A NEW MODIFIED GENERALIZED ODD LOG-LOGISTIC DISTRIBUTION WITH THREE PARAMETERS

Arbër Qoshja[1] & Markela Muça[2]
1. Department of Applied Mathematics, Faculty of Natural Science, Tirana, Albania
2. Department of Applied Mathematics, Faculty of Natural Science, Tirana, Albania
e-mail: qoshjaa@gmail.com

Abstract
Statistical distributions are very useful in describing and predicting real-world phenomena. Numerous extended distributions have been extensively used over the last decades for modeling data in many applied sciences such as medicine, engineering and finance. Recent developments focus on defining new families that extend well-known distributions and at the same time provide great flexibility in modeling data in practice. In this paper, we have introduced a new three-parameter exponential distribution called the generalized odd log-logistic-exponential distribution by using the generator defined by Cordeiro et al (2017). This model extends the odd log-logistic-exponential and exponential distributions. Several of its structural properties are discussed in detail. These include shape of the probability density function, hazard rate function, quantile function order statistics, and moments. The method of maximum likelihood is adopted to estimate the model parameters. The applicability of the new models is illustrated by using real data. The goodness-of-fits of the exponential, beta exponential, Kumaraswamy exponential and the generalized odd log-logistic-exponential distributions have been compared through the AIC, AICC, BIC and KS statistics and found that the generalized odd log-logistic-exponential distribution fits well the data.

Key Word: Exponential distribution, odd log logic distribution, maximum likelihood estimation, Monte Carlo.

Introduction

In 2017, Cordeiro et al. introduced a generator of continuous distribution called the generalized odd log-logistic family of distributions with pdf and cdf given by:

$$f(x,\alpha,\theta,\xi)=\frac{\alpha\theta g(x,\xi)G(x,\xi)^{\alpha\theta-1}[1-G(x,\xi)^{\theta}]^{\alpha-1}}{\left\{G(x,\xi)^{\alpha\theta}+[1-G(x,\xi)^{\theta}]^{\alpha}\right\}^{2}} \tag{1}$$

$$F(x,\alpha,\theta,\xi)=\frac{G(x,\xi)^{\alpha\theta}}{G(x,\xi)^{\alpha\theta}+[1-G(x,\xi)^{\theta}]^{\alpha}} \tag{2}$$

respectively.

The aim of this paper is to consider exponential distribution with three parameters called the generalized odd log-logistic-exponential distribution.

In this article we consider a new way of exponential distribution with three parameters by replacing $G(x,\lambda)=1-e^{-\lambda x},\quad x>0,\lambda>0$ in equation (2), called the generalized odd log-logistic-exponential distribution.

Definition 1. *A random variable* X *is said to have* the generalized odd log-logistic-exponential distribution *if it has the density:*

$$f(x,\alpha,\theta,\lambda)=\frac{\alpha\theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha\theta-1}[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha-1}}{\left\{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha}\right\}^{2}} \quad (3)$$

The cumulative distribution function associated with Equation (5) is given by

$$F(x,\alpha,\theta,\lambda)=\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}} \quad (4)$$

Figure 1, 2 and 3 illustrates some of the possible shapes of the pdf, cdf and hazard function of the generalized odd log-logistic-exponential distribution for selected values of the parameters α,θ, and λ respectively.

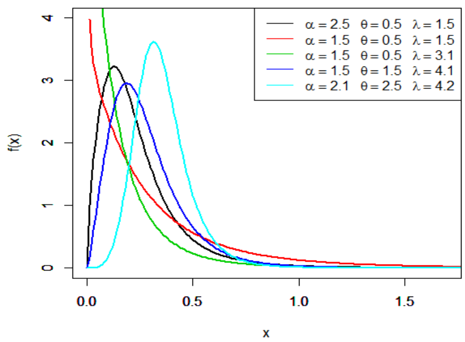

Figure 1. The pdf of the generalized odd log-logistic-exponential distribution for different values of parameters alpha, theta and lambda.

