On the Pure Mathematical Aspects in Mathematical Models of some Metal Forming Processes

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Abstract

We shortly present some aspects of the number theory, group theory and theory of partial differential equations as they are revealed by original mathematical models of typical metal forming processes. We show that the so called central embedding problems over a number field is especially concerned in some revised form that correlates with the wave equation operator (d´Alembert operator first of all). This is typical for metal forming process like the Mannesmann-piercing process, where a rotating round hot metal semiproduct is pierced by means of the piercing plug „between“ two rotating conical rolls. We shortly describe the main problem, namely - how a geometry of the plug is embedded into some space X = $L(U)$, $(R^n \times S^1) \supset U \not\subset R^n$ such that a logic $L(U)$ of the deformation space U (which „tunnels“ the space R^n) can exist.

Keywords: Central embedding problem, differential operators, telegraph equation

Mannesmann piercing process

Due to the Mannesmann process configuration with two rotating conical rolls C which are „skewed“ mutually by the angle α („schräg Geometrie“-System), we cannot directly introduce an orthogonality notion for the space R^n . Therefore instead of some measurable domain Ω bounded in the euclidean space $\mathbf{R}^n$, we consider such a $\Omega^* \not\subset R^n$ for which the space $L_2(\Omega)$ of all measurable real functions u on the Ω becomes a logic $L_2(\Omega^*)$ of Ω^* equipped with a complex measure z, such that $u(z) \in L_2(\Omega^*, z)$. (We will further use only $L_2(\Omega^*)$ and u.). We understand $L_2(\Omega^*)$ as an *orthogonal* logic by means of that we can observe and understand all „*skewed* configurations“ within the Mannesmann process.).

We consider a logical chain for an immersion u^φ of a circle S^1 into R^n

$$u^\varphi \in L(U) \supset u \in L_2(\Omega^*) \supset \boldsymbol{u} \in L_2(\mathbf{g}), \tag{1}$$

$$\partial_i u^\varphi \partial_k u^\varphi = g_{(ik)}\ ;\ i,\ k = 1,2,\ldots,n, \tag{2}$$

$$\partial_i \partial_i E(\mathrm{K/k}) = \partial^t g_a{}^b, \tag{3}$$

$$(\partial_i \partial_k(\mathrm{K/k}))u = 0 \quad \text{in } S^n \subset 2_\alpha \mathrm{C}\ , \tag{4}$$

with respect to an embedding E(K/k) of the plug geometry **g** creating the embedding problem for the „central sphere“ S^n involved in the two conical rolls-arrangement 2_αC.

Remark 1 (**the Mannesmann process independence problem**). We observe an independence of the piercing plug shape on a mechanism of the hollow initiation and formation in the Mannesmann process. We firstly outline such an independence leading to the arrangement 2_αC as follows:

Let exist a mapping A: K $\rightarrow$ K indicating a partial independence of K with respect to A. If A explains some meaning M of K, then it suppresses another references within which K as a phenomenon can be rooted. Say that within these references K possesses a meaning m so that we require an existence of such k $\in$ A that exists in the *same sense* as K and binds m in a reference frame of A. Thus we have k(m) existing over the K(M).

Say further that A can exist as a part of the number theory such that we can imagine k(m) as some polynomial existing over a number field K(M). Then an arrangement of the K(M)/k(m)-type, or generally K/k would be impossible for $k\in A$. However, we can choose equivalently an arrangement K/k (k is not over K) for $k\notin A$. In this case specially, k can represent neither polynomial of the k(m)-type, nor the field of K(M)-type. We are simply for $k\notin A$ „out of the number theory". Can we return back to the number theory, i.e. remove $k\notin A$ and establish $k\in A$ with K/k by some torsion? (We need a **notion of torsion within the Mannesmann process!**) - If however K and k would be number fields in the arrangement K/k, then any torsion group T(k(m)) of curves represented by some polynomials of the k(m)-type cannot exist in this case, since k is a field. (In the number theory we consider the torsion group of elliptic curves over the quadratic fields K specially.) We must therefore quit from the number theory again only with the notion of the k(m) polynomials representing some curves „outside any torsion group" and excluding K/k as an arrangement of number fields and search for them a „new torsion concept" inspired by the given technological configuration. This task can be managed as follows.

