

# Axiomatic Probability Theory and Dimensional Probability Spaces

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## Abstract

The aim of this research is to evaluate the main occurrences that blossoms in mathematics especially in the field of probability. To abstract the whole course generated in this context of both dimensional probability space and axiomatic probability theories then some few cases are discussed in it. These are the ways and the modes in which we can approach these core subjects of probability. Everything that pertain these two subjects is included in order to help in learning about these two prospective probability issues.

**Keywords:** axiomatic, axiomatic probability, axiomatic probability theories, dimensional probability

## 1. Introduction

Mathematics is the key to every aspect of science. As a science, several objectives were being addressed of which probability was the main aspect of it. Meanwhile, probability as a branch of mathematics had a general layman's meaning as the likelihood of something happening. As core figure in mathematics, probability was still given an absolute scientific meaning, which was almost the same but highly differentiated because in this case, probability took figures and numbers. However, probability being a huge scientific course in mathematics it had other several fields, which were being addressed to build up this probability. It is important to note that, under this probability, several objectives were met of which axiomatic probability theory and (Chen, Y. (2010) dimensional probability spaces were discussed. Between these two sectors of probability, we had other myriad chapters under it, but the major concern was entitled majorly here.

When we take mathematical assumptions of probability, then the actual figures and definitions crop out and then we firstly define what the term "space" under probability meaning. This term "space" is defined mathematically as a set with some added structures. The set is exclusively entitled with other names of which it can also be called a "universe," of which it can be used occasionally. When talking about probability spaces we actually talk about the parent space, which is the most prominent space of which it can also take other unlimited hierarchy of spaces. This hierarchy of spaces is formed from mathematical combination of spaces. An example to illustrate the above scenario can be induced into taking the following conceptions, all inner products spaces are also formed vector spaces (Good, I. (1950) because the inner product induces a norm on the inner product space. That is self-satisfying.

It is important to note that, scientific mathematical values for instance probability theory also have another name. It is also known as Lebesgue-Rokhlin probability space of just Lebesgue space. This is an ambiguous term relating this latter to taking an individual authentic beholder's name. The man who introduced this subject of probability theory was called Vladimir Rokhlin, and this was in the year 1940. This man gave mutual satisfying assumptions to this matter of probability theory, assumptions which were satisfied using probability space. However, for probability theory to be actually known, an informal definition was given to it when meant that probability theory is probability space, (Gudder, S. (1970). (Pg 53-129)) This was consisting of an interval of finite or countable number of atoms.

Furthermore, the subject spaces take a mathematical point of view. Mathematics has taken ages to grow and to cite its introduction will take up to years Before Christ, (BC). Those are the times when mathematics took the name of "ancient mathematics." Probability was still being adorned by the scientific scholars of those times. As per the term "space," it was defined scientifically as geometric dimensional space that is being observed every day. Just as it was cited as probability matters introduced in BC years, (Gudder, S. (1970). Pg 53-129, so as axiomatic methods which were being used as the main research tools especially since Euclid in about 300 BC. This dates mathematics especially this branch of probability as old enough in the field of science.

In addition, Rene Descartes adopted the method of coordinates, which is analytical geometry, in the year 1637. Those were times when geometric theorems were being treated as an absolute objective truth knowable through

intuition and reason, and these were similar to objects of natural science and axioms of which were being dealt with as obvious implications of definitions. Similarly, everything in these subjects had micro subjects and to reveal them all it actually takes extra efforts to enlist them. For the purpose of the above subject, geometric figures had two equivalence relations which were being used. These figures were “congruence” and “similarity.” When the subject congruence is taken into importance, we clearly find a multiple of components of figures that can be turned into congruent ones. These includes; translations, rotations and reflection transformation of figures into congruent figures, homothetic, into similar ones. A clearly working example will include, ((Gudder, S. (1970). Pg 53-129), take an example of circles, all circles are mutually similar, but eclipses are not similar to circles. A third equivalence relation which was introduced by Gaspard Monge in the year 1775 occurs in projective geometry. According to this mathematician, not only eclipses but also parabolas and hyperbolas turn into circles under appropriate transformations, because they are all protectively equivalent.

