzi, 2010). Empirical studies have established that the GARCH is a more parsimonious model than the ARCH model (Poon & Granger, 2003) and becomes a valuable model for volatility forecasting in financial time series. In particular the GARCH (1, 1) is the most popular model for estimating and forecasting volatility. The ARCH model incorporates the autore-gressive term in return series whereas the GARCH model is superior to ARCH because it adds the general feature of conditional heteroscedasticity terms. The conventional GARCH model, which is called symmetric GARCH model is not always a perfect model and could be improved because the error terms are assumed to be normally distributed. Consequently, the symmetric GARCH model is less than adequate to fully account for some stylized fact of returns. It is as a result of such limitation that the study incorporated EGARCH model in order to improve volatility forecasting in cases of financial stylized characteristics of return series. The objective of this study was to identify the optimal model for forecasting Kenya inflation data.

Material and Methods

Inflation data was obtained from the Kenya National Bureu of Statistics (KNBS) for the month of January 1990 to December 2015. Statistical model building was conducted followed by rigorous evaluation of the data to check for the properties of the financial time series.

ARCH(q) Model

let r_t be the mean corrected return or rate of inflation e $_t$ be the Gaussian white noise with zero mean and unit time t given by H_t=(r_1,r_2,...r_{t-1}). Then the process (r_t) is ARCH(q) (eagle,1982).

$$r_t = \delta_t \in_t \quad (1)$$

$$E(r_t \mid H_t) = 0 \quad (2)$$

$$Var(r_t \mid H_t) = \delta^2{}_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-1}{}^2 \quad (3)$$

$$E(\in_t H_t) = 0 \quad (4)$$

$$Var(\in_t \mid H_t) = 1 \quad (5)$$

Equations above show that the error term e $_t$ is a conditionally standardized martingale difference defined as follows: A stochastic series r_t is a martingale difference if its expectation with respect to past values of another stochastic series x_i is zero, that is $E(r_{t+i}|x_i, x_{i-1}...,)=0$ for i=1,2,...In this type of the impact of the past on the present volatility its assumed to be a quadratic function of lagged innovation.

GARCH (1,1) Model

GARCH (1,1) model is a particular case of GARCH(p,q) model where p and q are both equal to one. let r_t be the mean corrected return ,e $_t$, be a Gaussian white noise with mean zero and unit variance. Let also K_t be the information set or history at time t given by $K_t=r_1, r_2... t_{t-1}$. The the process r_t is a GARCH (1,1) if

$$r_t = \delta_t \in_t, \in_t (0,1) \quad (6)$$

$$\delta^2{}_t = \alpha_0 + \alpha_1 r^2{}_{t-1} + \beta_1 \delta^2{}_{t-1} \quad (7)$$

Clearly from equation 6 and 7 a large past mean corrected squared return $r^2{}_{t-1}$ or past conditional variance gives rise to large values of $\delta^2{}_t$ (Tsay,2002) where α_0,α_1 and β_1 are the parameters of the model such that $\alpha_0 \geq 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $(\alpha_i + \beta_i) \leq 1$. The constraints on the parameters are to ensure that the conditional variance $\delta^2{}_t$ is positive as observed by Mbeah (2013).

EGARCH (1,1)

The exponential GARCH model was proposed by Nelson (1991) to overcome some weakness of the GARCH model in dealing with financial time series. The significant advantage of the EGARCH model is that if the parameters are negative, δ_t will be positive. The nature logarithmic of the conditional variance is assumed as a linear function of its own lagged term and allowed to vary over time (Nelson, 1991).

$$\log \delta_t^2 = c + \sum_{i=1}^{p} g(Z_{t-i}) + \sum_{i=1}^{q} \beta_j \log \delta_{t-j}^2 \quad (8)$$

Model Selection

Model selection is a vital part of statistical forecasting. Accurate forecasting is only possible if one identifies an appropriate model. Model selection involved the use of information criteria to identify the best fitting model from a set of competing models. In this case the best fitting model was the model with the smallest value of the information criteria. The information criteria adopted for the study was AIC and BIC.

Model Evaluation

After selecting the best model, the adequacy of the model fit was conducted to determine if it fits the criteria. If a GARCH or EGARCH model is correctly specified then the estimated standardized residuals should not display serial correlation, conditional heteroscedasticity or any type of non-linear dependence (Zivot, 2009). Ljung Box, Engle Arch test and QQ plot was used for model diagnostic checking.