Let **g** be a geometry of the k(m) polynomials. Then the mapping **g**: $A \to S^n$ can exist such that S^n is a n-dimensional „sphere with a torsion" over the K/k-arrangement. If the two conical rolls-arrangement $2_\alpha C$ is not any geometrical projection of K/k, then $S^n \subset 2_\alpha C$ can be regarded as the basic (central) configuration of the Mannesmann process. Thus such an abstraction from the field-meaning of K and k indicates towards their possible „conical character", which cannot be of the C-type. We assign a responsibility for the hollow initiation and formation to the $2_\alpha C$-arrangement with respect to K fitted by the shape of the plug of geometry **g**. This geometry preserves an independence of K from these „hollow-phenomena" (fitted by S^n).

Remark 2. With a torsion within S^n, we have no measurability of functions $u\in L_2(\Omega^*)$ on Ω^*. Thus, we cannot either explain S^n as an object. We only say, that if S^n has a „tubular cover", then S^n explains a plug shape with respect to the geometry **g**. No definition can be given, since no measurable phenomena can be fitted for an explanation of S^n. Consequently, having no objects distinguishable with respect to the S^n, we have no (mathematical) structure with theorems pronounced about such objects. Even more, the fact, that there is no fitted structure in the Mannesmann process, is absolutely typical for it. In that sense we consider S^n as a typical entity for that process. Without a possible object language, we believe in the process of a model construction that an usage of the mathematical language combined instantaneously with its some physical connotations is possible, provided that an abstraction from the original meaning of the used terms leads to a collection of the relevant knowledge about the Mannesmann process.

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Since K and k are no fields, we consider the nonconvex cover of S^n (determining the rolling product – tube) as being bound by means of two polar cones

$K = \{\zeta\in R^n: \zeta\wedge\xi \le 0\ \forall\xi\in K\}$ for the outer surface,

and

$k = \{\xi\in R^n: \xi\wedge\zeta\le 0\ \forall\zeta \in k\}$ for the inner surface of a tube with respect to a plug axis.

The formations $\zeta\wedge\xi$ and/or $\xi\wedge\zeta$ are represented by negative winding numbers (when a plug is active) and by zero in an opposite case. Thus the lines ξ and ζ are being wound on the K or k respectively in a time t such that

$(\partial^2/\partial\xi_i\partial\zeta_k = \partial^2/\partial\xi_k\partial\zeta_i) \Rightarrow \partial^2/\partial x_i\partial x_i$ defined on R^n in $L_2(\Omega^*)\ni i, k$.

We say that a piercing process can proceed if there is an extension K^*/k of K/k by a fixation of the piercing plug by means of the knot $\kappa\subset S^n$ created via the immersion of the circle S^1 into R^n (2).

That recalls (Opolka 1990): „Every central embedding problem $E_m = E(G, Z/p^m, c)$ for the absolute Galois group $G_k = Gal(\underline{k}/k)$ of k, where G = Gal(K/k) is the Galois group of finite Galois subextensions K/k of $\underline{k}$/k which is ramified only at p and ∞ and where $k(\mu_{p^m})/k$ is cyclic, has exponent <2m + t." (In this case, K is a number field, p prime number, t = t(k,p) a smallest natural number depending only on k and p, the so called p-exponent of k.)

In our case, when K and k are not fields, extension K^*/k of K/k takes place as the following „atomic rearrangement“: The elements $g_a{}^b$ of deformation matrix in (3) with the indices a and b representing two, by an initiation and growth of the hollow („crack“) separated atoms of two parallel atomic planes A, B of the deformed metal matrix, take values characterizing a new coupling between another two atoms c and d of a developed metal structure preserving the mutual relation of A and B.

Hint. Take $\partial^t g_a{}^b \neq 0$ in (3). If the matrix (of numbers) $g_a{}^b$ represents an invariant relation between A and B in a time t, then $\partial^t g_a{}^b = 0$ should hold within some continuous constancy of this invariant relation. However, if this relation is not even partially continuous - due to a mutual separation of a and b- atoms and thus $\partial^t g_a{}^b=0$ cannot hold, then only $\partial^t g_a{}^b \neq 0$ can (continuously) hold, provided that these discontinuities are continuously developed during a hollow growth in a time t. Thus, consequently, if $g_a{}^b$ takes values from an open interval (0, 1), then for a mapping $t : (0, 1) \to X$ (X is not a topological space with a domain representation of Ω^*-type), **g** is embedded into X in a time $t \in L_2(\Omega^*) \ni i, k$ for $x \in \Omega^*$. (There is no $t \in [0, T]$ such that $[0, T] \times \Omega$ could exist.)