When we are addressing mathematics as a science, it will actually show the bigger branches it has. It all starts with geometry which is complete pure science, sketches, and drawings which will be including the hyperbolas not forgetting the natural science about probability which is main or major subject of the study in this. It is covering the major affluent of probability which includes axiomatic probability theory and dimensional probability spaces. Though this unique science has taken ages to grow from ancient times to date, it is more expressed mathematically using scientific expressions like alpha.

It is important to note that, mathematics takes out different core objects with different structures but in this case of geometry is not far much different. This is mainly because, the relations between two geometries, which are Euclidian and projective, show that mathematical objects are not given with their structures. However, they take a different view and this is; each mathematical theory describes objects by their properties, precisely those that are put as axioms at the foundation of this individual theory.

So many situations occurred in 19<sup>th</sup> century especially in this mathematical field of geometry. It claims that in some geometries, the sum of angles of a triangle is well defined but different from the classical value of 180 degrees. This argument was brought out by several mathematician scientists who supported their adventurous arguments with firm affirmatives. These scientists included; Nikolai Lobachevski back in the year 1829, Janos Bolyai in 1832. These two mathematicians argued that the sum depends on the triangle and is always less than 180 degrees. In addition, several other mathematicians argued on this “model” trying to justify it as right. These mathematicians included; (Chow, Y., & Teicher, H. (2012, n, p) Eugenio Beltrami in the year 1868 and a fellow other mutual mathematician called Felix Klein in the year 1871. These two mutual performing mathematicians contributed to this core model and obtained Euclidian “model” of the non-Euclidian hyperbolic geometry and therefore completely justifying this prospective theory.

Things became quite justifiable to quit the existence of probability in this field of the model above, the Euclidian model. This model took charges in the field as a whole covering geometry and space perspectives of probability. For instance, the Euclidean model of a non-Euclidean geometry was a clever choice of some objects existing in Euclidean space and some of those, relating these objects that were satisfying all axioms which meant “all theorems” of non-Euclidean geometry. To clearly exemplify the above relationship between these factors we ought to see how they interfere and mingle with each other. However, these Euclidean objects and relations “play” the non-Euclidean geometry like contemporary actors playing an ancient performance. This is mainly because; the relations between actors only mimic relations between the characters in the individual play. This nonprofessional’s comparison is relating to Euclidean relations to non-Euclidean relations to make the subject clearly understandable. (Chow, Y., & Teicher, H. (2012, n, p) Therefore, using this contemporary explanation, we can say that, chosen relations between the chosen objects of the Euclidean model only mimic the non-Euclidean relations. This is mainly because, is shows that the relations between objects are essential in mathematics, while the nature of objects is not.

Space seems to be a wide range of Intel with multifaceted definitions, especially with the term probability space. A probability space is also defined as a measure space such that the measure of the whole space is equal to one. Concerning this matter, we can say that a product of any family, whether finite or infinite, of probability spaces, is a probability space. By so doing, in the affirmed context obviously has a pullback contrasting factors. This is well suited and therefore, in contrasting this event, for measure spaces in generally, only the products of finitely many spaces are defined. Otherwise, there are those many infinite dimensional probability measures especially Gaussian measures but lack infinite-dimensional Lebesgue measure.

The modulus of standard probability spaces is also useful indeed. To paraphrase this concept, we can complement that every probability measure on standard measurable spaces leads to a well standard probability

space. This means that the product of a sequence, whether finite or infinite, of standard probability spaces is a standard probability space. All non-atomic standard probability spaces are mutually isomorphic where one of them is interval with Lebesgue spaces. By employing the concept of geometry, then this concept becomes invalid because the spaces are completely less geometric. Other words, the ideas of dimension, are applicable to all other spaces in either one form or another and this does not apply to measurable measures and probability.

