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Modeling Volatility in Nigeria Foreign Exchange Market Using GARCH-type Models

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Abstract
In this study, the performance of GARCH-type model is considered in modelling Nigeria foreign exchange returns. The datasets consists of the foreign exchange of Nigeria naira for the periods before recession and during recession. It is observed that volatility is higher during recession than when there was no recession. Model selection criteria based on Hannan-Quinn Information Criterion (HQIC) shows that Gaussian process is least considered model to capture the variability in foreign exchange rate returns in Nigeria, but student's t and Generalized Error distribution are more suitable, therefore forecast performance was used to access each of the Asymmetric models. The empirical analysis shows that GARCH (1, 1) and gjrGARCH (1, 1) with Student's t error distribution and iGARCH(1, 1), sGARCH(1,1), and csGARCH (1,1) are the best fitted models. Fifty days out-of-sample forecast shows that csGARCH (1, 1) based on Generalized Error distribution is the best predictive model based on Mean Square Error (MSE), and sGARCH based on Mean Absolute Error (MAE) and Directional Absolute Error (DAE). The study recommends that future study should consider alternative error distributions with a view to realizing a more robust volatility forecasting model that could guarantee sound policy choices.
Keywords: Volatility, foreign exchange, GARCH-type models, Error Distributions

1. Introduction

Financial time series data often consist of periods of calm behaviour alternating with periods of very wild fluctuations. The study on the volatility of exchange rate is closely linked to the risk of assets, as volatility measures exposure to risk. Higher volatility leads to large variations of return, hence higher risk. Volatility of exchange rate provide useful information in measuring risk, and a number models are applied in forecasting exchange rate movement and evaluating the performance of the local currencies in international market. Statement made by (Hamadu and Adeleke 2009) which cannot be ignored is that forecasting currency exchange rate rates is an important financial problem that has recorded a great deal of attention particularly because of intrinsic difficulty and practical applications.

The issue of modelling exchange rate volatility has gained considerable importance in the research studies since 1973, when many countries shifted towards floating exchange rate from fixed exchange rate regime. Part of the studies were conducted to understand the behaviour of exchange rate and to explain the sources of its movements and fluctuations. There has been excessive volatility of the Nigeria Naira against major foreign currencies in the exchange market since the adoption of flexible exchange–rate regimes in 1986. Therefore, continuous exchange rate volatility was thought to have led to currency crises, distortion of production patterns as well as sharp fluctuations in external reserve (Bala and Asemota 2013).

Exchange rate volatility is a major challenge facing development of an economy, making planning more problematic and investment more risky. Nigeria being a developing nation highly dependent on foreign trade, and these trades relies on exchange rate. This show that the impact of exchange rate variability on economies especially developing ones is not only in one direction.

Many studies have adopted diverse techniques in modelling exchange rate volatility. Hamadu and Adeleke (2009) modelled and compared Multilayer Perception Back Propagation Neural Network (MLPBPNN) model with several models, along with ARIMA generated by Expert Modeler System (EMS) to model Nigerian foreign exchange while Adeleke *et.* al., (2015) modelled daily exchange rate using extreme value theory, among others.

In modelling volatility, popular and frequently applied models to estimate exchange rate volatility are the autoregressive conditional heteroscedastic (ARCH) model advanced by Engle (1982) and generalized ARCH (GARCH) model developed independently by Bollerslev (1986) and Taylor (1987) considered to be symmetric. Extension of the symmetric GARCH is the like of EGARCH, IGARCH, TGARCH, fGARCH, GJRGARCH, CSGARCH, TGARCH, among others. The GARCH-type model is a popular type of model being used to model stock and exchange rate volatility. Lim and Sek (2013) used both GARCH-types to model and identify the superior model in capturing the characteristics of stock market at different type.

In the recent times, the GARCH-type of models has been adopted in various capacities. Hu and Tsay (2014) consider a sample estimate of generalized kurtosis matrix and proposed test statistics for detecting linear combinations that do not have conditional heteroscedascity, they applied the test to weekly log returns of seven exchange rates against US dollars. Kalli and Griffin (2015) proposed stochastic Volatility (SV) model drawing strength from autoregressive SV models, aggregation of autoregressive process, and Bayesian nonparametric modelling to model daily returns on stock, which can equally be applied to foreign exchange data. Musa and Abubakar (2014) studied volatility using GARCH and Asymmetric Models to model daily Dollar/Naira exchange rate using GARCH (1, 1), GJR-GARCH (1, 1), TGARCH (1, 1) and TS-GARCH (1, 1) models by using daily data.

Financial time series data are not normally distributed, they is inherently fat tailed, and measures needs to be taken to standardize the data before a modelling .A natural choice of transformation is the logarithm transformation. Barndoff-Nielson and Shephard (2005) showed that the finite sample distribution of log transformation of realized volatility is closer to the asymptotic standard normal distribution than that of a non-transformed version of realized volatility.

In this study GARCH-type models would be used to further investigate and compare the volatility of foreign exchange at different time frames using different distribution model of Gaussian, student's t and Generalized Error Distribution (GED). These models is used to measure volatility of Nigeria naira exchange rate against United State (US) Dollars for the period when Nigeria is considered to be in recession and the period when the economy was relatively not in recession, using the symmetric and asymmetric GARCH-type model and draw a comparison among the models using Hannan-Quinn Information Criteria for model selection, after which model performance are assessed using Mean Square Error (MSE), Mean Aboslute Error (MAE) and Directional Accuracy Error (DAC).

The remaining part of is paper is follows is as follows: in section 2, related studies are reviewed, in section 3, the Materials and methods used in carrying out the analysis is outlined, and section 4 contains data and estimation techniques. While in section 5 contains summary and conclusion.

