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# New Approach on Identification of Circular Cone

**Getachew Abiye Salilew**

Department of Mathematics, Madda Walabu University, Bale Robe, Ethiopia
Email: getachewabye@gmail.com

## Abstract

The objective of this paper is to provide elementary approach for identification of circular cone by using known results. In this paper, the author identified oblique circular cone by considering two different slant heights of a cone, the axis of a cone and larger angle of a triangle which is obtained from the lateral surface of a circular cone. The author found that there are four different types of circular cones namely right circular cone, acute oblique circular cone, right oblique circular cone and obtuse (critical) oblique circular cone by using very elementary ways.
**Keywords:** circle, circular cones, oblique circular cones, triangles.

## 1. Introduction

Circular cone is one of the interested areas in geometry, for the researcher since ancient time. But major contributions have been made on it during 19th centuries, which were initiated since 17th century. On this topic several contributions have been already done by the mathematician [1-7], but we believe that our approach for dealing current problems described in this article is different than others. A conic surface is generated by a straight line that always intersects a fixed curve and passes through a fixed point not in the same plane as the curve. A cone is a three-dimensional geometric shape that tapers smoothly from a flat base (frequently, though not necessarily, circular) to a point called the apex or vertex. In our study, we shall focus on the circular cones. Circular cone is one of a conic surface whose base is a circle. Circular cone is a cone and it is one of the basic shapes in geometry with circular base.

There are different circular cones. One is a right circular cone. It is a cone where the axis of cone is the line joining the vertex to the midpoint of the circular base. That is, the center point of the circular base is joined with the apex of the cone. A right circular cone is one whose axis perpendicular to the plane of the base. And the other is oblique circular cone whose axis is not perpendicular to the plane of the base. Gedefa and Chaudhary introduced a new technique for identification of the nature of triangles by using known results. Chaudhary and Getachew extended this identification of the nature of triangles to identification of the nature of trapezoids and recently Getachew introduced a new technique for similarity of these extended trapezoids by using known results [8-10]. In this paper, we applied these techniques to find necessary conditions, which enable to identify the nature of a circular cone that are presumably not exist before by using elementary ways. We presented that right circular cone, acute oblique circular cone, right oblique circular cone and obtuse oblique circular cone by using known results. Conical and other quadratic surfaces have been examined using the techniques of coordinate geometry [11-12].

## 2. Main Theorem

***Theorem 1:*** Find necessary conditions, which enable to identify the nature of a circular cone.

**Proof:** Our approach to derive necessary conditions, for identification the nature of a circular cone, motivated by the work [9] and also by using known results.

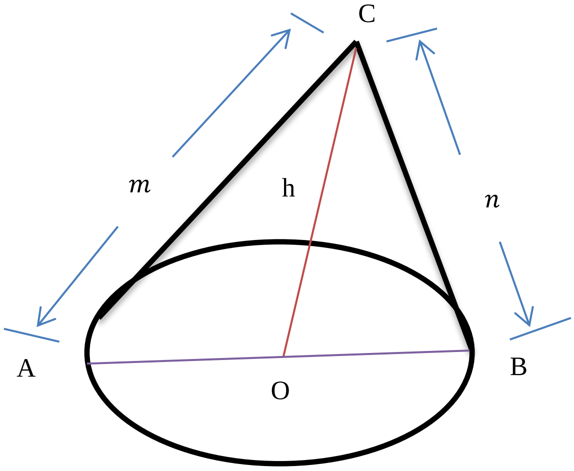


**Figure 1:** Circular cone

In figure 1 above, we consider a circular cone has largest slant height $m$ and smallest slant height $n$. Let the radius of a circular cone is $r$ and the axis of a cone, $OC = h$, is the line joining the vertex to the midpoint of the circular base. That is, the center point of the circular base is joined with the apex of the cone. Then from triangle ΔABC on figure 2 below which is obtained from a circular cone in figure 1 above, we derive basic conditions for identification of a circular cone.

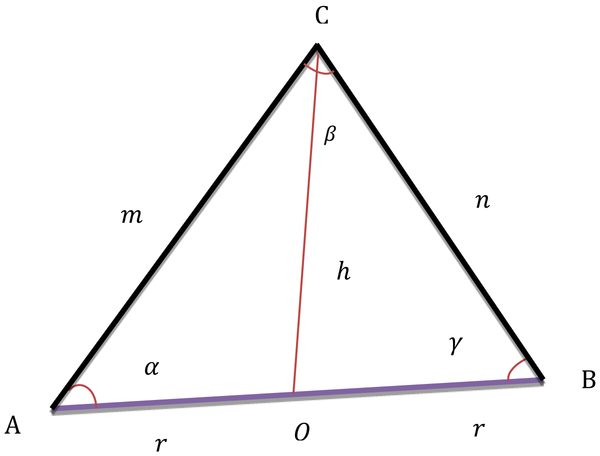


**Figure 2:** A triangle obtained from lateral surface of a circular cone

From triangle ΔABC in figure 2 above, we consider $\gamma$ is the largest angle, $AB = 2r$ is the diameter of a circle which is also one side of a triangle ΔABC, $h$ is axis of a cone and we use slant height $m$ as the largest side for triangle ΔABC then by the work done [9] and using elementary ways, we obtain the following.