Most are the times when we apply general knowledge to pledge the concepts of mathematical factors and figures and hence we can take mathematical methodologies to analyze mathematical terms. Using the mathematical logics and assumptions, we can bring out the concept of descriptive set theory as a major clause in history of probability. Mathematically, we say that descriptive set theory is the study of “well-behaved” subsets of the real line and other polished substantial spaces. As well as being, one of the most primary areas of research in set theory, it. This descriptive set theory is termed as one (Kolmogorov, A. (1950), n.p) of the most primary areas of research in set theory. There are so many distinctive areas of study below the latter. However, this descriptive set theory has also distinct applications to other fields of mathematics such as functional analysis, ergodic theory, the study of operator algebras and group actions and mathematical logic

## 2. Polish Spaces

The term space as addressed in the probability still carries out its value with prospective branches. Well termed above sub-topic “polished spaces” as a relevant key subject. However, to exemplify this subject with a proper definition several factors are ordained and therefore “polished spaces” will be equipped with special definition and well paraphrased properties distinction this space body. This highly termed “polished space” is a second countable topological space that is amortizable with a complete metric. Further explanations to confect the essence behind this “polished spaces” are as well elaborated as a complete separable metric space whose metric has been forgotten. For instance, we have the real line and the Baire space. The Cantor space and the Hilbert cube as major examples to explain and bring out the image of the subject “polished spaces.”

However, polished spaces have been credited as a major part of mathematics. It is, however, granted big chances especially in the mathematical discipline of general topology and it is in this region where the above golden definition of “polished space” comes from. They gained this name respectively because they were firstly extensively studied by topologists and logicians who included; Sierpinski, Kuratowski, Tarski, and others. It is important to note that, these kinds of spaces are major field of study especially nowadays because they are the primary settings of distinctive set theory, and also they include the study of Boreal equivalence relations. In addition to this matter, “polished spaces” are also a convenient setting for more advanced measure theory, particularly in probability theory. A major point of concern is about some spaces where they are not complete in the usual metric may be polished for example; the open interval. However, between any two infinite Polish spaces, there is a Boreal isomorphism, that is, a bijection that preserves the Boreal structure. Moreover, every uncountable Polish space has the cardinality of the continuum. To explain this further, we can term the generalizations of Polish spaces as Lusin spaces, Suslin spaces and finally Radon spaces.

Every individual figure in mathematics is termed to have a distinctive feature or features that accrue it different from the rest. However, these polished spaces have special features, which are universal and prompt to show other subjects of mathematics that it is different from them. As a result, these features and characteristics were identified by other different topologists whose arguments if brought together made “polished spaces” different.

Property number one was ordained in Alexandrov’s theorem. This property was expressed mathematically with digits and numbers to express the actual logic behind it. From this theorem, polished spaces were identified to have a special feature that if for example;  $X$  is a polish then so is any  $G$ , subset of  $X$ .

Secondly, another property of polished space was taken from Cantor-Bendixson theorem, which was also stated in terms of digits and numbers. This affirming property stated that; if  $X$  is a polish then any closed subset of  $X$  could be written as the disjoint of a perfect subset and a countable open subset.

The third property of this polished space was also claimed form conversing to Alexandrov’s theorem. However, this property was affirming the distinct nature of polished space by claiming that, a subspace of a polished space  $P$  is a polish if and only if  $Q$  is the intersection of a sequence of open subsets of  $P$ .

The fourth well-stated property of this polished space claimed that, a topological space  $X$  is polish if and only if  $X$  is homeomorphism to the intersection of a sequence of open subsets of the cube, where ‘ $I$ ’ is the unit interval and  $N$  is the set of natural numbers.

The final property accrued to this polished space is got relatively from Hilbert cube. This property states that

every G-subset of the Hilbert cube is a polish space. Conversely, every polish space is homeomorphism to a G-subset of the Hilbert cube.