2. Review of Related Works

Over the last few decades, exchange rate and fluctuations have become an important subject of macroeconomic analysis and have received a great deal of interest in the academics domain, financial analyst and policy makers, Abdalla (2012) identified that the study of exchange rate has become of great interest, particularly after the collapse of the Bretton Woods agreement of fixed exchange rates among major industrial countries. Abdalla (2012) pointed out that since that occurrence; there has been an extensive debate about the topic of exchange rate volatility and its potential influence on the economy and also its role in investment analysis, security valuation, profitability and risk management.

The need to model exchange rate volatility has brought about development of models in empirical finance, which have a solid background in statistics, to investigate this volatility across different regions and countries. Common and frequently applied models to model exchange rate volatility are the autoregressive conditional heteroscedastic (ARCH) model advanced by Engle (1982) and generalized (GARCH) model developed independently by Bollerslev (1986) and Taylor (1987).

ARCH model relates the conditional variance of the disturbance term to the linear combination of the squared disturbance in the recent past. ARCH model have been used to model financial time series because of its inherent potentials. In modelling volatility, is important to know the optimal lag length of the data, (Atoi 2014) pointed out that determining the optimal lag length is cumbersome; oftentimes engender over-parametrization, and Rydberg (2000) claimed that large lag values are required in ARCH models, thus the need for many parameters. However, Bollerslev (1986) and Taylor (1987) independently proposed the extension of ARCH model with an Autoregressive Moving Average (ARMA) formulation, with a view to achieving much more reasonable result. The model is called the Generalized ARCH (GARCH), which models conditional variance as a function of its lagged values as well as squared lagged values of the disturbance term.

As much as GARCH model has been proven to be effective in capturing symmetric effect of volatility, it has some inherent limitations, such as the violation of non-negativity constraints imposed on the parameters to be estimated. To overcome these constraints, some extensions of the original GARCH model were proposed. This includes asymmetric GARCH family models such as Threshold GARCH (TGARCH) proposed by Zakoian (1994), Exponential GARCH (EGARCH) proposed by Nelson (1991) and Power GARCH (PGARCH) proposed by Ding *et al*. (1993).The EGARCH which captures asymmetric properties between returns and volatility was proposed to address three major deficiencies of GARCH model to address (i) parameter restrictions that ensures conditional variance positivity; (ii) non-sensitivity to asymmetric response of volatility to shock and (iii) difficulty in estimating persistence in a strongly stationary series. The log of the conditional variance in the EGARCH model signifies that the leverage effect not quadratic but exponential. Specifying volatility in terms of its logarithmic transformation implies the non-restrictions on the parameters to guarantee that the variance is positive (MaJose, 2010), which is a key advantage of EGARCH model over the symmetric GARCH model.

Zakoian (1994) specified the TGARCH model by allowing the conditional standard deviation to depend on sign of lagged innovation. The specification does not show parameter restrictions to guarantee the positivity of the conditional variance. However, to ensure stationarity of the TGARCH model, the parameters of the model have to be restricted and the choice of error distribution account for the stationarity. TGARCH model is closely related to GJR-GARCH model developed by Glosten *et al.* (1993).

Mandelbrot (1963) and Fama (1965) deduce that daily stock index returns are non-normal and tend to have leptokurtic and fat-tailed distribution. For this reason, Bollerslev (1986) relaxed the traditional normality assumption to accommodate time varying volatility in high frequency data by assuming that such data follows student t-distribution. Bollerslev *et al.* (1994) establish that a GARCH model with normally distributed errors could not be a sufficient model for explaining kurtosis and slowly decaying autocorrelations in return series.

Empirical studies on modelling foreign exchange in the Nigeria context is observed in the study carried out by Adeleke *et. al.,* (2015), the authors carried out a study on foreign exchange returns of Nigeria naira against nine competing foreign currencies. GARCH type models are equally used in modelling stock returns, just as Jayasuriya (2002) examines the effect of stock market liberalization on stock return volatility using Nigeria and fourteen other emerging market data to estimate asymmetric GARCH model. The study inferred that positive (negative) changes in prices have been followed by negative (positive) changes. Ogum *et al.,*(2005) apply the Nigeria and Kenya stock data on EGARCH model to capture the emerging market volatility but the result of the study does not go along with that of Jayasuriya (2002). Dallah and Adeleke (2010) examine the volatility of daily stock returns of Nigerian insurance stocks using twenty six insurance companies' daily data. The result of ARCH (1), GARCH (1, 1) TARCH (1, 1) and EGARCH (1, 1) shows that EGARCH is more suitable in modelling stock price returns as it outperforms the other models in model evaluation and out-of-sample forecast. Okpara and Nwezeaku (2009) randomly selected forty one companies from the Nigerian Stock Exchange to examine the effect of the idiosyncratic risk and beta risk on returns using data from 1996 to 2005. By applying EGARCH (1, 3) model, the result shows less volatility persistence and establishes the existence of leverage effect in the Nigeria stock market, implying that bad news drives volatility more than good news.

3. Materials and Methods

3.1 Model Specification

Generalised Autoregressive Conditional Heteroscedasticity (GARCH) Models.

There are several GARCH specifications for modelling the conditional variance, or volatility, of a variable. This study considers different GARCH equations to model *Naira* exchange rate volatility during the study period. In the standard GARCH (1, 1) model, first derived by Bollerslev (1986) replaces the $AR(p)$ representation with an $ARMA(p,q)$ representation:

$$y_t = x_t'\gamma + \mu_t \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\mu_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

Where σ_t^2 is the conditional variance equation, ω is the mean, μ_{t-1}^2 is the ARCH term, and σ_{t-1}^2 is the GARCH term. GARCH models are usually estimated using the method of maximum likelihood estimation (MLE), from (1) above which follows that

$\mu_t = y_t - x_t'\gamma$ and $\mu_{t-1} = (y_{t-1} - x_{t-1}'\gamma)$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i(\gamma_{t-1} + x_{t-1}'\gamma)^2 + \sum_{j=1}^{q}\beta\sigma_{t-1}^2 \tag{3}$$

Where the coefficients $\alpha_i (i = 0,1,......p)$ and $\beta_i (i = 0,1,......p)$ are all assumed to

be positive, so as to ensure that the conditional variance σ_t^2 is always positive.