RESULTS

Data Description

The data employed in this study comprised of 312 monthly Kenya inflation data observation from January 1990 to December 2015. Jarque Bera test for normality for the inflation and return series had a significant p value hence null hypothesis of normality was rejected. The data does not follow a normal distribution which is line with characteristic of financial time series data. The value of ADF for both the inflation-raw data and the return series had a p-value of 0.02 and 0.01 respectively. This was statistically significant an indication that inflation return series was stationary. The standard deviation of the inflation data was 10.88 meaning there was a huge variation in inflation rate. Table 1 below gives summary statistics for the data.

Table 1: Data description of inflation and return series

Statistics	Inflation	Return
Mean	11.69533	-0.01638661
Standard Deviation	10.88689	0.3863861
Kurtosis	5.785278	23.66334
Skewness	2.265754	-2.626791
Maximum	61.54215	1.654138
Minimum	-3.662444	-3.324236
Jarque Bera P value	2.20E-16	2.20E-16
ADF	-3.7712	-6.1122
ADF P value	0.02082	0.01

Transformation

As stated earlier, variances of most financial time series changes with time and thus are non-stationary. In order to generate a stationary series, we convert the inflation prices to returns by logarithmic transformation. From fig 1 below, the return series is stationary.

Testing for Serial Correlation and ARCH effects in the return series

Box-Pierce-Ljung test and Engel ARCH test has been employed for the purposes of testing the presence of serial correlation in the data. Box Pierce-Ljung was developed by Box and Pierce (1970) and modified by Ljung and Box (1978). Results from table 2 and 3 below shows the presence of ARCH effects in the return inflation data. It is apparent that there is clustering in the variance of volatility. It means that periods of high volatility are followed by periods of high volatility while periods of low volatility and followed by periods of low volatility. This is a common characteristic of financial time series data. So the data under study exhibit the presence of ARCH effects. If the time series is an outcome of a completely random phenomenon then autocorrelation should be zero for all the time lags. Otherwise one or more of the autocorrelations will be significantly non-zero. The results from Ljung Box Q-test in table 4 and 5 below shows the presences of autocorrelations in the data since the p-value are significant.

