On New Forms of Generalized Homeomorphisms

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Abstract
The purpose of this paper is to introduce two new classes of homeomorphisms namely $\hat{\omega}$-homeomorphism and $\hat{\omega}^*$-homeomorphism and investigate some of their properties in topological spaces. Moreover we have shown that one of these classes has a group structure.

Keywords: $\hat{\omega}$-closed sets, $\hat{\omega}$-homeomorphisms, $\hat{\omega}^*$ - homeomorphisms.

1. Introduction

The notion homeomorphism plays a dominant role in topology. Many researchers have generalized the notion of homeomorphisms in topological spaces. Maki et al [7] introduced g-homeomorphism and gc-homeomorphism and Devi et al[2]introduced generalized semi-homeomorphism and semi-generalized homeomorphism in topological spaces. In this paper we introduce new classes of homeomorphisms namely $\hat{\omega}$-homeomorphism and $\hat{\omega}^*$-homeomorphism and investigate some of their properties in topological spaces. We prove that $\hat{\omega}$-homeomorphisms and $\hat{\omega}^*$-homeomorphisms are independent notions. It turns out that the set of all $\hat{\omega}^*$-homeomorphisms forms a group under composition of mappings.

2. Preliminaries

Throughout the paper (X,τ) and (Y,σ) and (Z,η) (or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed.
We recall the following definitions which are useful in the sequel.

Definition 2.1 A subset A of a topological space (X,τ) is called δ-closed [10] if A=cl$^{\delta}$(A) where cl$^{\delta}$(A) = {x$\in$ X :int (cl(U))$\cap$A$\neq\phi$, U$\in\tau$ and x$\in$U}.The complement of δ-closed set is called δ-open set.

Definition 2.2 A subset A of a topological space (X,τ) is called an a-open set [4] if A$\subset$int (cl (int$^{\delta}$(A))).The complement of an a-open set is called an a-closed set.The a-closure of a subset A of X is the intersection of all a-closed sets containing A and is denoted by acl(A).

Definition 2.3 A subset A of a topological space (X,τ) is called a
(i) generalized closed (briefly g-closed) [8] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X.
(ii) generalized semi-closed (briefly gs-closed) [8] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X.
(iii) α-generalized closed (briefly αg-closed) [8] if α cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X.
(iv)generalized α-closed (briefly $g\alpha$-closed) [8] if α cl(A)$\subseteq$U whenever A$\subseteq$U and U is α-open in X.
(v) $\hat{g}$-closed [9] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is semi-open in X.
(vi) $\alpha\hat{g}$-closed [3] if α cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\hat{g}$-open in X.
(vii) $\hat{\omega}$-closed [8] if acl(A)$\subseteq$U whenever A$\subseteq$U and U is $\alpha\hat{g}$-open in X.

The complement of $\hat{g}$-closed (resp.g-closed, gs-closed, αg-closed, $g\alpha$-closed, $\alpha\hat{g}$-closed and $\hat{\omega}$-closed) set is called $\hat{g}$-open (resp. g-open, gs-open, αg-open, $g\alpha$-open, $\alpha\hat{g}$-open and $\hat{\omega}$-open) .

Definition 2.4 A function f : (X, τ) $\rightarrow$ (Y, σ) is called
(i) a-continuous [4] if $f^{-1}(V)$ is a-open in X for every open set V in Y.
(ii) a-closed [6] if f (F) is a-closed in Y for every closed set F in X.
(iii) $\hat{\omega}$-closed [6] if f (F) is $\hat{\omega}$-closed in Y for every closed set F in X.
(iv) $\hat{\omega}$-irresolute [6] if $f^{-1}(V)$ is $\hat{\omega}$-closed in X for every $\hat{\omega}$-closed set V in Y.
(v) $\hat{\omega}$-continuous [5] if $f^{-1}(V)$ is $\hat{\omega}$-closed in X for every closed set V in Y.

