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COUPLED FIXED POINT THEOREMS IN ORDERED NON-ARCHIMEDEAN INTUITIONISTIC FUZZY METRIC SPACE USING k-MONOTONE PROPERTY

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Abstract: In this paper we define k-monotone property and proved coupled fixed point theorem in ordered non-Archimedean Intuitionistic fuzzy metric space.
Key words: Non- Archimedean property, k-monotone property, mixed monotone mappings, coupled fixed point, Fuzzy metric space, Intuitionistic Fuzzy metric space, Cauchy sequence, complete fuzzy metric space.

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1. Introduction:

Fuzzy set theory, a generalization of crisp set theory, was first introduced by Zadeh [21] in 1965 to describe situations in which data are imprecise or vague or uncertain. Kramosil and Michalek [11] introduced the concept of fuzzy metric spaces in 1975, which opened an avenue for further development of analysis in such spaces. Later on it is modified that a few concepts of mathematical analysis have been generalized by George and Veeramani [9].

Afterwards, many articles have been published on fixed point theorems under different contractive condition in fuzzy metric spaces.

Atanassov [1] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Coker [3] introduced the concepts of the so called "Intuitionistic fuzzy topological spaces". Park [18], using the idea of intuitionistic fuzzy sets, define the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [9].

Bhaskar and Lakshmikantham [3] discussed the mixed monotone mappings and gave some coupled fixed point theorems which can be used to discuss the existence and uniqueness of solution for a periodic boundary value problem.

Hu[10] studied common coupled fixed point theorems for contractive mappings in fuzzy metric space, and Park et.al.[18] defined an IFMS and proved a fixed point theorem in IFMS. Chandok alt el. [4], Choudhury at. Al[5],Cric and Laxmikantam [6], Nguyen at. Al.[16] studied and give the results on common coupled fixed point theorems in different metric spaces. Berinde [2] Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces, Recently Luong et.al.[12] proved coupled fixed points in partially ordered metric spaces . Mohinta and Samanta[15] and Park [19] prove the coupled fixed point theorem in non-Archimedean intuitionistic fuzzy metric space.

In this paper, we define non-Archimedean intuitionistic fuzzy metric space, and prove a coupled fixed point theorems for map satisfying the mixed monotone property in partially ordered complete non-Archimedean intuitionistic fuzzy metric space.

2. Preliminaries:

Dentition 2.1 [20] A binary operation $*[0,1]\times[0,1] \to[0,1]$ is a continuous t-norms if " * " is satisfying conditions:

(i) * is an commutative and associative

(ii) * is continuous

(iii) $a * 1 = a$ for all $a\in [0, 1]$

(iv) $a * b \le c*d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Basic example of t – norm are the Lukasiewicz t – norm T_1, $T_1 (a, b) = \max (a+b-1, 0)$,
t –norm T_p, $T_p (a,b) = ab$, and t – norm T_M, $T_M (a,b) = \min \{a,b\}$.

Definition 2.2[14] - A 3-tuple (X,M,*) is said to be non-Archimedean fuzzy metric space if X is an arbitrary set,$*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions ,for all $x,y,z\in X$, $s,t >0$,

(F_1) $M(x, y, t) > 0$

(F_2) $M(x, y, t) = 1$ if and only if $x = y$

(F_3) $M(x, y, t) = M(y, x, t)$

(F_4) $M(x, y, t) *M(y, z, s) \le M(x, z, t + s)$

(F_5) $M(x, y, \cdot)$**:** $(0, \infty) \to (0, 1]$ is continuous.

Then M is called a non-Archimedean fuzzy metric on X. Then M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Lemma 2.1. Let (X, M, *) non-Archimedean fuzzy metric space, then M is a continuous function on $X^2 \times (0, \infty)$.

Remark 2.1. Since * is continuous, if follows from (F_4) that the limit of the sequence in fuzzy metric space is uniquely determined.