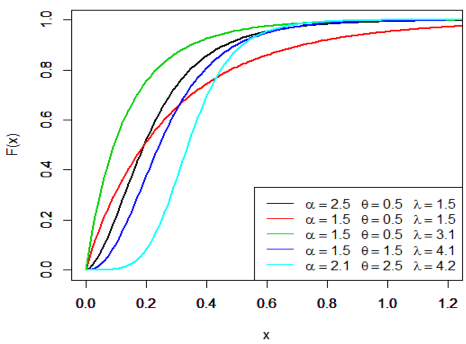

Figure 2. The cdf of the generalized odd log-logistic-exponential distribution for different values of parameters alpha, theta and lambda.

2 Reliability Analysis

2.1 Survival function

The reliability function (survival function) of the generalized odd log-logistic-exponential distribution is given by

$$R(x,\alpha,\theta,\lambda)=1-F(x,\alpha,\theta,\lambda)=1-\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}} \tag{5}$$

2.2 Hazard Rate Function

The hazard rate function (failure rate) of a life- time random variable X the generalized odd log-logistic-exponential distribution with three parameters is given by

$$h(x,\alpha,\theta,\lambda)=\frac{f(x,\alpha,\theta,\lambda)}{1-F(x,\alpha,\theta,\lambda)}=$$

$$=\frac{\alpha\theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha\theta-1}}{[1-\left(1-e^{-\lambda x}\right)^{\theta}]\left\{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha}\right\}} \tag{6}$$

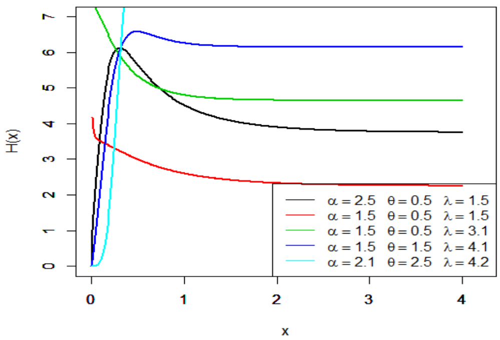

Figure 3. The hazard rate function of the generalized odd log-logistic-exponential distribution for different values of parameters alpha, theta and lambda.

2.3 Quantiles

The quantile of any distribution is given by solving the equation

$$F(x_p)=p,\quad 0<p<1.$$

The quantile of the generalized odd log-logistic-exponential distribution *is given by*

$$x\_p = -\frac{\ln\left(-e^{-\frac{\ln\left(e^{-\frac{\ln\left(-\frac{-1+p}{p}\right)}{\alpha}}+1\right)\alpha+\ln\left(-\frac{-1+p}{p}\right)}{\alpha\theta}}+1\right)}{\lambda} \tag{7}$$

3. Order statistics of the generalized odd log-logistic-exponential distribution Order statistics has an important role in quality control and reliability analysis, and also in hydrological and extreme value analysis. It is often used to identify the situations and parameter estimation. Here we assume that $X_1, X_2, ..., X_{n-1}, X_n$ *is a random sample from exponential distribution with pdf and cdf given in (3) and (4) respectively. Let* $X_{(1)}$, $X_{(2)}, ..., X_{(n-1)}, X_{(n)}$ *be the ordered values of the preceding sample in non-decreasing order of magnitude.*

The n^{th} order statistics of the generalized odd log-logistic-exponential distribution, $X_{(n)} = max(X_1, X_2, ..., X_{n-1}, X_n)$ is given by

$$f_{X_{(n)}}(x) = n[F(x)]^{n-1} f(x)$$

$$= n\left[\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}}\right]^{n-1}$$

$$\times\frac{\alpha\theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha\theta-1}[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha-1}}{\left\{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha}\right\}^{2}} \tag{8}$$

The smallest order statistic, $X_{(1)} = min(X_1, X_2, ..., X_{n-1}, X_n)$ has the pdf

$$f_{X_{(1)}}(x) = n[1-F(x)]^{n-1} f(x)$$

$$= n\left[1-\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}}\right]^{n-1}$$

$$\times\frac{\alpha\theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha\theta-1}[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha-1}}{\left\{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha}\right\}^{2}} \quad (9)$$