This implies that not all atoms of A and B could be separated only within one extension K^*/k of K/k, and that such extensions could be regarded as subextensions on the one side, but not within any Galois group of G = $Gal(K^*/k)$-type on another. Our extensions are cyclic, or there should exist cycles of extensions K^*/k of K/k such that a group of these cycles binds a notion of differentials used within differential operators in relations (1)-(4). Searching for this group, we start:

Let C be a differential group with the endomorphism ∂: $C \to C$, $\partial \circ \partial = 0$ such that there exist a dual differential ∂^+ to (one) ∂ and ∂_+ to another ∂ within $\partial \circ \partial$ for which

$$\partial^+ \circ (\partial \wedge \partial) \Leftrightarrow \partial_i \,, \; \partial_+ \circ (\partial \wedge \partial) \Leftrightarrow \partial_k \,.$$

We comment the operation $\wedge$ as a symbol inducing such a logic $L(Z(C))$ as a solution to the problem (3) with respect to C that

$$\partial_i \partial_i L = 0 \quad \text{on } Z(C) - B(C) \quad \text{and} \quad \partial_i \partial_k L(Z(C)) = 0 \quad \text{on } B(C)\,, \tag{5}$$

where the kernel $Z(C)$ of the given endomorphism is the group of cycles. Therefore we put $L_2(\mathbf{g}) \equiv L(Z(C))$ with respect to the piercing plug represented by $B(C)$.

Physically speaking, we manipulate with a symbol $\wedge$ of a *simultaneity*, having a notion of the *time* $t \in L_2(\Omega^*)$ and with a notion of an *induction* (via $\wedge$), having a symbol L satisfying the equation $\partial_i \partial_i L = 0$ formally known from the electromagnetic *field* theory.

If we look for differentials ∂ existing with respect to coordinates of a process and generally consider a notion of coordinates transformation $x_i{}' = (\partial x_i / \partial x_k) x_k$, then we cannot generally consider a differential object $(\partial x_i / \partial x_k)$ independently of the logic $L(Z(C))$ and thus neither the coordinates pair $(x_i{}', x_k)$. This implies that a pair (x_i, x_k) should be relevant for the logic $L(Z(C))$ as a function of independent variables x_i and x_k, for which the object $(\partial x_i / \partial x_k)$ is avoided. Instead the operator $\partial_i \partial_k$ ($\equiv \partial^2 / \partial x_i \partial x_k$) is a differential object independent of $L(Z(C))$. Further, if $\partial_i \partial_k L(Z(C)) = 0$ on the piercing plug of the Mannesmann process, then the plug must stay invariant, but invariant to what? – For a and b separated and c and d being „conjugate“ with respect to the plug, we should consider a notion of „conjugation compensated by“ a (a,b,c,d)-reordering process. Simply:

$$a, b \Rightarrow \exists \mathcal{a}, \mathcal{b} \in Z(C) \text{ for } g \text{ in } Z(C)\text{: } g\mathcal{a}g^{-1} = \mathcal{b}\,, \; c \Rightarrow \exists \mathcal{c} \in B(C)\text{: } g\mathcal{c}g^{-1} \in B(C),\; g\mathcal{c}g^{-1} \Rightarrow \exists d$$

for $a, b, c, d \notin Z(C)$ with $B(C)$ as its normal subgroup. $B(C)$ is invariant under conjugation: for each $\mathcal{c}$ in $B(C)$

and each g in $Z(C)$ the element gcg^{-1} is still in $B(C)$.

The piercing plug represented by $B(C)$ is now invariant with respect to g. Taking into the account that the plug has a rotational shape, but g is not a rotation at the same time, we will consider g only as a „rotational segment“ within the problem (2). We put $g \Leftrightarrow \varphi$, where we shall call an arc of symmetry of the group $B(C)$ the segment φ of the circle S^1 being immersed into R^n.