Definition 2.5 A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be
(i) g-homeomorphism [7] if f is bijective, g-open and g-continuous.
(ii) gs-homeomorphism [2] if f is bijective, gs-open and gs-continuous.
(iii) αg-homeomorphism [1] if f is bijective, αg-open and αg-continuous.
(iv) $g\alpha$-homeomorphism [1] if f is bijective, $g\alpha$-open and $g\alpha$-continuous.

3. $\hat{\omega}$- homeomorphisms

In this section we introduce the concept of $\hat{\omega}$-homeomorphisms and study some of their properties.

Definition 3.1 A bijective map f : (X, τ) $\rightarrow$ (Y, σ) is called $\hat{\omega}$-homeomorphism if f is both $\hat{\omega}$-continuous and $\hat{\omega}$-closed.

Example 3.2 Let X= {a,b,c,d} = Y, τ = { ϕ, {a}, {b}, {a,b}, {a,b,c}, X } and σ = { ϕ, {a},{b},{a,b},Y}.Let f : (X, τ) $\rightarrow$ (Y, σ) be the identity function. Then f is an $\hat{\omega}$-homeomorphism.

Theorem 3.3 Let f : (X, τ) $\rightarrow$ (Y, σ) be a bijective and $\hat{\omega}$-continuous map.
Then the following are equivalent.
(i) f is an $\hat{\omega}$-closed map.
(ii) f is an $\hat{\omega}$-homeomorphism.
(iii) f is an $\hat{\omega}$-open map.

Proof:
(i) $\Rightarrow$ (ii)Let f be an $\hat{\omega}$-closed map. By hypothesis f is bijective and $\hat{\omega}$-continuous.Hence f is an $\hat{\omega}$-homeomorphism.
(ii) $\Rightarrow$ (iii) Let f be an $\hat{\omega}$-homeomorphism. Then f is $\hat{\omega}$-closed. By theorem 3.31[6], f is $\hat{\omega}$-open.
(iii) $\Rightarrow$ (i) Let f be an $\hat{\omega}$-open map. By theorem 3.31[6], f is $\hat{\omega}$-closed.

Theorem 3.4 Every $\hat{\omega}$-homeomorphism is an αg–homeomorphism.

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an $\hat{\omega}$-homeomorphism. Then f is bijective, $\hat{\omega}$-continuous and $\hat{\omega}$-closed. Let V be a closed set in Y. Since f is $\hat{\omega}$-continuous, $f^{-1}(V)$ is $\hat{\omega}$-closed in X. Since every $\hat{\omega}$-closed set is αg-closed [8], $f^{-1}(V)$ is αg-closed in X which implies f is αg-continuous.

Let W be a closed set in X. Since f is $\hat{\omega}$-closed, f (W) is $\hat{\omega}$-closed in Y and so f (W) is αg-closed in Y which implies f is αg-closed. Thus f is an αg–homeomorphism.

Theorem 3.5 Every $\hat{\omega}$-homeomorphism is an $g\alpha$–homeomorphism.

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an $\hat{\omega}$-homeomorphism. Then f is bijective, $\hat{\omega}$-continuous and $\hat{\omega}$-closed. Let V be a closed set in Y. Since f is $\hat{\omega}$-continuous, $f^{-1}(V)$ is $\hat{\omega}$-closed in X. Since every $\hat{\omega}$-closed set is $g\alpha$-closed [8], $f^{-1}(V)$ is $g\alpha$-closed in X which implies f is $g\alpha$-continuous.

Let W be a closed set in X. Since f is $\hat{\omega}$-closed, f (W) is $\hat{\omega}$-closed in Y and so f (W) is $g\alpha$-closed in Y which implies f is $g\alpha$-closed. Thus f is an $g\alpha$–homeomorphism.

Theorem 3.6 Every $\hat{\omega}$-homeomorphism is an gs –homeomorphism.