Generally the distribution of the r^{th} order statistics with the generalized odd log-logistic-exponential distribution is as follows

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!}[F(x)]^{r-1}[1-F(x)]^{n-r} f(x)$$

$$= \frac{n!\beta}{(r-1)!(n-r)!(1+\beta)^{2n}}\left[\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}}\right]^{r-1}$$

$$\times\left[1-\frac{\left(1-e^{-\lambda x}\right)^{\alpha\theta}}{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+\left[1-\left(1-e^{-\lambda x}\right)^{\theta}\right]^{\alpha}}\right]^{n-r}$$

$$\times\frac{\alpha\theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha\theta-1}[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha-1}}{\left\{\left(1-e^{-\lambda x}\right)^{\alpha\theta}+[1-\left(1-e^{-\lambda x}\right)^{\theta}]^{\alpha}\right\}^{2}} \quad (10)$$

3.3. Useful expansions

Based on generalized binomial expansions, the pdf (1) of X can be expressed as (for more details see Cordeiro 2016)

$$f(x) = \alpha\theta\sum_{i,j,l=0}^{\infty}\sum_{k=0}^{l}\left(-1\right)^{j+k+l}\binom{-2}{i}\binom{-\alpha(i+1)}{j}\binom{\alpha\theta(i+1)+\theta j-1}{l}\binom{l}{k}G(x)^{k} \quad (11)$$

By using the same methodology, the pdf of the generalized odd log-logistic exponential distributions has the form

$$f(x) = \alpha\theta \sum_{i,j,l=0}^{\infty} \sum_{k=0}^{l} (-1)^{j+k+l} \binom{-2}{i} \binom{-\alpha(i+1)}{j} \binom{\alpha\theta(i+1)+\theta j-1}{l} \binom{l}{k} \left(1-e^{-\lambda x}\right)^{k}$$

$$= \alpha\theta \sum_{i,j,l=0}^{\infty} \sum_{k=0}^{l} \sum_{t=0}^{\infty} (-1)^{j+k+l} \binom{k}{t} \binom{-2}{i} \binom{-\alpha(i+1)}{j} \binom{\alpha\theta(i+1)+\theta j-1}{l} \binom{l}{k} \left(-e^{-\lambda x}\right)^{j}$$

Theorem. *Let* X *be a random variable with pdf* (6). *The expectation is given by:*

$$E(x) = \alpha\theta \sum_{i,j,l=0}^{\infty} \sum_{k=0}^{l} \sum_{t=0}^{\infty} (-1)^{j+k+l+1} \binom{k}{t} \binom{-2}{i} \binom{-\alpha(i+1)}{j} \binom{\alpha\theta(i+1)+\theta j-1}{l} \binom{l}{k} \frac{1}{j^2\lambda^2}$$

$$E(X) = \int_0^{\infty} x f(x) dx =$$

Proof. $$= \alpha\theta \sum_{i,j,l=0}^{\infty} \sum_{k=0}^{l} \sum_{t=0}^{\infty} (-1)^{j+k+l} \binom{k}{t} \binom{-2}{i} \binom{-\alpha(i+1)}{j} \binom{\alpha\theta(i+1)+\theta j-1}{l} \binom{l}{k} \int_0^{\infty} x \left(-e^{-\lambda x}\right)^{j} dx =$$

$$= \alpha\theta \sum_{i,j,l=0}^{\infty} \sum_{k=0}^{l} \sum_{t=0}^{\infty} (-1)^{j+k+l+1} \binom{k}{t} \binom{-2}{i} \binom{-\alpha(i+1)}{j} \binom{\alpha\theta(i+1)+\theta j-1}{l} \binom{l}{k} \frac{1}{j^2\lambda^2}$$

4. Estimation

In this section, we define the maximum likelihood estimation and Newton Raphson procedure to estimate the parametric values.

4.1. Maximum likelihood estimation

In this subsection, interest is to define the parameter estimation of the generalized odd log-logistic-exponential distribution by maximum likelihood estimation.