Let (X, M, *) be a fuzzy metric space with the following condition:

(F_6) $\lim_{t\to\infty} M (x, y, t) = 1$ for all $x, y \in X$

Remark 2.2. In the above definition 2.2, the triangular inequality (F_4) is replaced by

$$M (x, z, \max \{t, s\}) \ge M(x, y, t) * M (y, z, s) \quad \text{for all } x, y, z\in X ,\ s, t >0$$

More equivalently $M (x, z, t) \ge M(x, y, t)* M(y, z, t)$ for all $x, y, z\in X$, $s, t >0$

(NA)

Then the triple (X, M,*) is called a non-Archimedean fuzzy metric space.

It is easy to check that the triangular inequality (NA) implies (F_4), that is, every non-Archimedean fuzzy metric space is itself a fuzzy metric space.

Definition 2.3 [20]. A binary operation $\diamond:[0,1]\times[0,1] \to[0,1]$ is a continuous t-co norms if "$\diamond$" is satisfying conditions:

(i) $\diamond$ is commutative and associative;

(ii) $\diamond$ is continuous;

(iii) $a \diamond 0 = a$ for all $a\in [0, 1]$

(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Note. The concepts of *triangular norms* (*t*-norms) and *triangular conorms* (*t*-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [13] in his study of statistical metric spaces.

Definition-2.4[17]: A 5-tuple (X, M, N, *, ◊) is said to be *non Archimedean intuitionistic fuzzy metric space* if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all x, y, z ∈ X, s, t > 0,

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$

(IFM-2) $M(x, y, t) > 0$

(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$

(IFM-4) $M(x, y, t) = M(y, x, t)$

(IFM-5) $M(x, z, \max\{t, s\}) \geq M(x, y, t) * M(y, z, s)$ for all x, y, z∈X , s, t >0

(IFM-6) $M(x, y, .) : (0, \infty) \to (0, 1]$ is continuous

(IFM-7) $N(x, y, t) > 0$

(IFM-8) $N(x, y, t) = 0$ if and only if $x = y$

(IFM-9) $N(x, y, t) = N(y, x, t)$

(IFM-10) $N(x, z, \min\{t, s\}) \leq N(x, y, t) \lozenge N(y, z, s)$ for all x, y, z∈X , s, t >0

(IFM-11) $N(x, y, .): (0,\infty) \to (0, 1]$ is continuous

Then (M, N) is called an *non Archimedean intuitionistic fuzzy metric* on X , the function M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non nearness between x and y with respect to 't' respectively.

Remark 2.3: In the above definition the triangular inequality (IFM5) and (IFM10) are equivalent to

$$M(x, z, t) \geq M(x, y, t) * M(y, z, t)$$

and $$N(x, z, t) \leq N(x, y, t) \lozenge N(y, z, t) \quad \text{for all } x, y, z\in X, s, t>0 \qquad (NA)$$

Then the triple (X, M, N, *, ◊) is called a *non-Archimedean Intuitionistic fuzzy metric space*(NAIFMS).

Remark 2.4. It is easy to check that the triangular inequality (NA) implies, that every non-Archimedean Intuitionistic fuzzy metric space is intuitionistic fuzzy metric space.

Definition 2.5[18] Let (X, M, N, *, ◊) be a non-Archimedean Intuitionistic fuzzy metric space .

(a) A sequence $\{x_n\}$in X is called an *Cauchy sequence*, if for each ε ∈ (0,1) and t>0 there exists $n_0\in N$ such that $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_{n+p}, t) = 0$ for all p=0,1,2…..

(b) A sequence $\{x_n\}$ in a non-Archimedean Intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be *convergent* to x∈X

$$\lim_{n\to\infty} M(x_n, x, t)=1 \ , \ \lim_{n\to\infty} N(x_n, x, t)= 0 \quad \text{for all } t>0.$$

(c) A non-Archimedean Intuitionistic fuzzy metric space (X, M, N, *, ◊) is called *complete* if every Cauchy sequence is convergent in X.

Definition 2.6. [15] A partially ordered set is a set P and a binary relation ≼, denoted by (X, ≼) such that for all a, b, c ∈ P,

(a) a ≼ a(reflexivity),

(b) a≼ b and b≼ c implies a≼ c(transitivity),≼

(c) a≼ b and b≼ a implies a = b(anti-symmetry).