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an $\hat{\omega}$-homeomorphism. Then f is bijective, $\hat{\omega}$-continuous and $\hat{\omega}$-closed. Let V be a closed set in Y. Since f is $\hat{\omega}$-continuous, f^{-1}(V) is $\hat{\omega}$-closed in X. Since every $\hat{\omega}$-closed set is gs-closed [8], f^{-1}(V) is gs -closed in X which implies f is gs-continuous.

Let W be a closed set in X. Since f is $\hat{\omega}$-closed, f (W) is $\hat{\omega}$-closed in Y and so f (W) is gs -closed in Y which implies f is gs-closed. Thus f is a gs –homeomorphism

Remark 3.7 The converses of theorem 3.4, 3.5 and 3.6 are not true as shown by the following example.

Example 3.8 Let X={a,b,c}=Y, τ ={ ϕ ,{a,b},X} and σ ={ ϕ ,{a},{a,b},Y}.Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=a, and f(c)=c.Then f is not an $\hat{\omega}$-homeomorphism since there exists a closed set {c} of X such that f({c}) ={c} is not $\hat{\omega}$-closed in Y. However f is a αg-homeomorphism, $g\alpha$ -homeomorphism and gs- homeomorphism.

Definition 3.9 A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be an a-homeomorphism if f is both a-continuous and a-closed.

Theorem 3.10 Every a-homeomorphism is an $\hat{\omega}$-homeomorphism.

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an a-homeomorphism. Then f is bijective, a-continuous and a-closed. Let V be a closed set in Y. Since f is a-continuous, f^{-1}(V) is a-closed in X. Since every a-closed set is $\hat{\omega}$-closed [8], f^{-1}(V) is $\hat{\omega}$-closed in X which implies f is $\hat{\omega}$-continuous.

Let W be a closed set in X. Since f is a-closed, f (W) is a-closed in Y and so f (W) is $\hat{\omega}$-closed in Y which implies f is $\hat{\omega}$-closed. Thus f is an $\hat{\omega}$–homeomorphism

Remark 3.11 The converse of theorem 3.10 is not true as shown by the following example.

Example 3.12 Let X={a,b,c,d}=Y, τ ={ ϕ ,{a},{b},{a,b},X} and σ ={ ϕ ,{a,b},Y}.Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=a, f(c)=d and f(d)=c. Then f is an $\hat{\omega}$-homeomorphism but not an a-homeomorphism since there exists a closed set {c,d} of X such that f({c,d}) ={c,d} is not a-closed in Y.

Remark 3.13 The following examples shows that the concept of homeomorphism and $\hat{\omega}$-homeomorphism are independent of each other .

Example 3.14 Let X={a,b,c,d}=Y, τ ={ ϕ ,{a},{b,c},{a,b,c},X}and σ ={ ϕ ,{a,b,c},Y}.Define a function f :(X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=c, f(c)=a and f(d)=d.Then f is an $\hat{\omega}$-homeomorphism but not a homeomorphism since there exists an open set {b,c} of X such that f({b,c}) ={a,c} is not open in Y.

Example 3.15 Let X= {a,b,c}=Y, τ ={ ϕ ,{c},{a,c},{b,c},X} and σ ={ ϕ ,{b}, {a,b},{b,c},Y}. Define a function f :(X, τ) $\rightarrow$ (Y, σ) by f(a)=c, f(b)=a, and f(c)=b.Then f is a homeomorphism but not an $\hat{\omega}$-homeomorphism since there exists a closed set {b} of X such that f({b}) ={a} is not $\hat{\omega}$-closed in Y.

Remark 3.16 The following examples shows that the concept of g-homeomorphism and $\hat{\omega}$-homeomorphism are independent of each other .