Let X_1, $X_2, ..., X_n$ be i.i.d random variables of size n. Then the likelihood function for this distribution is

$$L(.) = \prod_{i=1}^{n} f(x_i, \alpha, \theta, \lambda) = \frac{(\alpha\theta\lambda)^n e^{-\lambda \sum_{i=1}^{n} x_i} \prod_{i=1}^{n} \left(1-e^{-\lambda x_i}\right)^{\alpha\theta-1} [1-\left(1-e^{-\lambda x_i}\right)^{\theta}]^{\alpha-1}}{\prod_{i=1}^{n} \left\{ \left(1-e^{-\lambda x_i}\right)^{\alpha\theta} + [1-\left(1-e^{-\lambda x_i}\right)^{\theta}]^{\alpha} \right\}^2} \quad (12)$$

the sample log-likelihood function

$$\ell(X;\alpha,\theta,\lambda) = \ln L(.) = n\ln\alpha + n\ln\theta + n\ln\lambda - \lambda\sum_{i=1}^{n} x_i$$
$$+(\alpha\theta-1)\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right) + (\alpha-1)\sum_{i=1}^{n}\ln\left(1-\left(1-e^{-\lambda x_i}\right)^{\theta}\right)$$
$$-2\sum_{i=1}^{n}\ln\left\{\left(1-e^{-\lambda x_i}\right)^{\alpha\theta} + [1-\left(1-e^{-\lambda x_i}\right)^{\theta}]^{\alpha}\right\}$$

The maximum likelihood estimates can be obtained as the simultaneous solutions of the following non-linear equations:

$$\frac{\partial\ell(X;\alpha,\theta,\lambda)}{\partial\alpha} = 0,\ \frac{\partial\ell(X;\alpha,\theta,\lambda)}{\partial\theta} = 0,\ \frac{\partial\ell(X;\alpha,\theta,\lambda)}{\partial\lambda} = 0.$$

The exact solution for unknown parameters is not possible analytically so the estimates are obtained by solving nonlinear equations simultaneously. The solution of nonlinear system is easier by iterative techniques common as Newton Raphson approach. By providing initial guess of the parameters, Newton Raphson used these initial values to calculate parameter estimates. Asymptotically these estimates of parameters approaches to normality and the z-score are approximately standard normal, which can be used to find the $100(1-\alpha)$ two sided confidence interval for the parameters.

4.2 Maximum product spacing estimates

The maximum product spacing (MPS) method has been proposed by Cheng et al (1983). This method is based on an idea that the differences (Spacing) of the consecutive points should be identically distributed. The geometric mean of the differences is given as

$$GM = \sqrt[n+1]{\prod_{i=1}^{n+1} D_i} \tag{13}$$

where, the difference D_i is defined as

$$D_i = \int_{x_{(i-1)}}^{x_{(i)}} f\left(x,\theta,\alpha,\lambda\right)dx;\ i = 1,2,\ldots,n+1. \tag{14}$$

where, $F(x_{(0)},\theta,\alpha,\lambda) = 0$ and $F(x_{(n+1)},\theta,\alpha,\lambda) = 1$. The MPS estimators $\hat{\alpha}_{PS},\hat{\theta}_{PS}$ and $\hat{\lambda}_{PS}$, of θ,α,λ are obtained by maximizing the geometric mean (GM) of the differences. Substituting pdf of the generalized odd log-logistic-exponential distribution and taking logarithm of the above expression, we will have

$$LogGM = \frac{1}{n+1}\sum_{i=1}^{n+1}\log\left[F(x_{(i)},\theta,\alpha,\lambda) - F(x_{(i-1)},\theta,\alpha,\lambda)\right] \tag{15}$$