Finally, let us have a look at the operator $\partial_i\partial_i$ within the problem (3) connotations:

We start with a notion of a growth rate of the hollow within $g_a{}^b$ suggesting an existence of the operator ∂^t ($\equiv \partial/\partial t$) with respect to a measure z. We therefore need a notion of a *logarithmic norm* $\mu(\mathbf{A})$ of a matrix $\mathbf{A}$ (introduced originally by (Dahlquist 1959, Lozinskij 1958), since with an induced matrix norm $\|\cdot\|$

$$d/dt^{+} (\log \|\cdot\|) \leq \mu(\mathbf{A})$$

and we have a maximal growth rate of ($\log \|\cdot\|$) given by $\mu(\mathbf{A})$. Let $g_a{}^b$ be the coefficients of deformation matrix $\boldsymbol{D}$ such that a measure z on Ω^* can be used as a norm $z(\boldsymbol{D})$. We suggest that

$$\partial/\partial t \,(\log \sqrt{|g_{(ik)}|}) \leq z(\boldsymbol{D}) \,, \tag{6}$$

where the growth rate of ($\log \sqrt{|g_{(ik)}|}$) determines the growth rate of the hollow with respect to the measure z. Since it cannot be simultaneously decided whether and why numerically possible instances $g_{(ik)} = |g_{(ik)}|$ would be excluded or allowed with respect to the hollow, we will exclude such decisions from a sense of the logic L by the putting $|g_{(ik)}| \neq 0$, $1 \notin L$ with respect to log within $\log\sqrt{|g_{(ik)}|}$, constructing in such a manner

$$(\log \sqrt{|g_{(ik)}|}) \wedge g_{(ik)} \Rightarrow \exists L(Z(C)) \text{ , or } (\log \sqrt{|g_{(ik)}|}) g_{(ik)} \Rightarrow L(Z(C))$$

for

$$\partial_i((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) = \partial_k((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) = 0 \,.$$

The last relation should represent a condition of the parallelity of the planes A, B with the hollow „between“ them and in the framework of $L_2(\Omega^*)$. It can be also rewritten into the form

$$\partial_i((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) \wedge \partial_k = 0 \;(\Leftrightarrow \partial \circ \partial = 0), \tag{7}$$

where we can observe the logical connection with the group C. Consequently, the object on the left hand side of this relation has no sense as an operator, being considered only within the logic $L_2(\Omega^*)$. Therefore we introduce an action distinguishing $L_2(\Omega^*)$ and $L(Z(C))$ into this relation, namely the embedding $E(\mathrm{K/k})$ of the geometry $\mathbf{g}$, obtaining thus at first

$$(\partial_i((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) \wedge \partial_k) E(\mathrm{K/k}) \neq 0 \text{ , or } \quad (\partial_i((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) \partial_k) E(\mathrm{K/k}) \neq g_a{}^b \,.$$

Taking into the account the operator $\partial/\partial t$, we can obtain

$$(\partial_i((\log \sqrt{|g_{(ik)}|}) g_{(ik)}) \partial_k) E(\mathrm{K/k}) = \partial^t g_a{}^b \tag{3a}$$

as the equation (3) with d´Alembert operator $\partial_i\partial_i$.

The embedding E(K/k) of the plug geometry **g** creating the embedding problem (3a), or (3) respectively for the „central sphere“ S^n involved in two conical rolls arrangement 2_αC cannot lead to any exponent <2m + t of the problem, since there is no smallest natural number t = t(k, p) depending only on k and p. In the more „technological words“, there is no hollow within the piercing process which could be associated with a natural number depending only on k and not K at the same time. As to numbers so far, we have only negative winding numbers as representations of the „commutation formations“ $\zeta\wedge\xi$ or $\xi\wedge\zeta$ so that we would generally say that S^n can act on k with respect to K by commuting coordinates within indices i, $k \in L_2(\Omega^*)$. Thus e.g.: $x_1x_2 = x_2x_1$, when $x_3x_n = x_nx_3$, etc. and n = 4, 6, ... The smallest natural number n depending logically on k and K is 4, but this number cannot be determined completely since K/k is not complete. Therefore we say that the space R^n is asymptotically 4-dimensional, where the winding (number) conditioned by an existence of the given commuting is not completely realized and should be realized via the extension K^*/k of K/k.