After having done up with the distinctive properties of this polish spaces then we can close down to put through some of the spaces that are ever a polish. Therefore, the following are spaces which are a polish; the closed subsets of a polish space. Secondly, open subsets of a polish space, products and disjoint unions of countable families of polish spaces. Another example is the locally compact spaces that are amortizable and countable at infinity. Of this finite definition, we cannot forget the countable intersections of polish subspaces of a Hausdorff topological space as a polish and finally the set of non-rational numbers with the topology induced by the real line. The above examples driven are a polish space. The above definitions of examples of existing polished spaces are far much distinct in their own nature.

### 3. Boreal Sets

The Boreal set is another huge factor of study upon this matter. We can clearly state Boreal sets and provide them with a complete definition and factors districting it from other factors and figures in mathematics as a science of probability. The definition of this subject, Boreal can have a mathematical definition too. Mathematically, we can say that a Boreal set is any set in a topological space that can be formed from open sets, which are equivalently from closed sets, though the operations of countable union, countable intersection and relative complement. The actual man to have a special naming of these Boreal set was Emile Boreal. This set's name took after him. This Boreal set can be illustrated through a multivariate collection of numbers or digits only of just confect the meaning behind it. To exemplify this, we can probably say that for a topological space X, the collection of all Boreal sets on X forms an  $\sigma$ -algebra known as Boreal algebra or Boreal  $\sigma$ -algebra. The Boreal algebra on X is the smallest [σ-algebra](#) containing all open sets either equivalent or all closed sets.

Just with mere special features of Boreal set, we can identify that; every open subset of X is a Boreal set and finally, if A is a Boreal set, so is. That is, the classes of Boreal sets are closed under complementation. However, if  $A_n$  is a Boreal set for each natural number n, then the union is a Boreal set. That is, the Boreal are sets under constable unions.

The above topic on Boreal sets is observed to take the wave of mathematical ideologies and definitions as compared to other (Wootters, W. K. (1981). N.p) subjects of study. However, to put polish sets in Boreal sets we can say that a fundamental result shows that any two uncountable polish spaces X and Y are Boreal isomorphic. This means that there is a bijection from X and Y such that the pre-image of any Boreal set is Boreal, and the image of any Boreal is Boreal. Therefore, according to this explanation, we can say that it gives additional justification to the practice of restricting attention to Baire space and Cantor space, since these and any other polish spaces are all isomorphic at the level of Boreal sets. By so doing, this respective area of effective descriptive set theory combines the methods of descriptive set theory with those of generalized recursion theory. This theory focuses on the lightface analogues of hierarchies of classical descriptive theory. Therefore, this gives an upper hand in studying of generalized recursion theory than Boreal hierarchy, and analytical hierarchy instead of projective hierarchy.

Likewise, in probability theory, we have a probability space or a probability triple. These terms can be illustrated as a mathematical construct that a real world process or experiment consisting of states that occur randomly. When this experiment is ongoing then we figure out the trend of occurrences in our minds, that is, we construct a probability space with a specific kind of situation in our minds. (Chow, Y., & Teicher, H. (2012, n, p) Once we experience an occurrence of a situation in a time, then we can predict the occurrence of the same situation and only find that the probabilities are all the same.

Probability is made of different parts, which are only three respective parts. One part of probability is a sample space; where a sample space is the set of all possible outcomes. The second part is a set of events, where each event is a set containing zero or more outcomes. Finally, is the assignment of probabilities to the events; which are a function from events to probabilities?

Every moment we carry out an event, an individual outcome result and that is like tossing up events. Outcomes generated maybe (Wootters, W. K. (1981). n.p) a little of practical use and more complex events are used to characterize these events outcomes and these collections of all such events is called [σ-algebra](#).