Definition 2.7[15]: Let (X, ≼) be a partially ordered *set* and F: X×X→X .The mapping F is said to have *k-monotone property* if

$$x_0 \preccurlyeq x_1 ,\ y_0 \succcurlyeq y_1 \Rightarrow F(x_0, y_0) \preccurlyeq F(x_1, y_1) \ \& \ F(y_0, x_0) \preccurlyeq F(y_1, x_1) \quad \text{for all } x_0,x_1,y_0,y_1\in X$$

Definition 2.8.[15]. Let (X, $\preccurlyeq$) be a partially ordered *set* and F: X×X→X .The mapping F is said to have mixed monotone property if F(x, y) is monotone non-decreasing in first coordinate and is monotone non-increasing in second coordinate . i.e. for any x,y ∈ X ,

$$x_0 \preccurlyeq x_1 \Rightarrow F(x_0, y) \preccurlyeq F(x_1, y)$$

& $$y_0 \preccurlyeq y_1 \Rightarrow F(x, y_0) \succcurlyeq F(x, y_1) \quad \text{for all } x_0,x_1,y_0,y_1 \in X$$

Remark 2.5. Thus mixed monotone property is particular case of k-monotone property.

Example 2.1. Let X=[2, 64] on the set X, we consider following relation $x \preccurlyeq y \Leftrightarrow x \le y$,Where $\preccurlyeq$ is a usual ordering, (X, $\preccurlyeq$) a partial order set .
We define F: X×X→X. as F(x, y) = x+[1/y] ,Where [k] represents greatest integer just less than or equal to k.
One can verify that F(x, y) follows k-monotone property.

Definition 2.9 [19]. An element (x, y)∈X×X→X is called a *coupled fixed point* of the mapping F: X×X→X if F(x, y)=x & F(y, x)=y .

3. Main Results:

Theorem 3.1: Let (X, $\preccurlyeq$) be a partially ordered set and (X, M, N, *, ◇) is a complete Non-Archimedean Intuitionistic fuzzy metric space. Let F: X×X→X be a continuous mapping having k-monotone property on X . Assume that for every ε∈ (0,1) with

$$M(F(x,y),F(u,v),t) \ge 1-\frac{\varepsilon}{2}\max\{M(F(x,y),x,t),M(x,F(u,v),t),M(F(x,y),u,t),M(u,F(u,v),t)\}$$

$$N(F(x,y),F(u,v),t) \le 1-\frac{\varepsilon}{2}\min\{N(F(x,y),x,t),N(x,F(u,v),t),N(F(x,y),u,t),N(u,F(u,v),t)\} \quad \text{(I)}$$

for all x, y, u, v ∈ X with $x \succcurlyeq u$ and $y \preccurlyeq v$.
If there exists $x_0, y_0, x_1, y_1 \in X$, such that $x_0 \preccurlyeq x_1$, $y_0 \succcurlyeq y_1$, where $x_1=F(x_0, y_0)$ & $y_1 = F(y_0,x_0)$ then there exists x, y, ∈ X such that F(x, y) = x & F(y, x)=y.

Proof: Let $x_0,x_1,y_0,y_1 \in X$ be such that $x_0 \preccurlyeq x_1$, $y_0 \succcurlyeq y_1$ .where $x_1=F(x_0, y_0)$ & $y_1 = F(y_0,x_0)$
we construct sequences $\{x_n\}$ & $\{y_n\}$ in X as follows

$$x_{n+1}=F(x_n, y_n) \ \&\ y_{n+1} = F(y_n, x_n) \quad \text{for all } n\ge 0$$

we shall show that $x_n \preccurlyeq x_{n+1}$ and $y_n \succcurlyeq y_{n+1}$ for all n≥0
Since $x_0 \preccurlyeq x_1$, $y_0 \succcurlyeq y_1$, therefore by k-monotone property

$$x_1=F(x_0, y_0) \preccurlyeq F(x_1,y_1)=x_2 \text{ and } y_1=F(y_0, x_0) \succcurlyeq F(y_1,x_1)=y_2$$

i. e. $x_1 \preccurlyeq x_2$, $y_1 \succcurlyeq y_2$,
again applying the same property we have

$$x_2=F(x_1, y_1) \preccurlyeq F(x_2,y_2)=x_3 \text{ and } y_2=F(y_1, x_1) \succcurlyeq F(y_2,x_2)=y_3$$