Remark 3 (**One mathematico-technological example of the winding number occurrence**). The logic $L(Z(C))$ itself does not depend on the coordinate transformations $x_i' = (\partial x_i/\partial x_k)x_k$, as it was showed above. Let G be a group of these transformations in the space (X, μ) with such a measure, which depends only on G (denoted as $\mu = \mu(G)$). If there is a vector field ρ_N on some process-fiber $f^{-1}(U)$ such that ρ_N is free of coordinates x_i', then it has no sense to say that ρ_N is G-invariant, i.e. ρ_N is completely $L(Z(C))$-consistent. Thus ρ_N cannot have any „one to one correspondence“ with the measure μ, i.e. there exists $\rho_N \leftrightarrow \{\mu\}_N$ correspondence with respect to the logic $L(Z(C))$. This fact enables fundamentally to distinguish μ from $z(\boldsymbol{D})$ as

$$\mu = \mu(G),\ \{\mu\}_N \supset \{1\} \not\subset \{z(\boldsymbol{D})\}, \tag{6a}$$

The mapping

$$\rho_N : \{1\} \rightarrow S^1 \tag{6b}$$

then can be regarded as a consequent „realization“ of the correspondence $\rho_N \leftrightarrow \{\mu\}_N$ „distinguishing“ the unit set {1} from the (unit) sphere Ω^* with the measure z. If now $\Omega^* \not\subset R^n$, then the circle S^1 can be (naturally) embedded into R^n. In this way we obtain the knot $K \subset S^n$ fitting the plug in R^n such that the fiber $f^{-1}(U)$ is wrapped around the plug. As a corresponding winding number we cannot use an usual isomorphism W: $\pi_1(S^1,1) \rightarrow Z$, where $\pi_1(S^1,1)$ is the fundamental group of the circle for which we have no „technological“ substantiation. We work with the isomorphism

$$W: \pi_1(R^n \times S^1 \backslash K) \rightarrow Z(C),\ W(\rho_N) = N, \tag{6c}$$

where $\pi_1(R^n \times S^1 \backslash K)$ represents the „**unknot group**“, when the so called fundamental group $\pi_1(S^n \backslash K)$ of the complement of the knot K must be excluded for $S^n \equiv \partial(R^n \times S^1)$.

Let further A (from the *Remark 1.*) be a function of action of the both conical rolls in the arrangement $2_\alpha C \supset S^n$. Generally it holds $\delta A = 0$. But there is the transport angle α such that $\delta A(\alpha) \neq 0$ for

$$A^* \equiv A(\delta A(\alpha)),$$
$$\delta^* A^* = 0. \tag{6d}$$

The extension A^* of A is given by the fitting of the piercing plug in R^n. It is very hard task to find an explicit expression for the action A^*. Nevertheless, we have found it and solving (6d) we have yielded

$$g = (\mathrm{atan}(2 \tan(N \operatorname{asin} (\sqrt{(1 - \cos^2\alpha)}/\cos \alpha))))/(N \operatorname{asin} (\sqrt{(1 - \cos^2\alpha)}/\cos \alpha)) \tag{6e}$$

for

$$\partial g/\partial U = \mathbf{u} \in L_2(\mathbf{g}) \ (\Leftrightarrow L(Z(C))).$$

The plug fitting („coupling") constant runs roughly within the open interval $g\in(1, 1.62)$ and creates the basis of the manifold $\mathsf{T}(U)$, $R^n\times S^1\supset U\not\subset R^n$, representing the finished rolled tube (see the picture 1.). In fact we work with the „tautology"

$$\lambda : \mathsf{T}(U) \to \mathsf{T}^*(U),\ R^n\times S^1\backslash R^n \ni \lambda \text{ for } \forall g, \tag{6f}$$

where λ is represented by the unique plug-shape in the process. This mapping we call the „tunneling" of the space R^n by means of a realization of the deformation space U such that, due to λ-existence, the space U can exist „over" R^n without destructing it.

Below is given the table of values of winding number N realized by the process fiber $f^{-1}(U)$ wrapped around the piercing plug for some practically used values of g (the „length" of the plug of the particular „λ-shape").

In fact, the winding number N cannot be determined as the whole number. We can observe in the picture 2., where g = 3/2, that the piercing process was realized with N = 8-13 with a few crossings. It means that the process was not completely realized with respekt to A^* and it was slightly „oversteering" (such undesirable, but practically typical phenomenon was caused by the slightly incorrect shape of the support roll that has intervened into the action A of two working rolls).

We can observe the slightly distorted manifold „$\mathsf{T}(U)$" in the picture 1., where a „distorsion" indicates a presence of so many pairs („z", „λ") $\in$ „$\mathsf{T}(U)$", how many „crossings" of the process fiber wrapping we observe in the picture 2. (= oversteering). The presence of the pairs of this type in „$\mathsf{T}(U)$" is a consequence of the partial suppression of the realization of the tunneling (6f).