The MPS estimators $\hat{\alpha}_{PS}, \hat{\lambda}_{PS}, \theta_{PS}$ of α, λ, θ can be obtained as the simultaneous solution of the following non-linear equations:

$$\frac{\partial LogGM}{\partial\theta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\left[\frac{F_{\theta}^{'}(x_{(i)},\theta,\alpha,\lambda) - F_{\theta}^{'}(x_{(i-1)},\theta,\alpha,\lambda)}{F(x_{(i)},\theta,\alpha,\lambda) - F(x_{(i-1)},\theta,\alpha,\lambda)}\right] = 0$$

$$\frac{\partial LogGM}{\partial\alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\left[\frac{F_{\alpha}^{'}(x_{(i)},\theta,\alpha,\lambda) - F_{\alpha}^{'}(x_{(i-1)},\theta,\alpha,\lambda)}{F(x_{(i)},\theta,\alpha,\lambda) - F(x_{(i-1)},\theta,\alpha,\lambda)}\right] = 0$$

$$\frac{\partial LogGM}{\partial\lambda} = \frac{1}{n+1}\sum_{i=1}^{n+1}\left[\frac{F_{\lambda}^{'}(x_{(i)},\theta,\alpha,\lambda) - F_{\beta}^{'}(x_{(i-1)},\theta,\alpha,\lambda)}{F(x_{(i)},\theta,\alpha,\lambda) - F(x_{(i-1)},\theta,\alpha,\lambda)}\right] = 0$$

4.3 Least square estimates

Let $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ be the ordered sample of size n drawn the generalized odd log-logistic-exponential distribution pdf. Then, the expectation of the empirical cumulative distribution function is defined as

$$E\left[F\left(X_{(i)}\right)\right] = \frac{i}{n+1}; i = 1,2,\ldots,n \tag{16}$$

The least square estimates (LSEs) $\hat{\alpha}_{LS}, \hat{\theta}_{LS}, \lambda_{LS}$ of α, θ, λ are obtained by minimizing

$$Z\left(\alpha,\theta,\lambda\right) = \sum_{i=1}^{n}\left(F\left(x_{(i)},\alpha,\theta,\lambda\right) - \frac{i}{n+1}\right)^{2} \tag{17}$$

Therefore, $\hat{\alpha}_{LS}, \hat{\theta}_{LS}, \lambda_{LS}$ of α, θ, λ can be obtained as the solution of the following system of equations:

$$\frac{\partial Z\left(\alpha,\theta,\lambda\right)}{\partial\alpha} = \sum_{i=1}^{n} F_{\alpha}^{'}(x_{(i)},\alpha,\theta,\lambda)\left(F\left(x_{(i)},\alpha,\theta,\lambda\right) - \frac{i}{n+1}\right) = 0$$

$$\frac{\partial Z\left(\alpha,\theta,\lambda\right)}{\partial\theta} = \sum_{i=1}^{n} F_{\theta}^{'}(x_{(i)},\alpha,\theta,\lambda)\left(F\left(x_{(i)},\alpha,\theta,\lambda\right) - \frac{i}{n+1}\right) = 0$$

$$\frac{\partial Z\left(\alpha,\theta,\lambda\right)}{\partial\lambda} = \sum_{i=1}^{n} F_{\beta}^{'}(x_{(i)},\alpha,\theta,\lambda)\left(F\left(x_{(i)},\alpha,\theta,\lambda\right) - \frac{i}{n+1}\right) = 0$$

5. Simulation algorithms

Since the probability integral transformation cannot be applied explicitly, we, therefore need to follow the following steps for generating a sample of size n from the generalized odd log-logistic-exponential distribution $GOLLE(\alpha,\theta,\lambda)$:

1. Set n, α,θ,λ and initial value x^0.
2. Generate $U \sim Uniform(0,1)$.
3. Update x^0 by using the Newton's formula
$x^* = x^0 - R(x^0,\alpha,\theta,\lambda)$
where, $R(x^0,\alpha,\theta,\lambda) = \dfrac{F_X\left(x^0,\alpha,\theta,\lambda\right)-U}{f_X\left(x^0,\alpha,\theta,\lambda\right)}$, $F_X(.)$ and $f_X(.)$ are cdf and pdf of the generalized odd log-logistic-exponential distribution, respectively.
4. If $|x^0 - x^*| \le \varepsilon$, (very small, $\varepsilon > 0$ tolerance limit), then store $x = x^*$ as a sample from $GOLLE(\alpha,\theta,\lambda)$.
5. If $|x^0 - x^*| > \varepsilon$, then, set $x^0 = x^*$ and go to step 3.
6. Repeat steps 3-5, n times for $x_1, x_2, \ldots, x_n$ respectively.