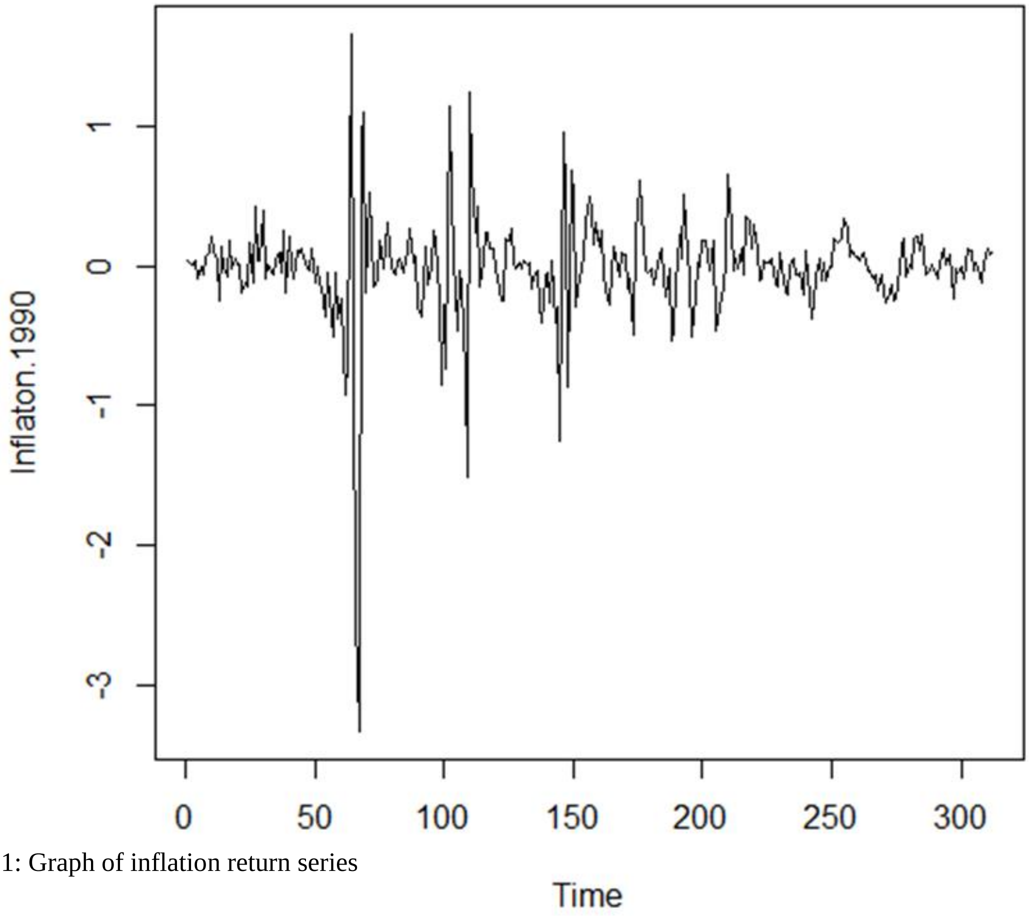

Figure 1: Graph of inflation return series

Table 2: Engle ARCH test for heteroscedasticity for return series

Lag	Chisq	Df	P-value
10	88.39	10	1.21E-14
15	87.327	15	3.118E-12
20	85.935	20	3.77E-10

Table 3: Engle ARCH test for squared for return series

Lag	Chisq	Df	P-value
10	29.654	10	0.0009757
15	29.175	15	0.01527
20	28.699	20	0.09386

Table 4: Ljung Box-Q-test for returns

Lag	Chisq	Df	P-value
10	40.441	10	0.00001417
15	52.542	15	0.00000459
20	54.134	20	0.00005526

Table 5: Ljung Box-Q-test for squared returns

Lag	Chisq	Df	P-value
10	111.04	10	2.20E-16
15	111.84	15	2.20E-16
20	113.42	20	4.66E-15

Model estimation and evaluation

Model selection and analysis

Appropriate models were selected based on the Akaike information Criteria (AIC) and Bayesian Information Criteria (BIC). R statistical software was used to perform trial and error tests in order to determine the best fitting model. The rational was to obtain a parsimonious model that captures as much variation in the data as possible. The best model should have smaller AIC and BIC values. Models with larger AIC and BIC values are considered unsuitable. Egarch model had largest negative values of the two methods adopted as the selection criteria i.e. the smallest AIC and BIC values of the fitted models as shown in table 6 below.

Table 6: Comparison of Garch and Egarch Models

Model	AIC	BIC	Log Likelihood
Garch 11	-0.11252977	-0.06465496	21.61091
Garch 12	-0.09852271	-0.0386792	20.4188
Garch 21	-0.10550257	-0.04565907	21.51115
Garch 22	1.124977	1.141961	-1104.352
Egarch 11	-0.16682	-0.10698	31.10721

Evaluation EGARCH Model

Ljung Box statistics has been used to test the null hypothesis of no autocorrelation up to a specified lag. And Engles LM statistic was also used to test null hypothesis of no remaining ARCH effects. This is clearly the case as shown in table 4.12 below which shows that there is no presence of correlation in the data. In addition, table 4.13 shows the absence of ARCH effects in the data.

Table 7: Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	Statistic	P-value
Lag[1]	0.003702	0.9515
Lag[2*(p+q)+(p+q)-1][5]	2.168663	0.5791
Lag[4*(p+q)+(p+q)-1][9]	6.067096	0.2906

Ho: No serial correlation

H1: No serial correlation

From the table 7 above there is no serial correlation exhibited by the model as the p-values are not significant at 95 percent confidence interval.

Table 8: Weighted ARCH LM Tests

Lag	Statistic	Shape	Scale	P-value
ARCH Lag[3]	2.296	0.5	2	0.12974
ARCH Lag [5]	3.636	1.44	1.67	0.20992
ARCH Lag[7]	6.879	2.32	1.54	0.09221

Ho: No presence of ARCH-effects

H1: There is presence of ARCH effects

Table 8 above shows that there is not presence of ARCH effect as the p-values are not significant at 95 percent confidence interval.

Conclusion

The study has presented us with the opportunity to have a deeper understanding of the theory of time series analysis and its application to real life situation. It is clear from the study that the Kenyan inflation rate data series is characterized with spikes, variations and trends, hence EGARCH model serves as the best in forecasting future inflation.

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