EGARCH Model

The *exponential* GARCH (EGARCH) model proposed by Nelson (1991) allows for asymmetric effects between positive and negative asset returns. The specification for conditional variance is:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{q}\beta_j \log(\mu^2{}_{t-j}) + \sum_{j=1}^{p}\alpha_i\left|\frac{\mu_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r}\gamma_k\frac{\mu_{t-k}}{\sigma_{t-k}} \tag{4}$$

IGARCH Model

Another view of GARCH-type model is; if parameters of GARCH models are restricted to sum to one, and the constant term is dropped, it gives the *integrated* GARCH (IGARCH) model which is given by:

$$\sigma_t^2 = \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \mu_{t-i}^2 \quad (5)$$

The conditional variance in a typical GARCH (1,1) model is given as:

$$\sigma_t^2 = \omega + \alpha({\mu^2}_{t-1} - \omega) + \beta({\sigma^2}_{t-1} - \omega) \quad (6)$$

which shows mean reversion to ω, and is a constant for all time.

CGARCH Model

Another model we are considering in this study is the CGARCH. The component model CGARCH allows mean reversion to a varying level, q_t, such that:

$$\sigma_t^2 - q_t = \alpha({\mu^2}_{t-1} - q_{t-1}) + \beta({\sigma^2}_{t-1} - q_{t-1})$$

$$(7)$$

$$q_t = \omega + \rho(q_{t-1} - \omega) + \phi({\mu^2}_{t-1} - {\sigma^2}_{t-1})$$

(Bala & Asemola 2013) further showed that combining the transitory and permanent above equations, we have:

$$\sigma_t^2 = (1-\alpha-\beta)(1-\rho)\omega + (\alpha+\phi){\mu^2}_{t-1} - (\alpha\rho + (\alpha+\beta)\phi)(\beta-\phi){\mu^2}_{t-2}$$
$$+(\beta-\phi){\sigma^2}_{t-1} - (\beta\rho - (\alpha+\beta)\phi){\mu^2}_{t-2} \quad (8)$$

The above equation shows that the component model is a restricted GARCH (2, 2) model. The asymmetric component model combines the component with asymmetric TARCH model. This equation introduces asymmetric effects in the transitory equation and estimates model of the form:

$$q_t = \omega + \rho(q_{t-1} - \omega) + \phi({\mu^2}_{t-1} - {\sigma^2}_{t-1}) + \xi_1 z_{1t} \quad (9)$$

$$\sigma_t^2 - q_t = \alpha({\mu^2}_{t-1} - q_{t-1}) + \gamma({\mu^2}_{t-1} - q_{t-1})d_{t-1} + \beta({\mu^2}_{t-1} - q_{t-1}) + \xi_2 z_{2t} \quad (10)$$

where z is the exogenous variable and d is the dummy variable indicating negative shocks. Where $\gamma > 0$ shows presence of transitory leverage effects in the conditional variance.

The standard GARCH Model ('sGARCH')

The standard GARCH model (Bollerslev (1986)) may be written as:

$$\sigma_t^2 = \left(\omega + \sum_{j=1}^{m} \varsigma_j \upsilon_{jt}\right) + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (11)$$

With σ_t^2 denoting the conditional variance, ω the intercept and ε_t^2 the residuals from the mean filtration process discussed previously. The GARCH order is defined by (p,q), with possibly m external repressors υ_j which are passed pre-lagged. If variance targeting is used, then ω is replaced by,

$$\bar{\sigma}^2(1-\hat{P}) \quad (12)$$

Where $\bar{\sigma}^2$ is the unconditional variance of ε^2 which is consistently estimated by its sample counterpart at every iteration of the solver following the mean equation filtration, and $\bar{\upsilon}_j$ represents the sample mean of the j^{th} external repressors in the variance equation (assuming its stationarity), and $\hat{P}$ is the persistence and defined below. One of the key features of the observed behaviour of financial data which GARCH models capture is volatility clustering which may be quantified in the persistence parameter $\hat{P}$. For the 'sGARCH' model this may be calculated as,

$$\hat{P} = \sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{q} \beta_j \quad (13)$$

The unconditional variance of the model $\hat{\sigma}^2$, and related to its persistence, is,

$$\hat{\sigma}^2 = \frac{\hat{\omega}}{1-\hat{P}}, \tag{14}$$

Where $\hat{\omega}$ is the estimated value of the intercept from the GARCH model.

The GJR- GARCH model ('gjrGARCH')
The GJR GARCH model of Glosten, Jagannathan and Runkle (1993) models positive and negative shocks on the conditional variance asymmetrically via the use of the indicator function I ,

$$\sigma_t^2 = \left(\omega + \sum_{j=1}^{m} \varsigma_j \upsilon_{jt}\right) + \sum_{j=1}^{p}\left(\alpha_j \varepsilon_{t-j}^2 + \gamma_j I_{t-j} \varepsilon_{t-j}^2\right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{15}$$

Where j now represents the 'leverage' term. Because of the presence of the indicator function, the persistence of the model now crucially depends on the asymmetry of the conditional distribution used. The persistence of the model $\hat{P}$ is,

$$\hat{P} = \sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{q} \beta_j + \sum_{j=1}^{q} \gamma_{j\kappa} \tag{16}$$

where κ is the expected value of the standardized residuals z_t below zero (effectively the probability of being below zero),

$$\kappa = E[I_{t-j} z_{t-j}^2] = \int_{-\infty}^{0} f(z,0,1....)dz \tag{17}$$

where f is the standardized conditional density with any additional skew and shape parameters (. . .). In the case of symmetric distributions the value of κ is simply equal to 0.5.

