Frechet Cascade Stress-Strength System Reliability

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Abstract

In this paper it has been discussed the reliability of n- cascade system when the stress and strength are Frechet distributed random variables. The n-Cascade system is a standby redundancy system, where the standby component taking the place of failed component with decreased value of stress and independently distributed strength. Cascade reliability model is a special type of Stress- Strength model. For n=2,3,4 as Marginal Reliabilities, The values of Marginal Reliabilities used to find the system Reliabilities R_2 , R_3 and R_4 of Cascade model, which studied for this model in different cases for the parameters of this distribution according to the Cascade variables. These cases are 12 special cases and the values of R_2 , R_3 and R_4 are discussed for each of these cases.

Keywords : Frechet Distribution, Stress-Strength Cascade Model, Reliability for Cascade Model.

1. Introduction

Cascade system that is a special type of standby system were first developed and studied by Pandit and Sriwastav (1975) have featured relevance of geometric distribution in the study of behavior of a cascade system [1]. Raghavachar, Rao and Ramacharyulu (1983) presented a closed form solution of stress attenuated reliability function for n-cascade system when both stress and strength follow identical distributions [2]. In this system, the first component activates and encounters the effect of random pressure. The (n-1) remains a constant support component in the pressure resistance standby mode. If the first component fails to resist the strength of the second component, it is reactivated and faces the effect of the process pressure in sequence for only one component at a time, [See [3] and [4]]. Moreover, there are scientists who have worked on this system Maheshwari, Rekha, Rao and Raghavachar (1993) studied stress attenuated reliability for n-cascade system whose stress and strength follow normal and exponential distributions respectively [5]. Rekha and Shyam Sunder (1997) have also highlighted a similar cascade system where stress and strength follow gamma and exponential distributions respectively. They showed that for higher parametric values and lower attenuation factors a high degree of reliability could be attained [6]. Rekha and ChechuRaju (1999) endeavored to present a closed form solution of stress attenuated reliability function for n-cascade system with exponential stress and standby strengths following Rayleigh and exponential distributions [7]. Shyam Sunder (2012) has studied stress attenuation for cascade system when both stress and strength follow Rayleigh distribution [8]. In most of the works mentioned in the literature on cascade model, study is carried out by considering the influence of stress attenuation factor only.

This observation has motivated the present study of attempting to design reliability model for a cascade system under joint effect of stress as well as strength attenuation factors. Further, reliability assessment (estimation of reliability function) is carried using the standard methods. The paper is organized as follows. In Section 1 introduction for n-cascade system. In section 2 the general model is developed with the reliability expressions of an n-cascade system is obtained when the stress strength of the components follow particular distributions. In section 2.1 the expressions of Rn, is obtained when both stress-strength are Frechet distribution. Some numerical values of R with R(1), R(2), R(3) and R(4) for particular values of the parameters for Frechet distribution are tabulated and discussed in section 3 .

2. General Model

Reliability is a measure of system performance and arises in the context of mechanical reliability of a system [9]. In stress-strength model the reliability R, of a component is defined as the probability that the strength of the component, X is not less than the stress Y on it. Symbolically; R = P (X ≥ Y)such that X and Y be random variables independent are non-negative representing to the strengths and stresses of the ith Components respectively and this system totally fails if and only if the pressure of stress is greater than its strength it mean the reliability of cascade system could survive with a loss of the first components $(n-1)$ if and only if

$X_i \leq Y_i$; $i = 1,2,3, \ldots, n-1$ and $X_n > Y_n$

In cascade system after every failure the stress is modified by a factor (k) which is called "attenuation factor" that is reason why the cascade model is a special case of standby system because it introduces an improvement factor for the subsequent component on the previous Such that ;

$Y_2 = kY_1$, $Y_3 = kY_2 = k^2Y_1$,…, $Yi = k^{i-1}Y_1$, i = 1, 2, . . . n and k >1 .

and the System Reliability of n-components for Cascade Model, R_n , of the system is given by :

$$R_n = \sum_{i=1}^{r} R(i) \qquad \ldots(1)$$

Where R(i) ; i= 1 , 2 ,...,r is the marginal reliability. Therefore, the th n marginal reliability is defined as,

$$R(\text{ r }) = P[\{\cap_{i=1}^{n-1}(Xi < Yi)\} \cap Xn > Yn \ ]$$

$$=P[X_1<Y_1, X_1<kY_1, \ldots, X_1< k^{r-2}Y_1, X_1 \geq k^{r-1}Y_1] \qquad \ldots(2)$$

A system is effective but it fails when all components are failed also .