But the observed „oversteering" of the process is not so dangerous like even partial realization of the torsion group of elliptic curves avoiding the action A, with profound destructive torsional phenomena on the plug surface. With the mapping $\mathbf{g}$: A $\to S^n$ (mentioned in the *Remark 1.*) the inherent torsion helps us to fit a dimensional completeness of the process with respect to dimensionally incomplete space R^n. We can namely regard the sphere S^n as „4-dimensionally complete", i.e. as if the S^n with a torsion were equivalent to S^4 without it. The individual existence of S^n and S^4 will be preserved by a considering all symmetries of the Mannesmann process with respect to the both spheres, i.e. we consider them as complete up to the torsion which cannot be determined within S^n and thus the symmetries are indeterminable with respect to S^4.

Consequently, the indices i, $k \in L_2(\Omega^*)$, by means of which we study the incomplete symmetries of the Mannesmann process, cannot be used for a definition of a torsion within the S^n like e.g. some torsion components

$$T_{ij}{}^k := \Gamma_{ij}{}^k - \Gamma_{ji}{}^k - \gamma_{ij}{}^k$$

since the index j in the Christoffel Γ-symbols and in the Lie γ-brackets is overdetermining here and the indices i, k are not carried by the ∂-type objects.

Such an abstract result have a very practical and pragmatic consequence. – Since there is no tangent bundle over a domain D, sections ($\mathbf{e}_1$, ..., $\mathbf{e}_n$) of which could be used for a definition of torsion components $T_{ij}{}^k$, there is also no domain D which could form an representation of the topological space X. So, the space X, into which is $\mathbf{g}$ embedded, is not a topological space. Thus a domain D is an anomaly of the Mannesmann process:

A domain representation D of X (Dahlgreen 2006) is a triple (D, D^R, δ), where D^R is a dense subset in D and δ: $D^R \to X$ is continuous and onto. For f: $D^R \to Y$ there can exist f ́: D → E, where E is an admissible domain representation of Y. If D is not sufficiently dense in D, then f ́is called a partial continuous function from D to E. We know that due to the continuous hollow growth, the condition $\partial^t g_a{}^b \neq 0$ holds under an invariant relation $g_a{}^b$ between *A*, *B* that is not even partially continuous with respect to indices *a*, *b*. Thus, if f ́exists, then ***D***f ́= λf ́can exist, where an eigenvalue λ of ***D*** corresponding with f ́represents a „plug shape of geometry **g**“. In this way a plug *inadmissibly* intervenes in the separation process of the atoms of *A* and *B* and many defects arise in the roll product „representing“ the space Y. – A lot of such continuously casted billets (semiproducts which are to be pierced within the Mannesmann process) are casted „dense“ with respect to their „axis“, so they are of a D-type and thus anomalous. Consequence: Some tangent bundle over D has sections ($\mathbf{e}_1, \ldots, \mathbf{e}_n$) existing in an eigentime τ such that $\partial^\tau g_a{}^b = 0$ and (3a), with a zero right side, becomes inadmissibly a scalar wave equation with d ́Alembert operator, while possibly the cones K and k become the so called *characteristic surfaces* (upper and lower cones) of this equation.

However, we do not completely resign on a wave-equation form, when we try to formulate a condition (7) of the parallelity of *A*, *B* by means of differential operators which would not be directly in the logic $L_2(\Omega^*)$ and thus „out of the pair *i*, $k \in L_2(\Omega^*)$ or trinity (*i*, *j*, *k*)“ of indices excluded by $T_{ij}{}^k$ from any consideration. – Let α and β be representations of *A* and *B* both with respect to k (when the atoms represented by *a* and *b* are mutually separated from each other) and with respect to K, when *c* and *d* conjugate (we thus operate with coefficients 2α and 2β with respect to *a*, *b*, *c*, *d*). Then we find a telegraphic equation

$$\partial^2\psi(x, t)/\partial t^2 - c^2\partial^2\psi(x, t)/\partial x^2 + 2\alpha\partial\psi(x, t)/\partial t + 2\beta\partial\psi(x, t)/\partial x + \gamma\psi(x, t) = F(\mathrm{K}/\mathrm{k})$$

$$\text{in } U,\ \forall x \in \Omega^* \text{ and } \exists t : (0, 1) \to \mathrm{X} = L(U), \tag{8}$$

where F(K/k) is a distribution of K and k in $R^n \times S^1 \supset U$. Its solution $\psi(x, t)$ is the (*a*, *b*, *c*, *d*)-configuration function defined as