**A.** If $m = n$, then $h^2 + r^2 = n^2 = m^2$.

Proof. Let $m = n$. Then $\alpha = \gamma$ implies that ΔABC is an isosceles triangle. And also the axis of a cone, $OC = h$, bisects the diameter of a circle which implies that ΔAOC~ΔBOC by side- angle- side theorem. Thus $\angle AOC + \angle BOC = 2\angle BOC = 180^{\circ}$ implies that both triangles ΔAOC and ΔBOC are right angle triangle. Thus using Pythagorean Theorem, we obtain $h^2 + r^2 = n^2 = m^2$. Therefore, for $m = n$ the axis of a cone is perpendicular to the diameter of its base at its center. Thus the cone is called right circular cone.

Next when the axis of a circular cone is not perpendicular to its base at center, then the cone is called oblique circular cone. Now we give the name of oblique circular cone based on the larger angle $\angle ABC = \gamma$ of a triangle ΔABC which is obtained from circular cone in figure 1 above.

**B.** If $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 < n^2 + r^2$, then $2h^2 - n^2 < h^2 + r^2 < m^2$.

Proof. Let $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 < n^2 + r^2$. This implies that $m$ is the largest side and $\gamma$ is an acute angle by [9] respectively. From $h^2 < n^2 + r^2$, we have: $h^2 - n^2 < r^2 \Longrightarrow 2h^2 - n^2 < h^2 + r^2$. Since $h^2 + r^2 < m^2$, we obtain $2h^2 - n^2 < h^2 + r^2 < m^2$. Therefore, for $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 < n^2 + r^2$ the axis of a cone is not perpendicular to a circle at its center and $\gamma$ is an acute angle so that the cone is called acute oblique circular cone.

**C.** If $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 = n^2 + r^2$, then $2r^2 + n^2 = h^2 + r^2 < m^2$.

Proof. Let $m \neq$ n, $h^2 + r^2 < m^2$ and $h^2 = n^2 + r^2$. This implies that $m$ is the largest side and $\gamma$ is right angle by [9] respectively. From $h^2 = n^2 + r^2$, we have: $h^2 + r^2 = n^2 + r^2 + r^2 = n^2 + 2r^2$. Since $h^2 + r^2 < m^2$, we obtain $2r^2 + n^2 = h^2 + r^2 < m^2$. Thus, for $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 = n^2 + r^2$ the axis of a cone is not perpendicular to a circle at its center and $\gamma$ is right angle so that the cone is called right oblique circular cone.

**D.** If $m \neq$ n, $h^2 + r^2 < m^2$ and $h^2 > n^2 + r^2$, then $2r^2 + n^2 < h^2 + r^2 < m^2$.

Proof. Let $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 > n^2 + r^2$. This implies that $m$ is the largest side and $\gamma$ is obtuse angle by [9] respectively. From $n^2 + r^2 < h^2$, we have: $n^2 + r^2 + r^2 < h^2 + r^2 \Longrightarrow n^2 + 2r^2 < h^2 + r^2$. Since $h^2 + r^2 < m^2$, we obtain $2r^2 + n^2 < h^2 + r^2 < m^2$. Therefore, for $m \neq n$, $h^2 + r^2 < m^2$ and $h^2 > n^2 + r^2$ the axis of a cone is not perpendicular to a circle at its center and $\gamma$ is obtuse angle so that the cone is called obtuse (critical) oblique circular cone.

Thus the proofs of [**A-D**] complete proof of main theorem.

## 3. Conclusions

We identified oblique circular cone by considering two different slant heights of a cone, the axis of a cone and larger angle of a triangle that is obtained from the lateral surface of a circular cone. The sides of a triangle are the two different slant heights and the diameter of a circle which is the base of a cone. We have found the following conditions from our main result.

**A.** For $m = n$ and $h^2 + r^2 = n^2 = m^2$, then the cone is called right circular cone.
**B.** For $m \neq n$ and $2h^2 - n^2 < h^2 + r^2 < m^2$, then the cone is called acute oblique circular cone.
**C.** For $m \neq n$ and $2r^2 + n^2 = h^2 + r^2 < m^2$, then the cone is called right oblique circular cone.
**D.** For $m \neq n$ and $2r^2 + n^2 < h^2 + r^2 < m^2$, then the cone is called obtuse oblique circular cone.

Significance of this study is to investigate frustum of a circular cone whose lateral surface form one form of the trapezoids [8] and also to identify the degree of oblique circular cone. The future work is by using the same technique to identify ellipsoid cone whose base is an ellipse.

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