However, there is the need to specify each event's likelihood of happening. Finally, there is the need to specify each event's likelihood of happening. This is actually done using a methodology called probability measure function. When doing probability we put down the probability space and more naturally, the outcome are

selected in a single outcome from a sample space. All the events in that contain the selected outcomes and are said to have occurred. The selection performed by nature is carried out in a way that if the given experiments were to be repeated or done severally an infinite number of times, the resulting frequencies of occurrence of each individual event would coincide with the probabilities prescribed by the function. This notion of probability space was introduced by a Russian mathematician called Andrey Kolmogorov alongside with the concept of axioms of probability. This was back in the years 1930s. Days have gone and still alternative approaches of axiomatic of probability theory exist

### 3.1 Discrete Case

Another concept of probability configured out using this concept of discrete case. However, discrete probability theory needs only at most countable sample spaces. Probabilities can be ascribed to points of by the probability mass function such that all subsets of can be treated as events. The probability measure takes the simple form. The greatest  $\sigma$ -algebra describes the complete information. In general,  $\sigma$ -algebra corresponds to a finite or countable partition, the general form of an event being. The case is permitted by the definition, but rarely used, since such can be excluded from the sample space.

Let  $Y$  be the random variable, which will represents the toss of a coin. In this case, there are two possible outcomes, which we can label as H and T. (Burks, A. W. (1979). N. p) unless we have reason to suspect that the coin comes up one way more often than the other way, it is natural to assign the probability of half to each of the two outcomes. In both of the above experiments, each outcome is assigned an equal probability. This would certainly not be the case in general. For example, if a drug is found to be active thirty percent of the time it is used, we might assign a probability 0.3 that the drug is effective the next time it is used and 0.7 that it is not effective. This last example illustrates the intuitive frequency concept of probability. That is, if we have a probability  $P$  that an experiment will result in outcome  $A$ , then if we repeat this experiment a large number of times we should expect that the fraction of times that  $A$  will occur is about  $P$ . To check intuitive ideas like this, we shall find it helpful to look at some of these problems experimentally. We could; For example toss a coin a large number of times and see if the fraction of times heads turns up is about half. We could also simulate this experiment on a computer.

We will be particularly interested in repeating a chance experiment a large number of times. Although the cylindrical die would be a convenient way to carry out a few repetitions, it would be difficult to carry out a large number of experiments. Since the modern computer can do a large number of operations in a very short time, it is natural to turn to the computer for this task

### 3.2 General Case

So many different points of views have been used when it comes to the concept of probability. However, credit goes solemnly to micro studies of this huge course of study. For example, to elaborate this concept of axiomatic theory and dimensional probability spaces we can figure out new methodologies and perceive this new study at a general point of view. That is exactly possible and below this sub unit micro study we can talk about the general cases of these two concepts; axiomatic theory and dimensional spaces of probability. Therefore this study can take the general shape of study as long as the concept is being supported with huge claims or working examples. That will be much better to stand. Take an example with the below statements and examples relating the matter with general cases.

In this case, we can look and embrace the concept of axiomatic probability theory and probability space is viable on the angles of a general case. This can actually be shown using a proper working example. For example, If  $\Omega$  is uncountable, it may happen that  $p(\omega) \neq 0$  for some  $\omega$ ; such  $\omega$  are called atoms. They are an at most countable set of which maybe empty, whose probability is the sum of probabilities of all atoms. Then, if this sum is equal to one, then all other points can be safely removed from the sample space, returning us to the discrete case. Otherwise, (Meakin, P. (1983). N. p) if the sum of all probabilities of atoms is between zero and one, then the probability space decomposes into a discrete or atomic part which maybe empty and a non-atomic part.

In the axiomatic approach to probability, random experiments are considered, sample space and also other events associate with different experiments. In our day to day encounter, we hear more of “chance” compared to “probability.” It is important to note that mathematics basically deals with qualifying things. Therefore, the probability theory quantifies chances of non-occurrence or occurrence of events. Also, one notable characteristic about probability is that it is mostly or can only be used to experiment a situation whereby the total number of



outcomes are known. Therefore, to apply probability in a situation, one should be aware of the total of possible outcomes from a given experience. Axiomatic probability is, therefore, another way of describing the possible probabilities of an event. In this case some axioms are predefined before probabilities are assigned. This is done to help in quantizing an event, therefore, easing the calculation of non-occurrence or occurrence of the event.