4.0 Data and Estimation Technique
In this study, data of Nigeria foreign exchange rate against the USD for the periods when Nigeria was not considered to be in recession (period 1) and the period when Nigeria is considered to be in recession (Period 2) was collected from Central Bank of Nigeria (CBN), which is downloadable at *www.cenbank.org/ExchangeRateByCurrency.asp.* Period 1 covers August 31, 2015 to August 14, 2016. Period 2 covers August 31, 2016 to August 14, 2017 (period 2), that is, the period when Nigeria was declared to be in recession, up till the reference date.

Data transformation follows that, if S_t be the exchange rate of USD against NGN and let $r_t = \log\left(S_t / S_{t-1}\right)$, then the exchange rate of NGN against USD is $1/S_t$. This will yield the log returns. Statistical tests of normality of Anderson Darling, Craver Von-Mises, Jargue Berra and Kolmogorov-Smirnov was implemented on the data as seen in table 3.

$$r_t = \log\left(\frac{1/S_t}{1/S_{t-1}}\right). \tag{18}$$

The returns follow a lag 4 which is implemented in R software. In order to achieve the objective of this study, software by R Core Team (2017); A language and environment for statistical computing. "fGarch" package in R by Diethelm Wuertz, Yohan Chalabi with contribution from Michal Miklovic, Chris Boudt, Pierre Chausse and others (2016) is used to fit symmetric GARCH model. Also "rugarch" package in R by Alexios Ghalanos (2015) is used for Asymmetric GARCH-type model.

Table 1: Unit Root Test for Returns at different time frame: Alternative hypothesis: stationary

	Period 2	Period 1		
Test	Test Statistics	Test Statistics	Lag	p-value
Phillips-Perron	-71.576	-66.253	4	0.01
Augmented Dickey-Fuller	-5.4509	-5.3848	6	0.01
KPSS	0.052027	0.075488	3	0.1

The exchange rate of Nigeria Naira against USD was logged to reduce the variance and was transformed to a continuously compounded daily foreign exchange as given in (18) above. The return series was tested to determine the order of integration using Phillips-Perron, ADF and KPSS, and the result in table 1 shows that the series is stationary at level.

Table 2: Summary Statistics of the Nigeria Naira FOREX against US Dollars

	Mean	Median	SD	Skew	Kurtosis
Period 2 Exchange	0.003278	0.003279	5.3158e-06	-1.726779	4.84812
Period 2 Returns	1.132e-05	0.000e+00	0.001056797	0.1948035	3.497958
Period 1 Exchange	0.004729	0.005089	0.0007080984	-1.466891	0.2061622
Period 1 Returns	0.007566	0.000000	0.0478415	6.797294	46.83032

Table 2 describes the summary statistics of the stationary foreign exchange rate and returns for period 1 (when there was no recession) and period 2 (during recession). The table reveals positive mean daily returns of 0.004729 with standard deviation 0.0007080984 there was no recession which measures the riskiness of investing in foreign exchange to be 0.071 percent. Also, mean daily returns of 0.00001132 with standard deviation of 0.001056797 during the period of recession shows that the riskiness of investing in foreign exchange to be 0.1057 percent. The higher the standard deviation, the higher the volatility of the market and the riskier the equity traded. Therefore, period 2 shows higher volatility than period 1.

Table 3: Normality Test

	Period 2		Period 1	
Test	Test Statistics	p-value	Test Statistics	p-value
Anderson-Darling(A)	4.7738	7.453e-12	71.177	< 2.2e-16
Cramer-von Mises (W)	0.81778	1.091e-08	14.954	7.37e-10
Kolmogorov-Smirnov (D)	0.12927	4.35e-10	0.12927	4.35e-10
Jargue Berra (χ^2)	123.61	2.2e-16	23403	< 2.2e-16

A very high Jarque Berra (J-B) value (123.61) in table 3 and a very small corresponding p-value, following Anderson-Darling, Cramer-von Mises, Kolmogorov-Smirnov the null of normality was rejected for the data. To support the inference on normality, the skewness (6.797294) and (0.1948035) for period 1 and 2 respectively are greater than 0 (skewness of a normal distribution is 0) and the kurtosis (3.497958) and (46.83032) is higher than 3 (kurtosis of a normal distribution is 3). The positive skewness is an indication that the upper tail of the distribution is thicker than the lower tail which implies that the returns rises more often than it drops, reflecting the renewed confidence in the market. Information arising from the descriptive statistics supports the subjection of the return series to volatility models.

Model Selection

The Hannan-Quinn Information Criteria (HQIC) is used to identify a most best performing model for symmetric and asymmetric model for eighteen models, GARCH (1, 1), EGARCH (1, 1), SGARCH (1, 1), GJRGARCH (1,1), and CSGARCH(1,1) of which each model is subjected to HQ information criteria. A model with lower value of HQIC, shows better ability in modeling volatility. Table 4 shows coefficients for the five models in the case of normal distribution, the asterisk (*) ones indicate significant coefficients in each model. Results for GARCH (1,1) is not included because there was no convergence. Information criteria for the all the eighteen models are presented in table 6. The result shows that student's t and Generalized Error Distribution model are improvements the Gaussian process; therefore, fitting the GARCH-type model with Gaussian process is not advisable.