2.1 When Stress-Strength are Frechet Distribution

Let us suppose that X is a strength Frechet variant with probability density function f(x) and cumulative distribution function F(x) with scale parameter α and shape parameter β

$$f(x_i) = \alpha_i \beta x_i^{-(\beta+1)} e^{-\alpha_i x_i^{-\beta}} \qquad x_i > 0;\ \alpha_i \ and\ \beta > 0,\ i = 1, 2, \ldots, n.$$

$$F(x_i, \alpha i, \beta) = e^{-\alpha_i x_i^{-\beta}} \qquad x_i > 0;\ \alpha \ and\ \beta > 0,\ i = 1, 2, \ldots, n.$$

and Y is also a stress Frechet variant with probability density function g(y) and cumulative distribution function G(y) with scale parameter λ and shape parameter β

$$g(y) = \lambda \beta\ y_1^{-(\beta+1)} e^{-\lambda y_1^{-\beta}} \qquad y > 0;\ \lambda \ and\ \beta > 0$$

$$G(y, \lambda, \beta) = e^{-\lambda y^{-\beta}} \qquad y > 0;\ \lambda \ and\ \beta > 0$$

To find the system Reliability, R_n, for the cases of n=2,3,4 using (1), for r =1 the 1-system Marginal Reliability can be drive from the following probability:

$$R(1) = p(X_1 \geq Y_1)$$

$$= \int_0^\infty [\int_{y_1}^\infty f(x_1)\, dx_1] g(y_1)\, dy_1$$

$$= \int_0^\infty \overline{F_1(y_1)} g(y_1)\, dy_1$$

Where $\overline{F_1(y_1)} = 1 - F_1(y_1)$, $F_1(y_1) = e^{-\alpha_1 y_1^{-\beta}}$ then

$$R(1) = \int_0^\infty [1 - F_1(y_1)]\ g(y_1)\, dy_1$$

$$= \int_0^\infty g(y_1)\, dy_1 - \int_0^\infty F_1(y_1)\, g(y_1)\, dy_1$$

$$= 1 - \int_0^\infty F_1(y_1)\, g(y_1)\, dy_1$$

$$= 1 - \int_0^\infty \lambda\, \beta\, y_1^{-(\beta+1)} e^{-(\alpha_1+\lambda) y_1^{-\beta}}\, dy_1$$

$$R(1) = 1 - \frac{\lambda}{(\alpha_1+\lambda)} \int_0^\infty \beta\, (\alpha_1+\lambda) y_1^{-(\beta+1)} e^{-(\alpha_1+\lambda) y_1^{-\beta}}\, dy_1$$

by comparing with probability density function of Frechet distribution then ;

We have $\int_0^\infty \beta\, (\alpha_1+\lambda) y_1^{-(\beta+1)} e^{-(\alpha_1+\lambda) y_1^{-\beta}}\, dy_1 = 1$, and we get

$$R(1) = 1 - \frac{\lambda}{\alpha_1+\lambda} = \frac{\alpha_1}{\alpha_1+\lambda}$$

And when r =2 the second Marginal Reliability can be drive from the following probability:

$$R(2) = p(X_1 < Y_1\, , X_2 \geq kY_1)$$

$$= \int_0^\infty [\, \int_0^{y1} f(x_1)\, dx_1\,][\int_{ky1}^\infty f(x_2)\, dx_2\,]g(y_1)dy_1$$

$$= \int_0^\infty F_1(y_1)\, \overline{F_2(ky_1)}\, g(y_1)dy_1$$

where $\overline{F_2(ky_1)} = 1 - F_2(ky_1)$ and $F_2(ky_1) = e^{-\alpha_2\, (k\, y_1)^{-\beta}}$, and

$$R(2) = \int_0^\infty F_1(y_1)[1 - F_2(ky_1)\,]\, g(y_1)dy_1$$

$$= \int_0^\infty e^{-\alpha_1 y_1^{-\beta}} \left[1 - e^{-\alpha_2\, (k\, y_1)^{-\beta}}\right] \lambda\, \beta\;\, y_1^{-(\beta+1)} e^{-\lambda\, y_1^{-\beta}} dy_1$$

$$= \frac{\lambda}{(\alpha_1+\lambda)} \int_0^\infty (\alpha_1+\lambda)\beta\, y_1^{-(\beta+1)}\, e^{-(\alpha_1+\lambda) y_1^{-\beta}} dY_1 - \frac{\lambda}{(\alpha_1+\alpha_2\, k^{-\beta}+\lambda)} \int_0^\infty \left(\alpha_1 + \alpha_2\, k^{-\beta} + \lambda\right)\beta\;\, y_1^{-(\beta+1)}\, e^{-(\alpha_1+\alpha_2\, (k)^{-\beta}+\lambda)}\;\, e^{-\lambda\, y_1^{-\beta}} dy_1$$