$$\psi(x, t) := \exp(\mathrm{i}\ asin(\alpha/\beta)) \text{ for } \quad 2_\alpha C \supset S^n,\ \alpha := acos(\alpha/\beta),\ \mathrm{i} = \sqrt{-1} \tag{9}$$

and propagating with the sound velocity c in the deformation space *U* in such a manner that, through following the cycles from the group *Z*(*C*), it does not create any patterns leading to some forms on *U*. – The reason is that $U \not\subset R^n$, or *U* cannot be an open subset of R^n and thus it excludes an existence of any *form* on *U*. Recall that a *form* on $U \subset E^n$ were a correspondence ω (Auslander 1977) that assigns to each point *x* of *U* a differential ω(*x*) from a cotangent space. Since the cotangent space at *x* is dual to the tangent space at *x*, we would have a tangent bundle (of these spaces) over D and thus a torsion of $T_{ij}{}^k$-type within the system $2_\alpha C$. And that is an inadmissible state as it is showed above. That is why we strictly require $\Omega^* \equiv S^4$ for $x \in \Omega^*$ and at $S^4 \Leftrightarrow S^n$ we call *U* of a cylindrical shape a nonconvex cover of S^n for $x \notin U$. No $S^4 \cap U \subset \mathbf{g}$ are naturally possible.

Since no patterns of ψ-propagation are admissible, so that no such pattern could be preserved during the time *t*, we say that there are no symmetry-requirements on the configuration function ψ(*x*,*t*) with respect to $\gamma \subset X$ representing the plane („between“ *A* and *B*), in which the hollow initiation can be recognized. We say equivalently that the function ψ(*x*, *t*) is invariant with respect to the mutual interchange of *acos* and *asin* for $\alpha/\beta \neq 1/\sqrt{2}$ within it. And specially:

$$\gamma: <\neq> \mathrm{Q}(\sqrt{2}),\ r + s\sqrt{2} \in \mathrm{Q}(\sqrt{2}) \mid r, s \in \mathrm{Q},$$

where, as usual, the number field is an extension Q(√2) of the field Q of rational numbers. There is no extension of Q into which the field Q(√2) could be embedded within the problem (8), since any rational numbers obtained by the process of differentiation of the $\psi(x, t)$ cannot belong to Q.

The number √2 is the symbol of an inadmissible crack-initiation and the γ-plane of an admissible hollow initiation is the plane of a „schräg-Geometrie“ of the Mannesmann process.

Shortly concluding, if any central embedding problem motivated by the number theory is *combined* with a notion of the shape fixing K (like a shape of the piercing plug here), then K loses its meaning as a number field with respect to k (fixing a hollow) and a requirement on the existence of the geometry **g** of such a *combination* is induced, for which K/k is a conical reference system. We call the embedding problem (3a) for the central sphere S^n solvable after cyclical extensions K^*/k of K/k within the Mannesmann piercing process applying the plug of the geometry **g** (being embedded into X).

This could imply that no number field can obtain a physical meaning within the models of deformation processes built by means of PDEs, since there is no way within every particular logic of an (infinitesimal) process, how to (finitely) extend the field Q. The real worth of number fields in such models is seemingly that they are preserved as symbols and we specially study a way how they lose their „field meaning“ with respect to the particular deformation process. Then, for the numbers on which „this meaning is particularly vanished“, we construct special structures like the deformation matrix ***D*** here.

Or, vice versa, if the symbols like γ in (8) possess a meaning of the field, then no number field could be considered within such a problem, or more specially, no embedding problems over number field can be studied with respect to a pure preserving of their symbols within a given particular logic, since no such embeddings exist in this case.

Example. If the symbol γ in the equation (8) possesses a meaning of the slip line *field*, then we can shortly read this equation from the metal forming processes point of view as follows: there are two rolls of the α-type and two rolls of the β-type. The objects k, K correlate α and β-type rolls with respect to the field γ of slips, via which a „flat plastic deformation“ of metal occurs. An attribute „*flat*“ comes from γ being complementarily a representation of the plane, or with one pair being a pair of work rolls and another of support rolls we have the system of the *plate* rolling called „Quarto“. In that system we have no distribution of K/k in $R^n\times S^1\supset U$ so that a notion of a distribution function F(K/k) loses its sense in (8) within the „Quarto“. K and k are not symbols but objects now and we put instead F(K/k) the Frege ́s concept F[K, k] = K/k. It means that the relation F is a 2-place-function whose value is always a truth value. More specially: instead of $t : (0, 1) \to X$ we will have the interval (0, 1) within a cylinder $S^1\times(0, 1)$ along that such ψ propagates, via which the values of F are mediated. Or, the ψ is a symbol of the truth preserved by the logic L_F(Quarto) which is induced by the Frege ́s F[K/k] = K/k within a telegraph equation of the (8)-form.