Continue in this manner we shall have,

$$x_0 \preccurlyeq x_1 \preccurlyeq x_2 \ldots \preccurlyeq x_n \preccurlyeq x_{n+1} \preccurlyeq \ldots \text{ and } y_0 \succcurlyeq y_1 \succcurlyeq y_2 \ldots \succcurlyeq y_n \succcurlyeq y_{n+1} \succcurlyeq \ldots$$

Since $x_{n-1} \preccurlyeq x_n$ and $y_{n-1} \succcurlyeq y_n$, from (1) we have,

$$M(F(x_n,y_n),F(x_{n-1},y_{n-1}),t) \ge 1-\frac{\varepsilon}{2}\max\left\{\begin{array}{l} M(F(x_n,y_n),x_n,t),M(x_n,F(x_{n-1},y_{n-1}),t), \\ M(F(x_n,y_n),x_{n-1},t),M(x_{n-1},F(x_{n-1},y_{n-1}),t) \end{array}\right\}$$

$$=1-\frac{\varepsilon}{2}\max\left\{\begin{array}{l} M(x_{n+1},x_n,t),M(x_n,x_n,t), \\ M(x_{n+1},x_{n-1},t),M(x_{n-1},x_n,t) \end{array}\right\}$$

$$=1-\frac{\varepsilon}{2}\max\{M(x_{n+1},x_n,t),1,M(x_{n+1},x_{n-1},t),M(x_{n-1},x_n,t)\}$$

$$=1-\frac{\varepsilon}{2}>1-\varepsilon$$

i.e. $M\left(x_{n+1},x_{n,},t\right)>1\text{-}\varepsilon$

and $N\left(F\left(x_{n,},y_n\right),F\left(x_{n-1},y_{n-1}\right),t\right)\ \le 1\text{-}\frac{\varepsilon}{2}\min\left\{\begin{matrix} N\left(F\left(x_{n,},y_n\right),x_{n,},t\right),N\left(x_{n,},F\left(x_{n-1},y_{n-1}\right),t\right), \\ N\left(F\left(x_{n,},y_n\right),x_{n-1},t\right),N\left(x_{n-1},F\left(x_{n-1},y_{n-1}\right),t\right)\end{matrix}\right\}$

$$=1\text{-}\frac{\varepsilon}{2}\min\left\{\begin{matrix} N\left(x_{n+1},x_{n,},t\right),N\left(x_{n,},x_n,t\right), \\ N\left(x_{n+1},x_{n-1},t\right),N\left(x_{n-1},x_n,t\right)\end{matrix}\right\}$$

$$=1\text{-}\frac{\varepsilon}{2}\min\left\{N\left(x_{n+1},x_{n,},t\right),0,N\left(x_{n+1},x_{n-1},t\right),N\left(x_{n-1},x_n,t\right)\right\}$$

$$=1-\frac{\varepsilon}{2}<1-\varepsilon$$

i.e. $N\left(x_{n+1},x_{n,},t\right)<1\text{-}\varepsilon$

Similarly we can show that $M(x_{n+1},x_{n+2},t) > 1\text{-}\varepsilon$

So for all $\varepsilon>0$, there exists $n_0\in N$ such that for all $m > n > n_0$ and $t >0$ we have

$M(x_n, x_m, t) \quad \ge M(x_n,x_{n+1},t)*M(x_{n+1},x_{n+2},t)*\ldots\ldots M(x_{m-1},x_m,t)$

$M(x_n, x_m,t) \quad \ge(1\text{-}\varepsilon)*(1\text{-}\varepsilon)*(1\text{-}\varepsilon)*\ldots\ldots*(1\text{-}\varepsilon)$

$\Rightarrow \quad M(y_{n+1},y_{n+2},t) \quad > 1\text{-}\varepsilon$

And $\quad N(x_n,x_m,t) \quad \le N(x_n,x_{n+1},t)\lozenge M(x_{n+1},x_{n+2},t)\lozenge\ldots\ldots \lozenge M(x_{m-1},x_m,t)$

$N(x_n,x_m,t) \quad \le (1\text{-}\varepsilon)\lozenge(1\text{-}\varepsilon)\lozenge(1\text{-}\varepsilon)\lozenge\ldots\ldots\lozenge(1\text{-}\varepsilon)$