From the pdf of the Frechet distribution ,we get :

$$\int_0^\infty (\alpha_1+\lambda)\, \beta Y_1^{-(\beta+1)} e^{-(\alpha_1+\lambda) Y_1^{-\beta}} dy_1 = 1$$

Also ;

$$\int_0^\infty (\alpha_1 + \alpha_2\, (k)^{-\beta} + \lambda)\, \beta Y_1^{-(\beta+1)} e^{-(\alpha_1+\alpha_2\, (k)^{-\beta}+\lambda) Y_1^{-\beta}} dy_1 = 1$$

Then

$$R(2) = \frac{\lambda}{\alpha_1+\lambda} - \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\lambda}$$

Now when r=3, then the reliability R(3) can be obtained from the following probability as:

$$R(3) = p(X_1 < Y_1\, , X_2 < kY_1, X_3 \geq k^2 Y_1\,)$$

$$= \int_0^\infty [\int_0^{y_1} f(x_1)\, dx_1]\, [\, \int_0^{ky_1} f(x_2)\, dx_2\,][\int_{k^2 y_1}^\infty f(x_3)\, dx_3\,]g(y_1)dy_1$$

$$= \int_0^\infty F_1(y_1)F_2(k\,y_2)\,\overline{F_3(k^2y_1)}\,g(y_1)dy_1$$

Where $\overline{F_3(k^2y_1)} = 1 - F_3(k^2y_1)$, $F_3(k^2y_1) = e^{-\alpha_3\,(k^2\,y_1)^{-\beta}}$; then

$$R(3) = \int_0^\infty e^{-\alpha_1 y_1{}^{-\beta}} e^{-\alpha_2\,(k\,y_1)^{-\beta}} \left[1 - e^{-\alpha_3\,(k^2\,y_1)^{-\beta}}\right] \lambda\,\beta\; y_1{}^{-(\beta+1)} e^{-\lambda\,y_1{}^{-\beta}} dy_1$$

$$= \frac{\lambda}{(\alpha_1+\alpha_2\,(k)^{-\beta}+\lambda)} \int_0^\infty (\alpha_1 + \alpha_2\,(k)^{-\beta} + \lambda)\beta\, y_1{}^{-(\beta+1)}\, e^{-(\alpha_1+\alpha_2\,(k)^{-\beta}+\lambda) y_1{}^{-\beta}} dy_1 - \frac{\lambda}{(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\lambda)} \int_0^\infty (\alpha_1 + \alpha_2\,(k)^{-\beta} + \alpha_3\,(k)^{-2\beta} + \lambda)\beta\; y_1{}^{-(\beta+1)}\, e^{-(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\lambda)\,y_1{}^{-\beta}}\; e^{-\lambda\,y_1{}^{-\beta}} dy_1$$

by comparing with probability density function of Frechet distribution then ;

$$R(3) = \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \lambda} - \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \alpha_3 k^{-2\beta} + \lambda}$$

Finally , when r =4 ,the 4th marginal reliability of , R(4) , can be obtained as :

$$R(4) = p(X_1 < Y_1\,, X_2 < kY_1,\; X_3 < k^2Y_1\,, X_4 \geq k^3Y_1)$$

$$= \int_0^\infty [\int_0^{y_1} f(x_1)\,dx_1][\,\int_0^{ky_1} f(x_2)\,dx_2\,][\int_0^{k^2y_1} f(x_3)\,dx_3\,][\int_{k^3y_1}^\infty f(x_4)\,dx_4]\,g(y_1)dy_1$$

$$= \int_0^\infty F_1(y_1)F_2(ky_1)F_3(k^2y_1)\,\overline{F_4(k^3y_1)}\,g(y_1)dy_1$$