Table 4: Testing Volatility in Nigeria Stock Market using GARCH Model using Gaussian Process

USD Period 2						**USD Period 1**		
Model	Equation	Parameter	Coeff.	P-value	Log-Lik	Coeff.	P-value	Log-Lik
GARCH (1,1)	Mean	Intercept	-1.132e-04	0.00000*		5.790e-04	0.7697	
	Var.	Intercept	-1.132e-04	0.31	1356.86	6.292e-04	<2e-16 *	509.237
		ARCH	4.331e-01	<2e-16*		5.374e-01	0.0526	
		GARCH	6.313e-01	<2e-16*		1.000e-08	-	
EGARCH (1,1)	Mean	Intercept	-0.000274	00.00000*		0.00000	1.00000	
		Ar1	0.578653	0.012960		0.69983	0.114420	
		Ma1	0.076402	0.594160		0.27556	0.592024	2034.21
	Var.	Intercept	-0.191034	0.452706	1429.16	-0.71610	0.00048*	
		ARCH	-0.124836	0.097051		0.32016	0.174222	
		GARCH	0.981663	0.00000*		0.94677	0.00000*	
		Asymmetry	0.706504	0.002403		2.18644	0.221746	
IGARCH (1,1)	Mean	Intercept	0.000043	0.57424		-	-	
		Ar1	0.593438	0.00000		-	-	
		Ma1	0.152390	0.20813		-	-	
	Var	Intercept	0.000000	0.99998	1393.55	-	-	
		ARCH	0.114851	0.00000			-	
		GARCH	0.885149	0.00000		-	-	
SGARCH (1,1)	Mean	Intercept	-0.000261	0.98939		-0.003073	0.956149	
		Ar1	0.563169	0.95817		0.956149	0.0000*	
		Ma1	0.016425	0.99947		0.963342	0.0000*	
	Var	Intercept	0.000000	1.00000	1409.21	0.000000	1.00000	783.308
		ARCH	0.239421	0.98780		0.184813	0.93643	
		GARCH	0.744271	0.97802		0.811561	0.84267	
GJR GARCH (1,1)	Mean	Intercept	0.000098	0.99956		0.75599	0.00000*	
		Ar1	0.626623	0.97686		1.00000	0.00000*	
		Ma1	0.098530	0.99915		0.95151	0.00000*	
	Var	Intercept	0.000000	1.00000	1401.71	0.00000	0.99101	1082.77
		ARCH	0.046901	0.99957		0.62941	0.91141	
		GARCH	0.853301	0.98585		0.000000	1.00000	
		Asymmetry	0.163222	0.99851		0.73884	0.96291	
CSARCH (1,1)	Mean	Intercept	-0.000007	0.908000		0.000048	0.99903	
		Ar1	0.627946	0.00000*		0.712214	0.99528	
		Ma1	0.153491	0.083440		0.032829	0.99998	
	Var	Intercept	0.000000	0.998306	1396.28	0.000000	0.99945	969.843
		ARCH	0.050000	0.002695*		0.320499	0.99057	
		GARCH	0.700000	0.000000*		0.522957	0.99057	
		Asymmetry	0.000023	0.000000*		1.000000	0.99692	

Table 5: Weighted Ljung-Box Test on Standardized Squared Residuals

	Period 2		**Period 1**	
Model	Test	p-value	Test	p-value
GARCH(1,1)				
Q (10)	19.76463	0.03155	0.06091574	1.0000*
Q (15)	22.03144	0.10698*	0.07893673	1.0000*
Q (20)	26.84633	0.13964*	0.0913382	1.0000*
EGARCH(1,1)				
Lag 1	0.9351	0.3335*	0.004442	0.9469*
Lag 5	3.7879	0.2817*	0.013560	1.0000*
Lag 9	7.2784	0.1769*	0.02296	1.0000*
SGARCH(1,1)				
Lag 1	0.0523	0.81911*	0.0002186	0.9882*
Lag 5	6.1477	0.08301*	0.0105707	0.9995*
Lag 9	11.5856	0.02251	0.0262539	1.0000*
GJRGARCH(1,1)				
Lag 1	0.2279	0.6331*	0.02931	0.8641*
Lag 5	1.9328	0.6344*	0.03885	0.9997*
Lag 9	3.3296	0.7035*	0.05275	1.0000*
CSGARCH(1,1)				
Lag 1	0.1901	0.66282*	0.009834	0.9210*
Lag 5	7.9447	0.03042	0.429515	0.9680*
Lag 9	13.4302	0.00847	0.73426	0.9947*

Table 5 consist of Weighted Ljung-Box Test on Standardized Squared Residuals for the models in the 2 periods, the null hypothesis of no serial correlation serial correlation is retained at 5% level of significance. The asterisk (*) shows the model which do not have serial correlation at a given Lag. Table 5 shows that all the models are not serially correlated in period one, but in period 2, Q (10) for GARCH (1, 1), lag 9 for SGARCH (1,1), lag 5 and lag 9 for CSGARCH (1,1) are found not to be serial correlated.

Table 6: Hannan-Quinn Information Criteria for Symmetric and Asymmetric models

Period 2				Period 1		
Model	Normal distribution	Student t	Generalized Error Distribution	Normal distribution	Student t	Generalized Error Distribution
GARCH(1,1)	-11.63867	-	-	-4.3315	-	-
EGARCH(1,1)	**-12.218**	**-12.865**	-12.251	**-7.4340**	-12.865	-19.718
IGARCH(1,1)	-11.940	-12.215	-15.003	-	-15.707	-17.915
SGARCH(1,1)	-12.061	-12.256	-18.678	-6.6650	-9.2320	**-20.316**
GJRGARCH(1,1)	-11.981	-12.184	-11.905	-9.2320	-14.122	-9.7694
CSGARCH(1,1)	-11.872	-12.550	**-20.337**	-8.2438	**-17.948**	-19.579

Table 7: Forecast Performance Measures for Asymmetric Models for (Period 2)

	EGARCH(1,1)	IGARCH(1,1)	SGARCH(1,1)	GJRGARCH(1,1)	CSGARCH(1,1)
MSE	1.199e-06	1.076e-06	1.209e-06	1.084e-06	**1.056e-06**
MAE	7.444e-04	7.599e-04	**7.286e-04**	7.669e-04	7.594e-04
DAC	5.200e-01	5.200e-01	**4.800e-01**	5.200e-01	5.200e-01

Forecast performance is sought for the models in period 2 based on student-t and GED, picking the least value across table 6, and assessing their forecast performance based on the Mean Square Error (MSE), Mean Absolute Error (MAE) and Directional Accuracy Error (DAC).