$\Rightarrow \quad N(y_{n+1},y_{n+2},t) \quad < 1\text{-}\varepsilon$

This shows that the sequence $\{x_n\}$ is Cauchy sequence in X and since X is complete fuzzy metric space it converges to a point $x\in X$ i.e. $\lim_{n\to\infty} x_n = x$

again since $y_{n-1} \succcurlyeq y_n$, $x_{n-1}\preccurlyeq x_n$, from (1) we have,

$M\left(F\left(y_{n-1},x_{n-1}\right),F\left(y_{n,},x_n\right),t\right)\ \ge 1\text{-}\frac{\varepsilon}{2}\max\left\{\begin{matrix} M\left(F\left(y_{n-1},x_{n-1}\right),y_{n-1},t\right), \\ M\left(y_{n-1},F\left(y_{n,},x_n\right),t\right),M\left(F\left(y_{n-1},x_{n-1}\right),y_{n,},t\right),M\left(y_{n,},F\left(y_{n,},x_n\right),t\right)\end{matrix}\right\}$

$$=1-\frac{\varepsilon}{2}\max\left\{\begin{matrix} M\left(y_{n,},y_{n-1},t\right),M\left(y_{n-1},y_{n+1},t\right), \\ M\left(y_{n,},y_{n,},t\right),M\left(y_{n,},y_{n+1},t\right)\end{matrix}\right\}$$

$$=1-\frac{\varepsilon}{2}\max\left\{M\left(y_{n,},y_{n-1},t\right),M\left(y_{n-1},y_{n+1},t\right),1,M\left(y_{n,},y_{n+1},t\right)\right\}$$

$$=1-\frac{\varepsilon}{2}>1-\varepsilon$$

$M\ (y_{n+1},\ y_{n,}\ t) >1\text{-}\varepsilon$

similarly we can show that $M(y_{n+1},\ y_{n+2,}\ t) >1\text{-}\varepsilon$

So for all $\varepsilon > 0$, there exists $n_0\in N$ such that for all $m> n> n_0$ and $t>0$ we have

$M(y_n,\ y_{m,}\ t) \ge M\ (\ y_n,y_{n+1},t)*M\ (\ y_{n+1},y_{n+2},t)*\ldots\ldots*M\ (\ y_{m-1},y_m,t)$

$M\ (y_n,\ y_m,\ t)> (1\text{-}\varepsilon)*(1\text{-}\varepsilon)*\ldots\ldots*(1\text{-}\varepsilon)$

And $y_1= F(y_0,x_0)$

$N(y_n,\ y_{m,}\ t) \ge N\ (\ y_n,y_{n+1},t)\lozenge N\ (\ y_{n+1},y_{n+2},t)\lozenge\ldots\ldots\lozenge N\ (\ y_{m-1},y_m,t)$

$N(\ y_n,\ y_m,\ t)< (1\text{-}\varepsilon)\lozenge(1\text{-}\varepsilon)\lozenge\ldots\ldots\lozenge(1\text{-}\varepsilon)$

This shows that the sequence $\{y_n\}$ is Cauchy sequence in X and since X is complete fuzzy metric space it converges to a point $y\in X$ i.e. $\lim_{n\to\infty}y_n=y$

Since F is given continuous therefore using convergence of $\{x_n\}$ and $\{y_n\}$ we have, $F(x, y)=x$ & $F(x, y)=y$.

Now we shall define a partial order relation over non-Archimedean fuzzy metric space and prove a coupled fixed point theorem using that relation.

Lemma6.3.2: Let (X, M, N, *, ◊) be a a non-Archimedean Intuitionistic fuzzy metric space with $a*b \geq \max\{a+b-1, 0\}$ and $a \lozenge b \leq \min\{a+b-1,0\}$ with $\phi: X\times X\times[0,\infty)\to R$, define the relation "$\preccurlyeq$" on X as follows $x \preccurlyeq u$, $y \succcurlyeq v \Leftrightarrow M(x, u, t)M(y, v, t)\geq 1+\phi(x, y, t)-\phi(u, v, t)$ for all t>0 then "$\preccurlyeq$" is partial order on X, called the partial order induced by ϕ.