Example 3.17 Let X={a,b,c}=Y, τ ={ ϕ ,{a},X} and σ ={ ϕ ,{a},{b},{a,b},{a,c},Y}.Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a)=a, f(b)=c,and f(c)=b.Then f is a g-homeomorphism but not an $\hat{\omega}$-homeomorphism since there exists a closed set {b} of Y such that f^{1}({b}) ={c} is not $\hat{\omega}$–closed in X.

Example 3.18 Let X={a,b,c,d}=Y, τ ={ ϕ ,{a},{b},{a,b},{a,b,c},X} and σ ={ ϕ ,{a}, {b}, {a,b},{a,b,c},{a,b,d},Y}.Let f : (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is an $\hat{\omega}$-homeomorphism but not a g-homeomorphism since there exists an open set {a,b,d} of Y such that f^{1}({a,b,d}) = {a,b,d} is not g-open in X.

Remark 3.19 From the above discussions we have Figure -1 where

A ⟶ B represents A implies B and A ⟶̸ B represents A does not imply B .

4. $\hat{\omega}^*$ –homeomorphisms

In this section we introduce another class of homeomorphisms called $\hat{\omega}^*$ –homeomorphisms and investigate some of their properties.

Definition 4.1 A bijective map f : (X, τ) $\rightarrow$ (Y, σ) is called $\hat{\omega}^*$-homeomorphism if both f and f^{-1} are $\hat{\omega}$-irresolute.

We denote the family of all $\hat{\omega}^*$-homeomorphisms of a topological space (X, τ) onto itself by $\hat{\omega}^*$-h(X, τ) .

Example 4.2 Let X={a,b,c}=Y, τ ={ ϕ ,{a},{a,b},X}and σ ={ ϕ ,{a},{a,b},{a,c},Y}.Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a)=a, f(b)=c and f(c)=b. Then f is an $\hat{\omega}^*$ -homeomorphism

Theorem 4.3 The composition of two $\hat{\omega}^*$ -homeomorphisms is a $\hat{\omega}^*$ -homeomorphism.

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) and g:(Y, σ) $\rightarrow$ (Z, η) be two $\hat{\omega}^*$ -homeomorphisms. Let V be a $\hat{\omega}$-closed in Z. Since g is $\hat{\omega}$-irresolute, $g^{-1}(V)$ is $\hat{\omega}$-closed in Y. Since f is $\hat{\omega}$-irresolute, $f^{-1}(g^{-1}(V)) =(g \circ f)^{-1}(V)$ is $\hat{\omega}$-closed in X which implies $g \circ f$ is $\hat{\omega}$-irresolute.

Let W be a $\hat{\omega}$-closed in X. Since f^{1} is $\hat{\omega}$-irresolute, $(f^{1})^{-1}(W) =f(W)$ is $\hat{\omega}$-closed in Y. Since g^{-1} is $\hat{\omega}$-irresolute,$(g^{-1})^{-1}(f(W))=g(f(W))= (g \circ f)(W)=((g \circ f)^{-1})^{-1}(W)$ is $\hat{\omega}$-closed in Z which implies $(g \circ f)^{-1}$ is $\hat{\omega}$-irresolute. Hence $g \circ f$ is an $\hat{\omega}^*$ -homeomorphism.

Remark 4.4 The following example shows that $\hat{\omega}$-homeomorphisms and $\hat{\omega}^*$ -homeomorphisms are independent notions.

Example 4.5 Let X= {a,b,c,d}=Y, τ ={ ϕ ,{a},{b},{a,b},X} and σ ={ ϕ ,{a,b},Y}. Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=a, f(c)=d and f(d)=c. Let f : (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is an $\hat{\omega}$-homeomorphism but not an $\hat{\omega}^*$ -homeomorphism since there exists an $\hat{\omega}$-closed set {c} of X such that f({c}) = {d} is not $\hat{\omega}$-closed in Y.

Example 4.6 The function f defined in example 4.2 is an $\hat{\omega}^*$ -homeomorphism but not an $\hat{\omega}$-homeomorphism since there exists a closed set {c} of X such that f({c}) = {b} is not $\hat{\omega}$-closed in Y.