Subsequently, last fifty out days 'out of sample forecast' was carried for the using GED. Figures 1 to 5 shows actual and forest charts representing eGARCH (1,1) using student t, iGARCH(1,1) using GED, sGARCH (1,1) using GED, gjrGARCH (1,1) using student t and csGARCH using GED based on performance recommended by HQIC.

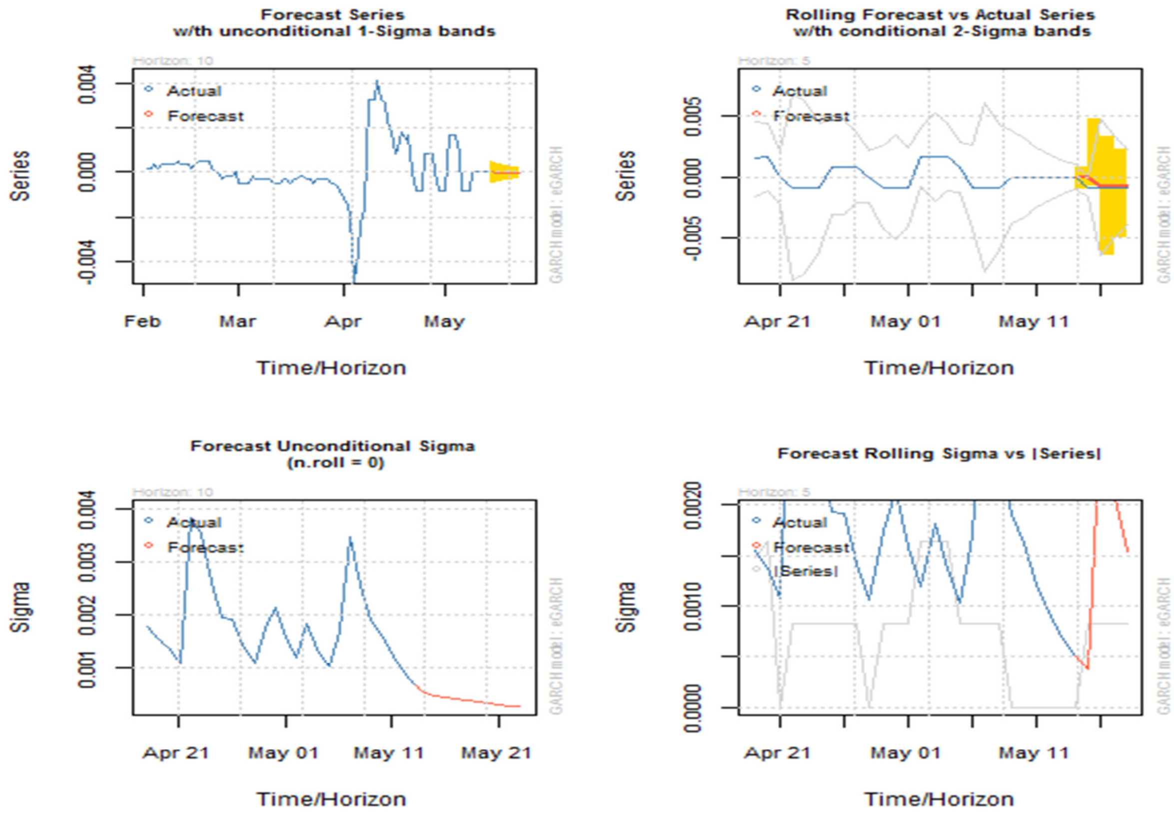

Figure 1: Actual and Forest Charts for eGARCH (1,1)

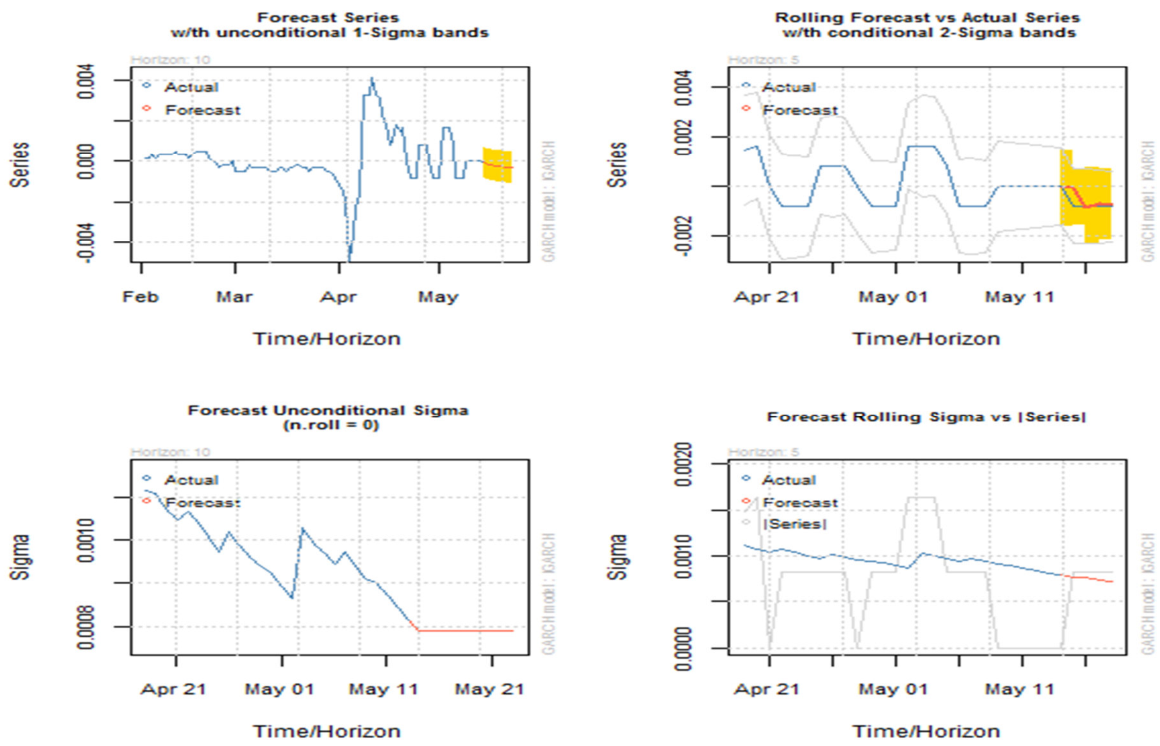

Figure 2: Actual and Forest Charts for iGARCH (1,1)