### 3.3 Non-atomic case

When it comes to the word atomic, everything feels like falling apart for non-mathematics lovers but the concept behind it is very cheap and is grossing perfectly to understand. Although when it comes to these strange micro scientific terms feels like heavy concepts and it is truly supported with heavy claims of working probability and scientific mathematical terms. Take a look at these concepts supporting the non-atomic cases of axiomatic probability theory and dimensional space probability. In this case, (Meakin, P. (1983). N. p) to approach this concept of axiomatic probability theory and space is through or using the concept of non-atomic case. This can be illustrated through an example to help exemplify the case study. For example; if  $B(\omega) = 0$  for all  $\omega \in \Omega$ , then in this case,  $\Omega$  ought to be uncountable. This is because otherwise  $B(\Omega) = 1$  which could not be satisfied, therefore, equation (\*) fails. Here, the probability of a given set is not the sum total over its elements, as the summation is only defined for the countable amount of elements. This helps in making the probability space theory much more technical. A formulation stronger than summation, measure theory is then applicable. Initially, probability is regarded to some “generator” sets.

Then a limiting procedure allows assigning probabilities to sets that are limits of sequences of generator sets, or limits of limits, and soon. All these sets are the  $\sigma$ -algebra. A set belonging to this group is considered measurable. Generally, they perceived to be are much more complicated compared to the generator sets, but again much better than non-measurable sets.

## 4. Complete probability space

Another major concept induced in the study of axiomatic probability theory and probability space is about a complete probability space. A probability space is said to be a complete probability space if for all with and all one has. Most times, the study of probability spaces is basically restricted to complete probability spaces. In probability theory, (Parzen, E. (1960, n. p) a standard probability space, also called Lebesgue–Rokhlin probability space or just Lebesgue space which is a probability space that satisfies certain assumptions which were introduced by Vladimir Rokhlin in the year 1940. Informally, it is a one of the probability space that consist of an interval or countable or a finite number of atoms.

The theory of standard probability spaces was started by Von Neumann in 1932 and shaped out later by Vladimir Rokhlin in the year 1940. It is important to note that Rokhlin showed the unit interval endowed by the Lebesgue measure has major advantages over the general probability space, yet it can be effectively substituted for most of these in probability theory. Additionally, the dimensions of unit interval is not an obstacle, as it was clear already to Norbert Wiener. Thereafter, he constructed the Wiener process, which was also called the Brownian motion, in the form of a measurable map from the unit interval to the space of continuous functions isomorphism. An isomorphism between any two probability spaces is an invertible map such that and both are measures preserving map. Two probability spaces are isomorphic, if there is an isomorphism between them.

## 5. Isomorphism module zero

This is still another scientific explanation of this concept of axiomatic theory and dimensional spaces of probability. This concept “isomorphism module zero” is clearly elaborated here below.

This is a concept which takes part as a modulus of study in this probability axiomatic theory and space of which is analyzed as a mere concept of isomorphism module zero. To paraphrase this better, then we need to talk about probability space as a point of relations between its concepts with isomorphism. We can say that two probability spaces, are isomorphic, if only there exist null sets, such that the probability space are isomorphic and if only they are being endowed naturally with sigma-fields and probability measures.

## 6. Standard probability space

With still another major point of concern when it comes to axiomatic probability theory and dimensional space

probability is glued hand in hand with this concept of standard probability space and therefore it does not exclude itself from it. However, this is majorly discussed here below how it is a factor that is playing with the two concepts of probability. It also contains the scientific terminologies just like any other science and a branch of probability.

This subject has a very short cited dating in the concept of probability yet it is another concept which is still brought out in terms of standard probability space. This concept claims that a probability space is standard, if it is isomorphic to an interval with Lebesgue measure, a finite or countable set of atoms, or a combination of both, especially the disjoint union. The measure is assumed finite, not necessarily probabilistic.

### **7. A criterion of standard**

We are not forgetting about this elated aspect of both axiomatic probability theory and dimensional probability space. Still on the same subject several things accrue to latter to validate it. For instance, a standard of a given probability space is equivalent to a certain property of a measurable map to a measurable space. The funniest bit is that the answer does not always depend on the choice. (Rényi, A. (1955). 6(3-4), 285-335.) This fact is quite useful and one may adapt the choice or be given but there is no need to examine all those cases.