Since $\overline{F_4(k^3y_1)} = 1 - F_4(k^3y_1)$ and $F_4(k^3y_1) = e^{-\alpha_4\,(k^3\,y_1)^{-\beta}}$; then

$$= \int_0^\infty e^{-\alpha_1 Y_1{}^{-\beta}} e^{-\alpha_2\,(k\,y_1)^{-\beta}} e^{-\alpha_3\,(k^2\,y_1)^{-\beta}} \left[1 - e^{-\alpha_4\,(k^3\,y_1)^{-\beta}}\right] \lambda\,\beta\; y_1{}^{-(\beta+1)} e^{-\lambda\,y_1{}^{-\beta}} dy_1$$

$$= \int_0^\infty \lambda\,\beta\,\frac{(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\lambda)}{(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\lambda)}\, y_1{}^{-(\beta+1)}\, e^{-(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\lambda)y_1{}^{-\beta}} dy_1$$

$$- \int_0^\infty \lambda\,\beta\,\frac{(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\alpha_4\,(k)^{-3\beta}+\lambda)}{(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}+\alpha_4\,(k)^{-3\beta}+\lambda)}\, y_1{}^{-(\beta+1)}$$

$$.\, e^{-(\alpha_1+\alpha_2\,(k)^{-\beta}+\alpha_3\,(k)^{-2\beta}++\alpha_4\,(k)^{-3\beta}+\lambda)y_1{}^{-\beta}}\; dy_1$$

by comparing with probability density function of Frechet distribution the final formula will sd:

$$R(4) = \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \alpha_3 k^{-2\beta} + \lambda} - \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \alpha_3 k^{-2\beta} + \alpha_4 k^{-3\beta} + \lambda}$$

Then the total reliability R_2 of the 2-cascade system will found to be as :

R_2 =R(1)+ R(2)

$$R_2 = \frac{\alpha_1}{\alpha_1 + \lambda} + \frac{\lambda}{\alpha_1 + \lambda} - \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \lambda}$$

Also; The total reliability R_3 of the 3-cascade system will be found as :

R_3 =R(1)+ R(2)+R(3)

$$R_3 = \frac{\alpha_1}{\alpha_1 + \lambda} + \frac{\lambda}{\alpha_1 + \lambda} - \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \lambda} + \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \lambda} - \frac{\lambda}{\alpha_1 + \alpha_2 k^{-\beta} + \alpha_3 k^{-2\beta} + \lambda}$$

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And the total reliability R_4 of the 4-cascade system will be found as :

R_4 =R(1)+ R(2)+R(3)+R(4)

$$R_4 = \frac{\alpha_1}{\alpha_1+\lambda} + \frac{\lambda}{\alpha_1+\lambda} - \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\lambda} + \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\lambda} - \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\alpha_3 k^{-2\beta}+\lambda}$$

$$+ \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\alpha_3 k^{-2\beta}+\lambda} - \frac{\lambda}{\alpha_1+\alpha_2 k^{-\beta}+\alpha_3 k^{-2\beta}+\alpha_4 k^{-3\beta}+\lambda}$$

3. Numerical and Graphical Study of Marginal and System Reliability

For some specific values of the parameters involved in the expressions of R(r), r =1,2,3,4. we perform numerical and graphical study by calculating marginal reliabilities R(1), R(2), R(3) and R(4) with system reliabilities R_2, R_3 and R_4 for Frechet Distribution for different values of strength parameters as ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$) and different values for stress parameter (β). The results recorded as in the following tables:

Table (1): Marginal Reliabilities R(1) and R(2) with system Reliability R_2 according to strength parameter *α*.

α	*β*	*λ*	*R(1)*	*R(2)*	*R_2*
1	2	.2	0.8333	0.0287	0.8621
2	2	.2	0.909	0.0168	0.9259
3	2	.2	0.9375	0.0119	0.9494
4	2	.2	0.9524	0.0092	0.9615
1	3	.3	0.7692	0.0202	0.7895
2	3	.3	0.8696	0.0128	0.8824
3	3	.3	0.9091	0.0093	0.9184
4	3	.3	0.9302	0.0073	0.9375
1	4	.4	0.7143	0.0122	0.7265
2	4	.4	0.8333	0.0083	0.8416
3	4	.4	0.8824	0.0061	0.8885
4	4	.4	0.9091	0.0049	0.9140

Table (2): Marginal Reliabilities R(1), R(2) and R(3) with system Reliability R_3 according to strength parameter α.