Theorem 4.7 The set $\hat{\omega}^*$ -h(X, τ) is a group under the composition of mappings.

Proof: Define a binary operation *: $\hat{\omega}^*$ -h(X, τ) X $\hat{\omega}^*$ -h(X, τ) $\rightarrow$ $\hat{\omega}^*$ -h(X, τ) by f*g = $g \circ f$ for all f,g $\in$ $\hat{\omega}^*$ -h(X, τ) where $\circ$ is the usual operation of composition of mappings. By theorem 4.3, f* g= $g \circ f \in \hat{\omega}^*$ -h(X, τ).We know that composition of mappings is associative and the identity map I : (X, τ) $\rightarrow$ (X, τ) $\in$ $\hat{\omega}^*$ -h(X, τ).Also if f $\in$ $\hat{\omega}^*$ -h(X, τ),then $f^{-1} \in \hat{\omega}^*$ -h(X, τ) such that f* f^{-1}= f^{-1} * f =I and so inverse exists for every f $\in$ $\hat{\omega}^*$ -h(X, τ).Thus $\hat{\omega}^*$ -h(X, τ) a group under the composition of mappings.

Theorem 4.8 Let f : (X, τ) $\rightarrow$ (Y, σ) be an $\hat{\omega}^*$ -homeomorphism. Then f induces an isomorphism from the group $\hat{\omega}^*$ -h(X, τ) onto the group $\hat{\omega}^*$ -h(X, τ).

Proof: Using the map f, define a map ψ_f : $\hat{\omega}^*$ -h(X, τ) $\rightarrow$ $\hat{\omega}^*$ h(X, τ) by ψ_f (h) = $f \circ h \circ f^{-1}$ for every h $\in$ $\hat{\omega}^*$ -h(X, τ).Then ψ_f is a bijection. Also for all $h_1, h_2 \in \hat{\omega}^*$ -h(X, τ), $\psi_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{1} =(f \circ h_1 \circ f^{1}) \circ (f \circ h_2 \circ f^{1})= \psi_f (h_1) \circ \psi_f (h_1)$.Hence ψ_f is a homomorphism and so it is an isomorphism induced by f.

5. Applications

Definition 5.1[6] A topological space (X, τ) is said to be an $aT_{\hat{\omega}}$ -space if every $\hat{\omega}$-closed set in X is a-closed.

Theorem 5.2 Every $\hat{\omega}$-homeomorphism from an $aT_{\hat{\omega}}$ -space into another $aT_{\hat{\omega}}$ -space is an a-homeomorphism.

Proof: Let f : (X, τ) $\to$ (Y, σ) be an $\hat{\omega}$-homeomorphism where X and Y are $aT_{\hat{\omega}}$ -spaces. Let V be a closed set in Y. Since f is $\hat{\omega}$-continuous, $f^{-1}(V)$ is $\hat{\omega}$-closed in X. Since X is an $aT_{\hat{\omega}}$ -space, $f^{-1}(V)$ is a-closed in X and hence f is a-continuous.

Let W be a closed set in X. Since f is $\hat{\omega}$-closed, f (V) is $\hat{\omega}$-closed in Y. Since Y is an $aT_{\hat{\omega}}$ -space, f (V) is a-closed in Y and hence f is a-closed. Thus f is an a-homeomorphism.

Definition 5.3 A topological space (X, τ) is said to be a $\delta T_{\hat{\omega}}$ -space if every $\hat{\omega}$-closed set in X is δ -closed.

Theorem 5.4 Every $\hat{\omega}$-homeomorphism from a $\delta T_{\hat{\omega}}$ -space into another $\delta T_{\hat{\omega}}$ -space is an $\hat{\omega}^*$ -homeomorphism.