Otherwise, it may be convenient to examine a random variable vector of a random sequence or a sequence of events treated as a sequence of two-valued random variables. The question of how its existence will be addressed afterwards. The probability space is assumed to be complete, otherwise it cannot be standard.

### **8. Regular conditional probabilities**

Another factor behind the back of everything about axiomatic probability theory and dimensional probability space is regular conditional probabilities. Actually these regular conditional probabilities have a good back ground to this exclusive subject. Therefore, we mainly consider the main or the discrete set up as the most prominent factor of consideration. However, (Pistone & Sempi, C. (1995). Pg 1543-1561.) It is from this concept where the conditional probability is another kind of probability measure. Generally, this conditional expectation may be treated like or as the usual expectation with respect to the conditional measure.

However, we also have the non-discrete part petition where the conditioning is actually often treated indirectly. That is, the conditional expectations. Therefore, as a result of this conditional expectation, a number of well-known facts which have special conditional counter parts. This can also be well illustrated prospectively using a working example. This can well showed down with the, following examples including; linearity of the expectation; Jensen's inequality; holder's inequality; the monotone convergence theorem etc. Take a situation where you are granted with a random variable on a probability space, (Pistone & Sempi, C. (1995). Pg 1543-1561.) it is natural to try constructing a conditional measure that is from the conditional distribution given from a class. Sometimes, under different occasions you can find different possibilities or impossibilities and this is too finite to trust. Take an instance with standard probability space; this is directly possible as compared to other events and a well-known canonical system of measures, which are basically the same as conditional probability measures. The conditional Jensen's inequality is the usual Jensen's inequality applied to the conditional measure. The same holds for many other facts.

### **9. Measurable preserving transformation**

On the final prospect of this subject axiomatic probability theory and dimensional probability space we consider the subject measurable preserving transformation. Just like any other subject regarding to this issue, everything has been classified into its accordance and measurable preserving transformation deals majorly with the impacts of null sets and its relations to general probability space. This preserving measure nature has a wide coverage and the subject is illustrated as follows; for example, imagine that you are given two probability spaces and on top of that measure preserving map is also handed over to you. Then what conclusion does it occur lastly? Of course that is the biggest question you can ask yourself but to answer it, then you can consider that the image will never cover the whole, it will indeed miss a null set. Sometimes we can also drive other conclusions from this example. (Rényi, A. (1955). 6(3-4), 285-335.)

This is mainly because according to the results from that probability event will seem to be equal to one but that is not actually so. The reality of outer measurements will be equal to one but the inner measures will be greatly differentiated otherwise, differ. We can also come up with our own conclusions assumptions to this event and say

that if the probability spaces are standard then it is one to one hence it satisfies fully the concept. Therefore it qualifies it to be a measurable preserving transformation measure. There are multivariate ways which are perfectly coherent ways to ignore these measures. Striving extremely hard to get rid of these null sets, mathematicians of ancient's times had to come up with a best way to get rid of these null sets. However, to achieve this task they had to come up with use of equivalence classes of measurable sets or functions. These equivalence classes of measurable subsets of a probability space formed complete Boolean algebra called the measure algebra or the metric structure as a synonym. Therefore, every measure of preserving transformation map leads to a (Chen, Y. (2010), n.p) homeomorphism of measures of algebra. It may seem that every homomorphism of measuring algebras has to correspond to some or at least one measure preserving map. Contrary, it is not so. However, for the standard probability spaces each corresponds to some.

## 10. Conclusion

In conclusion, this Study on probability is addressed widely especially the core subjects of dimensional probability space and axiomatic probability theory. It is discussed perfectly with severely well managed working scientific mathematical samples. These examples are situated in each and every core study in this research and these main study divisions included, the general case of probability, the subject of polished spaces, the non-atomic case, discrete cases, isomorphism module zero, standard probability zero, a probability criterion of standard, regular conditional probabilities and finally measurable transformation conditional. These case studies explain this question context intensively

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