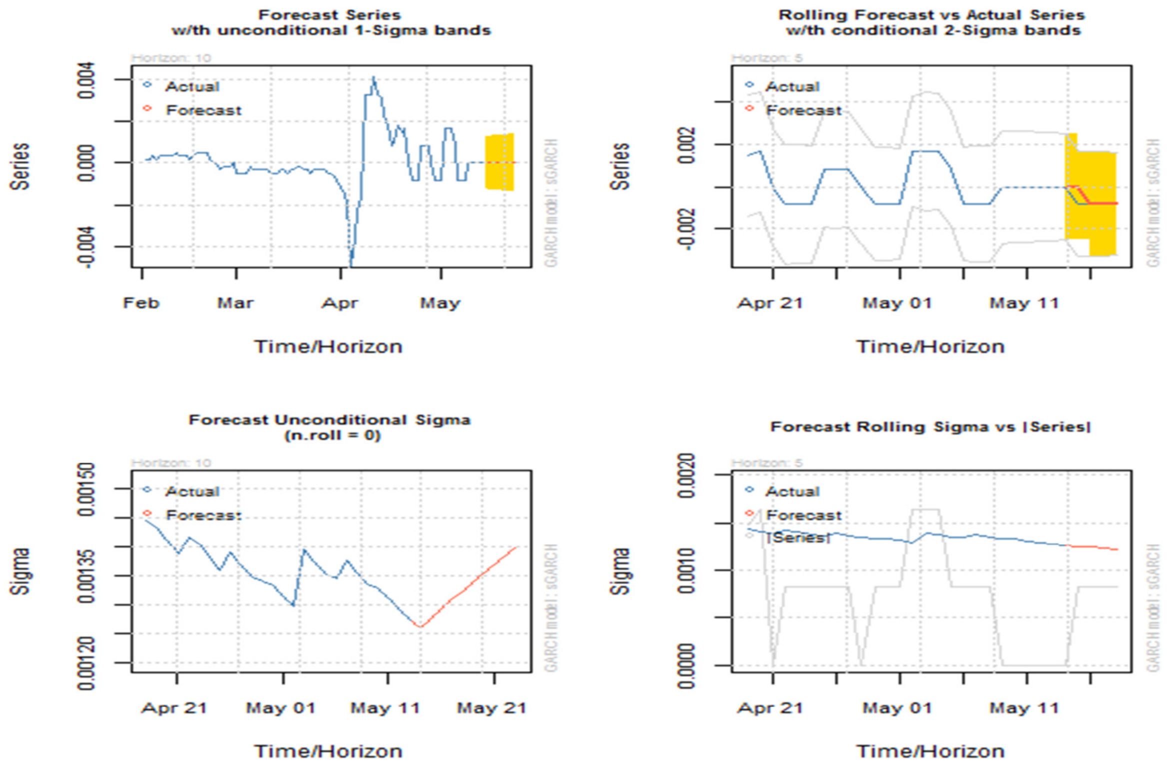

Figure 3: Actual and Forest Charts for sGARCH (1,1)

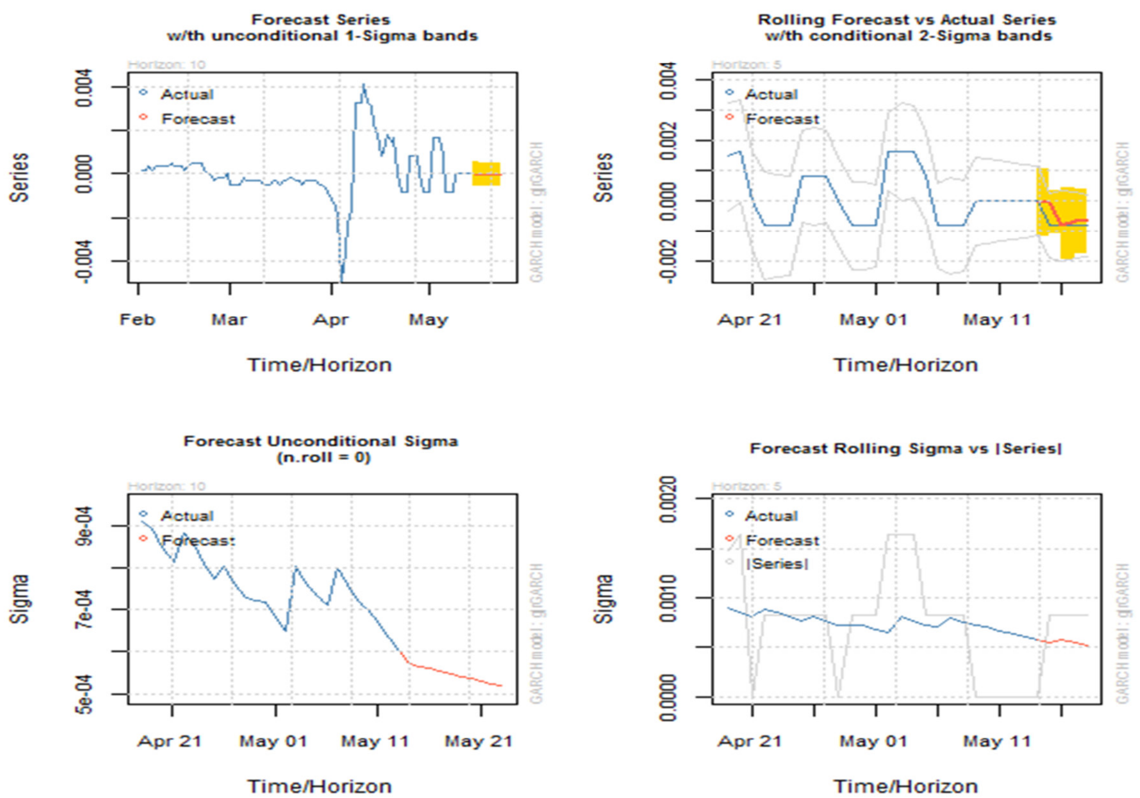

Figure 4: Actual and Forest Charts for gjrGARCH (1,1)