6. Application

Now we use a real data set to show that the generalized odd log-logistic-exponential distribution (GOLEE) can be a better model than the beta-exponential, Kumaraswamy-exponential and exponential distribution.
We consider a data set of the life of fatigue fracture of Kevlar 373/epoxy that are subject to constant pressure at the 90% stress level until all had failed, so we have complete data with the exact times of failure.

These data are:
0.0251,0.0886,0.0891,0.2501,0.3113,0.3451,0.4763,0.5650,0.5671,0.6566,0.6748,0.6751,0.6753,0.7696,0.8375,
0.8391,0.8425,0.8645,0.8851,0.9113,0.9120,0.9836,1.0483,1.0596,1.0773,1.1733,1.2570,1.2766,1.2985,1.3211,
1.3503,1.3551,1.4595,1.4880,1.5728,1.5733,1.7083,1.7263,1.7460,1.7630,1.7746,1.8275,1.8375,1.8503,1.8808,
1.8878,1.8881,1.9316,1.9558,2.0048,2.0408,2.0903,2.1093,2.1330,2.2100,2.2460,2.2878,2.3203,2.3470,2.3513,
2.4951,2.5260,2.9911,3.0256,3.2678,3.4045,3.4846,3.7433,3.7455,3.9143,4.8073,5.4005,5.4435,5.5295,6.5541,
9.0960

Table 1. . Estimated parameters of the GOLLE, BE, KWE and exponential distribution for data set.

Model	ML Estimate	Standard Error	Log-Likelihood	LSE	PS Estimator
Generalized odd log-logistic exp.dist.	Alpha=2.636 theta= 0.453 lambda= 0.155	0.380 0.114 0.07	120.752	2.697 0.480 0.170	3.038 0.356 0.098
Kumaraswamy Exponential	a=1.556 b=2.448 Lambda=0.328	0.401 6.065 0.691	122.094	1.987 2.228 0.453	1.520 1.082 0.598
Beta Exponential	a=1.679 b=1.508 Lambda=0.484	0.374 6.760 1.981	122.227	2.235 1.558 0.586	1.520 1.082 0.598
Exponential	Lambda=0.510	0.058	127.114	0.981	0.926

In order to compare the two distribution models, we consider criteria like Kolmogorov-Smirnov (K-S) statistics, -2ℓ, AIC (Akaike information criterion), and CAIC (corrected Akaike information criterion). Table 1 shows the MLEs under both distributions, Table 2 shows the values of KS, -2ℓ, AIC, AICC, and BIC values for the data set. The better distribution corresponds to smaller KS, -2ℓ, AIC and CAIC values. The values in Table 2 indicate that the GOLLE leads to a better fit than the beta exponential, Kumaraswamy exponential and exponential distribution.

Table 2. Criteria for comparison.

Model	K-S	-2ℓ	AIC	CAIC	BIC
GOLLE	0.0924	241.505	247.505	248.005	254.497
Beta-E	0.0962	244.455	250.455	250.621	257.447
Kw-E	0.0988	244.188	250.188	250.521	257.180
Exp.	0.512	254.228	248.643	249.143	258.559

The P-P plots, fitted distribution function and density functions of the considered models are plotted in Figures 4 and 5, respectively, for the data set.