Proof: The relation "$\preccurlyeq$" is a reflexive relation: let $x, y \in X$ be any element

Since $M(x, x, t)M(y, y, t)=1=1+\phi(x, y, t)-\phi(u, v, t)$ for all $x, y \in X$

Therefore "$\preccurlyeq$"is a reflexive relation (i)

For any $x, y, u, v \in X$ suppose that $x \preccurlyeq u$, $y \succcurlyeq v$, $x \succcurlyeq u$, $y \preccurlyeq v$ then we have.

$$x \preccurlyeq u,\ y \succcurlyeq v \Leftrightarrow M(x, u, t)M(y, v, t)\geq 1+\phi(x, y, t)-\phi(u, v, t) \quad \text{(I)}$$

and

$$x \succcurlyeq u,\ y \preccurlyeq v \Leftrightarrow M(u, x, t)M(v, y, t)\geq 1+\phi(u, v, t)-\phi(x, y, t) \quad \text{(II)}$$

Adding (I) & (II), we get,

$$2M(x, u, t)\, M(y, v, t) \geq 2$$

Or

$$M(x, u, t)\, M(y, v, t) \geq 1$$

$$M(x, u, t)\, M(y, v, t)=1 \Rightarrow M(x, u, t)=1 ,\ M(y, v, t)=1$$

i.e. x=u & y=v

Therefore "$\preccurlyeq$" is antisymmetric relation. (ii)

If $x \preccurlyeq u$, $y \succcurlyeq v$, $u \preccurlyeq u$, $v \succcurlyeq v$,

We have, $M(x, u', t)\, M(y, v', t) \geq M(x, u, t)M(y, v, t)*M(u, u', t)\, M(v, v', t)$

$$=\max[M(x, u, t)M(y, v, t)+ M(u, u', t)\, M(v, v', t)-1,0]$$
$$=\max[1+\phi(x, y, t)-\phi(u, v, t)+1+\phi(u, v, t)-\phi(u', v', t)-1,0]$$
$$=\max[1+\phi(x, y, t)-\phi(u', v', t),0]$$
$$=1+\phi(x, y, t)-\phi(u', v', t) \text{ i.e. } x \preccurlyeq u',\ y \succcurlyeq v'$$

And

$N(x, u', t)\, N(y, v', t)\leq N(x, u, t)N(y, v, t)\lozenge N(u, u', t)\, N(v, v', t)$

$$=\max[N(x, u, t)N(y, v, t)+ N(u, u', t)\, N(v, v', t)-1,0]$$
$$=\max[1+\phi(x, y, t)-\phi(u, v, t)+1+\phi(u, v, t)-\phi(u', v', t)-1,0]$$
$$=\max[1+\phi(x, y, t)-\phi(u', v', t),0]$$
$$=1+\phi(x, y, t)-\phi(u', v', t) \text{ i.e. } x \preccurlyeq u',\ y \succcurlyeq v'$$

Thus "$\preccurlyeq$" is transitive relation. (iii)

Theorem 3.3: Let (X, M, N, *, ◊) be a a non-Archimedean Intuitionistic fuzzy metric space With $a*b \geq \max\{a+b-1,0\}$ and $a \lozenge b\leq \min\{a+b-1,0\}$ with $\phi: X\times X\times[0,\infty) \to R$, bounded from above "$\preccurlyeq$" the partial order induced by ϕ if $F:X\times X\to X$ follows k-monotone property over X and there are $x_0, y_0, x_1, y_1 \in X$,such that $x_0 \preccurlyeq x_1$, $y_0 \succcurlyeq y_1$, where $x_1=F(x_0, y_0)$ & $y_1 = F(y_0,x_0)$ then there exists $x, y, \in X$ such that $F(x, y) = x$ & $F(y, x)=y$.