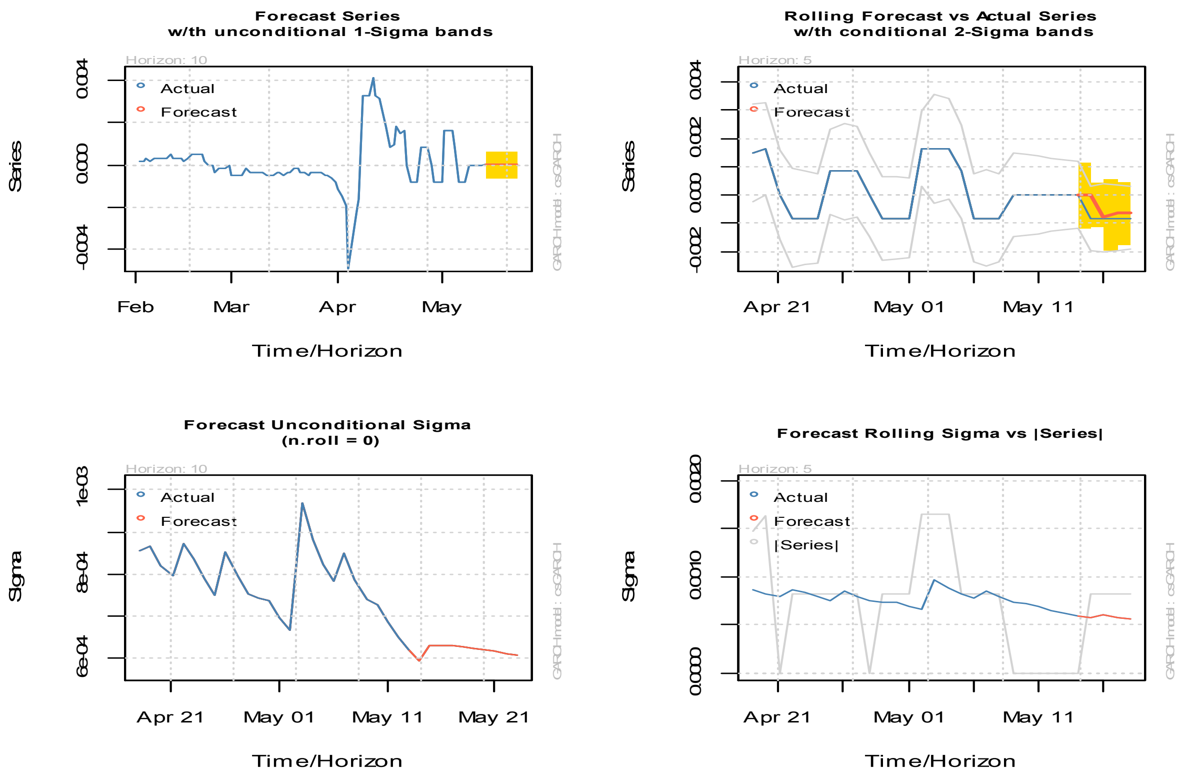

Figure 5: Actual and Forest Charts for csGARCH (1,1)

5. Summary and Conclusion

In this study, the performance of Symmetric and Asymmetric GARCH-type model in modelling foreign exchange returns is sought. The data set consist of two periods; before recession (period 1) and during recession (period 2). Explorative data analysis shows that the datasets are stationary; results shows exchange rate is more volatile in the period of recession than the period when there was no recession in Nigeria. Weighted Ljung-Box test on standardized squared residuals also indicate that there is no presence of serial correlation using the models to fit the data based on the objective of the study.

Model selection criteria based on HQIC shows that Gaussian process is least considered to capture the variability in foreign exchange rate, using Nigeria foreign exchange returns, while student's t and Generalized Error distribution are more suitable. Mean Square Error (MSE), Mean Absolute Error (MAE) and Directional Accuracy Error (DAC) are used to determine forecast performance of the models selected resulting from HQIC in table 6. Fifty out-of-sample forecast was carried out for ten horizons, figures 1-5 charts shows the relative difference between actual and forecast are indicated.

The underlying models are applied to model the volatility of exchange returns of daily closing data of foreign exchange of Nigeria naira against USD. GARCH (1, 1) and gjrGARCH (1, 1) with Student's t error distribution and iGARCH(1,1), sGARCH(1,1), and csGARCH (1,1) were selected to be the best fitted models based on Hannan-Quinn Information Criterion. The symmetric GARCH only conforms to the Gaussian process as provided in R "rugarch package". This supports previous studies that Gaussian process is inadequate for volatility modeling.

The applicable signs, as indicated in section 3.1 above and statistical significance asymmetric parameters at 5% in table 4 is used to establish the existence of leverage effect indicating that the volatility does not respond to equal magnitude of positive and negative shocks equally. The ARCH and GARCH terms in the models explain the volatility persistence of foreign exchange market returns.

From this study, it is hereby recommended that student-t and/ or Generalized Error distribution be used to model volatility in the two periods relative to Gaussian process.

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