Proof: Let f : (X, τ) $\to$ (Y, σ) be an $\hat{\omega}$-homeomorphism where X and Y are $\delta T_{\hat{\omega}}$ -spaces. Let V be an $\hat{\omega}$-closed set in Y. Since Y is a $\delta T_{\hat{\omega}}$ -space,V is δ -closed in X and so V is closed in Y. Since f is $\hat{\omega}$-continuous, $f^{-1}(V)$ is $\hat{\omega}$-closed in X and hence f is $\hat{\omega}$-irresolute.

Let W be a $\hat{\omega}$-closed set in X. Since X is a $\delta T_{\hat{\omega}}$ -space,W is δ -closed in X and hence W is closed in X. Since f is $\hat{\omega}$-closed, $f(W)=(f^{-1})^{-1}(W)$is $\hat{\omega}$-closed set in Y and hence f^{-1} is $\hat{\omega}$-irresolute.Thus f is an $\hat{\omega}^*$ -homeomorphism.

Remark 5.5. The following example shows that the composition of two $\hat{\omega}$-homeomorphisms need not be a $\hat{\omega}$-homeomorphism.

Example 5.6 Let X= {a,b,c,d} = Y, τ = { ϕ, {a}, {b},{a,b}, {a,b,c},X}, σ = { ϕ, {a},{b},{a,b},Y} and η ={ ϕ ,{a,b},Z}. Let f : (X, τ) $\to$ (Y, σ) and g: (Y, σ) $\to$ (Z, η) be the identity maps.Then both f and g are $\hat{\omega}$-homeomorphisms but $g\circ f$: (X, τ) $\to$ (Z, η) not an $\hat{\omega}$-homeomorphisms since ($g\circ f$) ({d})={d} is not $\hat{\omega}$- closed in Z where {d} is closed in X.

Theorem 5.7 Let f : (X, τ) $\to$ (Y, σ) and g: (Y, σ) $\to$ (Z, η) be $\hat{\omega}$-homeomorphisms. Then $g\circ f$: (X, τ) $\to$ (Z, η) is an $\hat{\omega}$-homeomorphism if Y is a $\delta T_{\hat{\omega}}$ -space.

Proof: Let V be a closed set in X. Since f is $\hat{\omega}$-closed, f (V) is $\hat{\omega}$-closed in Y. Since Y is a $\delta T_{\hat{\omega}}$ -space, f (V)is δ -closed in Y and so f (V) is closed in Y. Since g is $\hat{\omega}$-closed, g(f(V) =($g\circ f$) (V)is $\hat{\omega}$-closed in Z. and hence $g\circ f$ is $\hat{\omega}$-closed.

Let W be a closed set in Z. Since g is $\hat{\omega}$-continuous, $g^{-1}(W)$ is $\hat{\omega}$-closed set in Y. Since Y is a $\delta T_{\hat{\omega}}$ - space , $g^{-1}(W)$ is δ -closed in Y and hence $g^{-1}(W)$ is closed in Y. Since f is $\hat{\omega}$-continuous, $f^{-1}(g^{-1}(W)) =(g\circ f)^{-1}(W)$ is $\hat{\omega}$-closed in X and hence $g\circ f$ is $\hat{\omega}$-continuous. Thus $g\circ f$ is an $\hat{\omega}$-homeomorphism.

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1. $\hat{\omega}$-homeomorphism 2. a–homeomorphism 3. homeomorphism 4.g –homeomorphism
5. αg –homeomorphism 6. $g\alpha$ –homeomorphism 7.gs-homeomorphism.

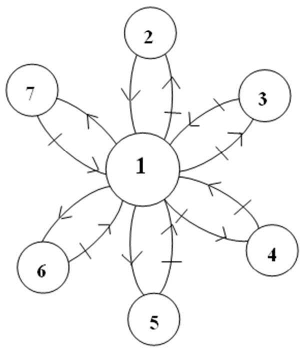

Figure-1