Proof: Let $x_0, y_0, x_1, y_1 \in X$,such that $x_0\preccurlyeq x_1$, $y_0 \succcurlyeq y_1$, where $x_1=F(x_0, y_0)$ & $y_1 = F(y_0,x_0)$.we construct sequences $\{x_n\}$ & $\{y_n\}$ in X as follows $x_{n+1}= F(x_n, y_n)$ &$y_{n+1}= F(y_n, x_n)$ for all $n \geq 0$.

we shall show that $x_n\preccurlyeq x_{n+1}$ and $y_n\succcurlyeq y_{n+1}$ for all $n\geq 0$

Since $x_0\preccurlyeq x_1$, $y_0 \succcurlyeq y_1$, therefore by k-monotone property

$$x_1=F(x_0, y_0) \preccurlyeq F(x_1,y_1)=x_2 \text{ and } y_1=F(y_0, x_0) \succcurlyeq F(y_1,x_1)=y_2$$

i. e. $x_1\preccurlyeq x_2$, $y_1 \succcurlyeq y_2$,

again applying the same property we have

$$x_2=F(x_1, y_1) \preccurlyeq F(x_2,y_2)=x_3 \text{ and } y_2=F(y_1, x_1) \succcurlyeq F(y_2,x_2)=y_3$$

Continue in this manner we shall have,

$$x_0\preccurlyeq x_1\preccurlyeq x_2\ldots\ldots \preccurlyeq x_n\preccurlyeq x_{n+1}\preccurlyeq\ldots \text{ and } y_0\succcurlyeq y_1\succcurlyeq y_2\ldots\ldots \succcurlyeq y_n\succcurlyeq y_{n+1} \succcurlyeq\ldots$$

By the definition of "$\preccurlyeq$" we have , for all t>0 $\phi(x_0, y_0,t) \preccurlyeq \phi(x_1, y_1,t) \preccurlyeq \phi(x_3, y_3,t) \preccurlyeq$……….. In other words, for all t>0, the sequence$\{\phi(x_n,y_n,t)\}$ is non decreasing in R. Since $\phi$is bounded above, and $\{\phi(x_n,y_n,t)\}$ is convergent and hence it is a Cauchy sequence . So, for all $\varepsilon>0$, there exists $n_0 \in N$ so that for all $m>n>n_0$ and t>0 we have,

$$\left|\phi(x_m, y_m, t)- \phi(x_n, y_n,t)\right|<\varepsilon$$

Since $x_n \preccurlyeq x_m$ & $y_n \succcurlyeq y_m$, we have

$$x_n \preccurlyeq x_m \;\&\; y_n \succcurlyeq y_m \Leftrightarrow M(x_n, x_m, t)\, M(y_n, y_m, t) \geq 1+\phi(x_n, y_n,t)- \phi(x_m, y_m,t) \text{ for all } t>0$$

$$1-[\phi(x_m, y_m, t) -\phi(x_n, y_n, t)] >1-\varepsilon$$

$$x_n \preccurlyeq x_m \;\&\; y_n \succcurlyeq y_m \Leftrightarrow N(x_n ,x_m, t)\, N(y_n, y_m, t) \leq 1+\phi(x_n,y_n,t)- \phi(x_m, y_m,t) \text{ for all } t>0$$

$$1-[\phi(x_m,y_m,t) -\phi(x_n,y_n,t)] <1-\varepsilon$$

We claim that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X, if not then there exists some ε_1 , ε_2 such that$\varepsilon_1 < \varepsilon_2$ and

$$M (x_n, x_m,t) \leq (1-\varepsilon_1) \;\&\; M (y_n, y_m,t) \leq (1-\varepsilon_2)$$

$$\text{Then } M (x_n,x_m,t)\, M (y_n, y_m,t) \leq (1-\varepsilon_1)) (1-\varepsilon_2) < (1-\varepsilon_1))^2 < (1-\varepsilon_1))$$

And

$$N (x_n,x_m,t) \leq (1-\varepsilon_1) \;\&\; N(y_n,y_m,t) \leq (1-\varepsilon_2)$$

$$\text{Then } N (x_n,x_m,t)\, N (y_n,y_m,t) \leq (1-\varepsilon_1)) (1-\varepsilon_2) < (1-\varepsilon_1))^2 < (1-\varepsilon_1))$$

Which is a contradiction.

This shows that the sequence $\{x_n\}$ & $\{y_n\}$ a Cauchy sequence in X, since X is complete , these converges to points x, y respectively in X consequently, by the continuity of F, we have F(x,y)=x & F(y,x)=y.

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