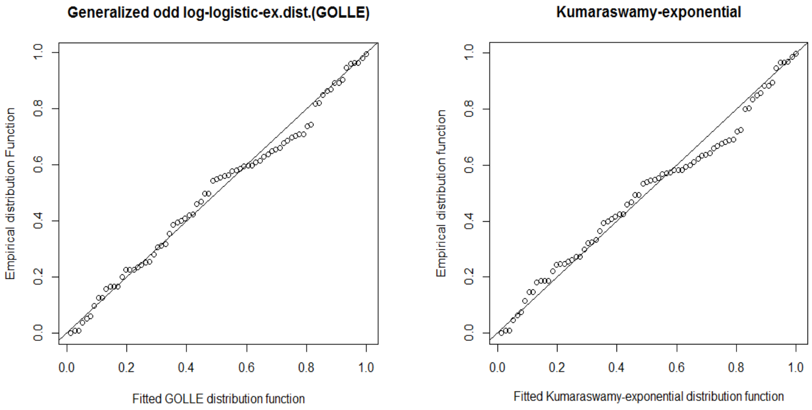

Figure 4. The P-P plots for the real data set

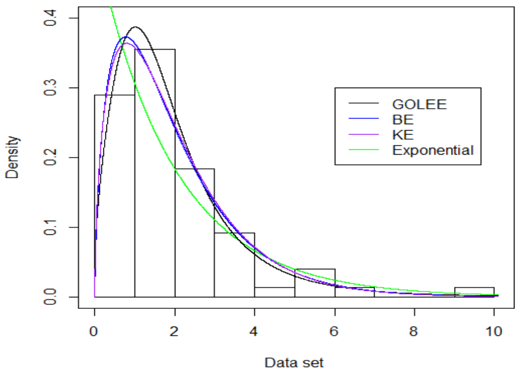

Figure 5. Fitted pdf's plots of the considered distribution for the real data set

7. Conclusion

In this article, we propose a new model, the so-called generalized odd log-logistic-exponential distribution which extends the exponential distribution in the analysis of data with real support. An obvious reason for generalizing a standard distribution is that the generalized form provides larger flexibility in modeling real data. We study shape of the probability density function, hazard rate function, quantile function order statistics, and moments. The estimation of parameters is approached by the method of maximum likelihood, maximum product spacing and least square estimators. The goodness-of-fits of the exponential, beta exponential, Kumaraswamy exponential and the generalized odd log-logistic-exponential distributions have been compared through the AIC, AICC, BIC and KS statistics and found that the generalized odd log-logistic-exponential distribution fits well the data. Finally, it is concluded that the generalized odd log-logistic-exponential distribution can be quit effectively used to model the real problems and so we can recommend the use of the generalized odd log-logistic-exponential distribution in various fields of science.

Reference

[1] Marshall AN, Olkin I. A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Biometrika. 1997;84:641–652.

[2] Eugene N, Lee C, Famoye F. Beta-normal distribution and its applications. Commun Stat, Theory Methods. 2002;31:497–512.

[3] Cordeiro GM, de CastroM. A new family of generalized distributions. J Statist Comput Simul. 2011;81:883–898.

[4] Alexander C, Cordeiro GM, Ortega EMM, Sarabia JM. Generalized beta-generated distributions. Comput Stat Data Anal. 2012;56:1880–1897.

[5] Alzaatreh A, Lee C, Famoye F. A new method for generating families of continuous distributions. Metron.2013;71:63–79.

[6] Bourguignon M, Silva RB, Cordeiro GM. The Weibull-G family of probability distributions. J Data Sci. 2014;12:53–68. JOURNAL OF STATISTICAL COMPUTATION AND SIMULATION 23

[7] Cordeiro GM, Alizadeh M, Ortega EMM. The exponentiated half-logistic family of distributions: properties and applications. J Probab Stat. 2014;0:1–21. doi:10.1155/2014/864396.

[8] Tahir MH, Cordeiro GM, Alzaatreh A, Zubair M, Mansoor M. The Logistic-X family of distributions and its applications. Commun Stat – Theory Methods. 2014;

[9] Cordeiro GM, Ortega EMM, Popović B, Pescim RR. The Lomax generator of distributions: properties, minification process and regression model. Appl Math Comput. 2014;247:465–486.

[10] Cordeiro, Gauss Moutinho, Morad Alizadeh, Gamze Ozel, Bistoon Hosseini, Edwin Moises Marcos Ortega, and Emrah Altun. "The generalized odd log-logistic family of distributions: properties, regression models and applications." Journal of Statistical Computation and Simulation 87, no. 5 (2017): 908-932.