We technically require: If N is a number of passes of the metal semiproduct between the rolls, then L_F(Quarto) should be N-valued. We can say that L_F(Quarto) is a ψ-preserving logic in (8), provided that „degrees of truth ψ“ are mediated via deformation ratios per N passes. Any such a deformation ratio should let vanish S^n preserving $x\in\Omega^*$ at the same time. In this sense we have ψ over the domain D with the subset D^R within which a N-valued plastic deformation takes place in such a manner that D^R is sufficiently dense. Thus we can obtain D as the domain representation of the topological space X⇔“Quarto“$\neg\exists U$ = L(X). Consequently, if we denote as φ_D the so called feeding angle, then φ_{D^R} is a phase (angle) of $\psi(x, t)$ as the solution of (8) with F.

Remark 4. As we can see, it is not easy to distinguish the cones K and k as symbols and as objects, avoiding their field interpretation at the same time t for $x\in\Omega^*$. – We can follow the given way putting $f(\varphi_D, \varphi_{D^R}) = \mu$ for μ as a friction coefficient. But we are not able to assign some special meaning to f, since the friction coefficient is widely dependent via f, starting with the deformation temperature, chemical composition of deformed metal, etc. (E.g., if f will be a field, then μ would be dominantly temperature dependent through the heat equation $\partial_t f = \nabla^2 f$). For some very simplified, steady-state conditions of $\partial f/\partial t = 0$ at the processes like a cold wire-drawing with an usage of lubricant, where K and k could be possibly combined into some „conical die“, the influence of a lubricant on the deformation characteristics should be clarified (Tittel 2011).

Conclusions

Simple comment about in the particle used method: *Observing the technological process of metal forming* $\rightarrow$ *distinguishing operators in a participation* $\rightarrow$ *which operators are of partial* ∂ *or* ∂^2*-types acting on what in which logic chain and their comparing in a supposingly allowed arrangement, which could correspond with a sense of the differential group* (*structure*) $\rightarrow$ *yielding their substantiated mathematical meaning* $\rightarrow$ *possibly allowed abstraction of such their meaning into their possible physical interpretation* $\rightarrow$ *within such interpretation to try to find their inherent and original technological position in the given observed process and their final rearrangement* into the **meaningful mathematical model** *of an observed technological process*. This implies that any mathematical model of an arbitrary metal forming processes cannot avoid relative high degree of an understanding of a directly relevant sense of pure mathematical aspects involved.

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Table 1. The values of N for $\alpha = 3^o$

g	N
1.0 (not used)	29.95
1.1 (not recommended)	24.89
1.2 (unstable)	21,00
1.3	17.89
1.4	15.22
1.5	12.87
1.6	10.74
1.62 (not used)	10.33

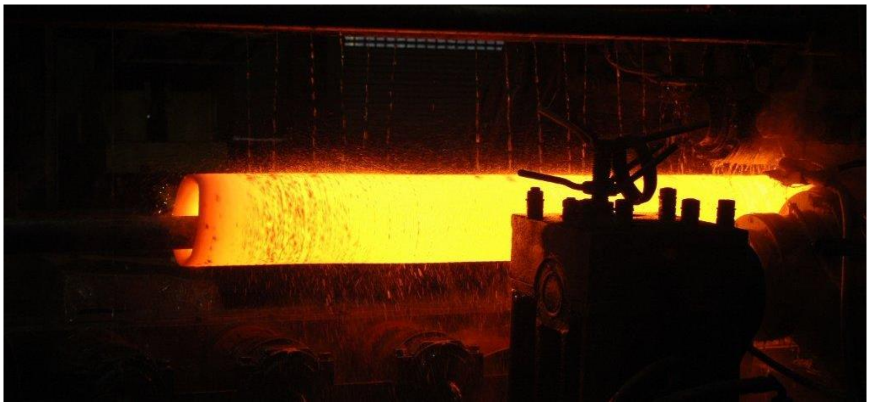

Figure 1. The rolled tube of the Mannesmann process

Figure 2. The winding number N realized by the process fiber wrapped around the plug