α	β	λ	R(1)	R(2)	R(3)	R_3
1	2	.2	0.8333	0.0287	0.0057	0.8678
2	2	.2	0.909	0.0168	0.0033	0.9292
3	2	.2	0.9375	0.0119	0.0023	0.9517
4	2	.2	0.9524	0.0092	0.0018	0.9633
1	3	.3	0.7692	0.0202	0.0023	0.7918
2	3	.3	0.8696	0.0128	0.0014	0.8838
3	3	.3	0.9091	0.0093	0.0010	0.9194
4	3	.3	0.9302	0.0073	e-048.03	0.9383
1	4	.4	0.7143	0.0122	7.29e-04	0.7272
2	4	.4	0.8333	0.0083	4.89e-04	0.8421
3	4	.4	0.8824	0.0061	3.6 e-04	0.8889
4	4	.4	0.9091	0.0049	2.88e-04	0.9143

Table (3): Marginal Reliabilities R(1), R(2), R(3) and R(4) with system Reliability R_4 according to strength parameter α.

α	β	λ	R(1)	R(2)	R(3)	R(4)	R_4
1	2	.2	0.8333	0.0287	0.0057	0.0025	0.8703
2	2	.2	0.909	0.0168	0.0033	0.0010	0.9303
3	2	.2	0.9375	0.0119	0.0023	6.62 e-04	0.9523
4	2	.2	0.9524	0.0092	0.0018	4.84 e-04	0.9638
1	3	.3	0.7692	0.0202	0.0023	8.27 e-04	0.7926
2	3	.3	0.8696	0.0128	0.0014	2.97 e-04	0.8841
3	3	.3	0.9091	0.0093	0.0010	1.80 e-04	0.9196
4	3	.3	0.9302	0.0073	e-048.03	1.28 e-04	0.9384
1	4	.4	0.7143	0.0122	7.29e-04	2.19 e-04	0.7274
2	4	.4	0.8333	0.0083	4.89e-04	6.50 e-05	0.8421
3	4	.4	0.8824	0.0061	3.6 e-04	3.73 e-05	0.8889
4	4	.4	0.9091	0.0049	2.88e-04	2.61 e-05	0.9143

Table (4): Marginal Reliabilities R(1), R(2), R(3) and R(4) with system Reliability R_4 according to stress parameter β.

α	β	λ	*R(1)*	*R(2)*	*R(3)*	*R(4)*	R_4
1	2	.2	0.8333	0.0287	0.0057	*0.0025*	*0.8703*
1	3	.3	0.7692	0.0202	0.0023	*8.27e-04*	*0.7926*
1	4	.4	0.7143	0.0122	7.29e-04	*2.19 e-04*	*0.7274*
2	2	.2	0.909	0.0168	0.0033	*0.0010*	*0.9303*
2	3	.3	0.8696	0.0128	0.0014	*2.97 e-04*	*0.8841*
2	4	.4	0.8333	0.0083	4.89e-04	*6.50 e-05*	*0.8421*
3	2	.2	0.9375	0.0119	0.0023	*6.62 e-04*	*0.9523*
3	3	.3	0.9091	0.0093	0.0010	*1.80 e-04*	*0.9196*
3	4	.4	0.8824	0.0061	3.6 e-04	*3.73 e-05*	*0.8889*
4	2	.2	0.9524	0.0092	0.0018	*4.84 e-04*	*0.9638*
4	3	.3	0.9302	0.0073	e-048.03	*1.28 e-04*	*0.9384*
4	4	.4	0.9091	0.0049	2.88e-04	*2.61 e-05*	*0.9143*

4. Conclusions

From Table(1) , it observed that the 2-system Reliability, R_2, is increasing when the strength parameter value(α) is increasing from (1to 4) for certain value of the stress parameter (β) , where R(1) is increasing and R(2) is decreasing also as in table (2) , the 3-system Reliability , R_3 ,is increasing with increasing in R(1) values and decreasing in R(2) and R(3) values.

similarly in table (3),the 4-system Reliability, R_4 , is increasing with to increasing in R(1) and decreasing in each of R(2), R(3) and R(4) values.

In table (4) ,clearly shown that the 4-system Reliability, R_4 , is decreasing according to increasing in the stress parameter value(β) for a certain value of strength parameter (α) with decreasing in each of R(1), R(2), R(3) and R(4) values.

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