

Common Fixed Point Theorems for Four Self Maps on A Menger Space, Satisfying Common E. A. Property

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ABSTRACT

In this paper, we prove common fixed point theorems for four self maps by using weak compatibility in Menger spaces. Our result extend, generalized several fixed point theorems on Menger spaces.

Keywords— *Common fixed points, Metric space, Menger space, weak compatible mappings and E. A. property.*

AMS subject classification— *47H10, 54H25.*

1. INTRODUCTION AND PRELIMINARIES

In 1922, Banach proved the principal contraction result [1]. As we know, there have been published many works about fixed point theory for different kinds of contractions on some spaces such as quasi-metric spaces [2], cone metric spaces [3], convex metric spaces [4], partially ordered metric spaces [5-9], G-metric spaces [10-14], partial metric spaces [15-16], quasi-partial metric spaces [17], fuzzy metric spaces [18], and Mengerspaces [19]. Also, studies either on approximate fixed point or on qualitative aspects of numerical procedures for approximating fixed points are available in the literature; see [4,20,21]. Jungck and Rhoades [22] weakened the notion of compatibility by introducing the notion of weakly compatible mappings (extended by Singh and Jain [23] to probabilistic metric space) and proved common fixed-point theorems without assuming continuity of the involved mappings in metric spaces. In 2002, Aamri and Moutawakil [24] introduced the notion of property (E.A) (extended by Kubiacyk and Sharma [25] to probabilistic metric space) for self-mappings which contained the

class of non compatible map-pings due to Pant [26]. Further, Liu et al. [27] defined the notion of common property (E.A) (extended by Ali et al. [28] to probabilistic metric space) which contains the property (E.A) and proved several fixed point theorems under hybrid contractive conditions. Since then, there has been continuous and intense research activity in fixed-point theory and by now there exists an extensive literature (e.g. [29-33] and the references there in).

Many mathematicians proved several common fixed-point theorems for contraction mappings in Menger spaces by using different notions viz. compatible mappings, weakly compatible mappings, property (E.A), common property (E.A) (see [18, 34-48]).

The important development of fixed point theory in Menger spaces were due to Sehgal and Bharucha-Reid [21]. A probabilistic metric space shortly PM-Space, is an ordered pair (X, F) consisting of a non empty set X and a mapping F from $X \times X$ to L , where L is the collection of all distribution functions (a distribution function F is non decreasing and left continuous mapping of reals in to $[0,1]$ with properties, $\inf F(x) = 0$ and $\sup F(x) = 1$).

1. The value of F at $(x, y) \in X \times X$ is represented by $F_{x,y}$. The function $F_{x,y}$ are assumed satisfy the following conditions;
2. (FM-0) $F_{x,y}(t) = 1$, for all $t > 0$, iff $x = y$;
3. (FM-1) $F_{x,y}(0) = 0$, if $t = 0$;
4. (FM-2) $F_{x,y}(t) = F_{y,x}(t)$;
5. (FM-3) $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$ then $F_{x,z}(t + s) = 1$.
6. A mapping $T: [0,1] \times [0,1] \rightarrow [0,1]$ is a t -norm, if it satisfies the following conditions;
7. (FM-4) $T(a, 1) = a$ for every $a \in [0,1]$;
8. (FM-5) $T(0, 0) = 0$,
9. (FM-6) $T(a, b) = T(b, a)$ for every $a, b \in [0,1]$;
10. (FM-7) $T(c, d) \geq T(a, b)$ for $c \geq a$ and $d \geq b$
11. (FM-8) $T(T(a, b), c) = T(a, T(b, c))$ where $a, b, c, d \in [0,1]$.
12. A Menger space is a triplet (X, F, T) , where (X, F) is a PM-Space, X is a non-empty set and a t - norm satisfying instead of (FM-8) a stronger requirement.
13. (FM-9) $F_{x,z}(t + s) \geq T(F_{x,y}(t), F_{y,z}(s))$ for all $x \geq 0, y \geq 0$.

14. For a given metric space (X, d) with usual metric d , one can put $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. where H is defined as:

$$H(t) = \begin{cases} 1 & \text{if } t > 0, \\ 0 & \text{if } t \leq 0. \end{cases}$$

and t -norm T is defined as $T(a, b) = \min\{a, b\}$.

For the proof of our result we required the following definitions.

Definition 1.1 :- A triangular norm $*$ (shortly t -norm) is a binary operation on the unit interval $[0,1]$ such that for all $a, b, c, d \in [0,1]$ the following conditions are satisfied:

- (1) $a * 1 = a$,
- (2) $a * b = b * a$,
- (3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
- (4) $a * (b * c) = (a * b) * c$.

Examples of t -norms are $a * b = \min\{a, b\}$, $a * b = ab$ and $a * b = \max\{a + b - 1, 0\}$.

Definition 1.2 :- Let $(X, F, *)$ be a Menger space and $*$ be a continuous t -norm.

(a) A sequence $\{x_n\}$ in X is said to be converge to a point x in X (written $x_n \rightarrow x$) iff for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists an integer $n_0 = n_0(\varepsilon, \lambda)$ such that $F_{x_n, x}(\varepsilon) > 1 - \lambda$ for all $n \geq n_0$.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists an integer $n_0 = n_0(\varepsilon, \lambda)$ such that $F_{x_n, x_{n+p}}(\varepsilon) > 1 - \lambda$ for all $n \geq n_0$ and $p > 0$.

(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Remark 1.3:- If $*$ is a continuous t -norm, it follows from (FM - 4) that the limit of sequence in Menger space is uniquely determined.

Definition 1.4:- Self maps A and B of a Menger space $(X, F, *)$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

Weakly Compatible Maps

In 1982, Sessa [17], weakened the concept of commutativity to weakly commuting mappings. Afterwards, Jungck [4] enlarged the concept of weakly commuting mappings by adding the

notion of compatible mappings. In 1991, Mishra [16] introduced the notion of compatible mappings in the setting of probabilistic metric space.

Definition 1.5 :- Self maps A and B of a Menger space $(X, F, *)$ are said to be compatible if $F_{ABx_n, BAx_n}(t) \rightarrow 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n \rightarrow x$, $Bx_n \rightarrow x$ for some x in X as $n \rightarrow \infty$.

Definition 1.6:- Let S and T be weakly compatible of a Menger space $(X, M, *)$ and $Su = Tu$ for some u in X then

$$STu = TSu = SSu = TTu.$$

Example 1.7:- Let $X = [0, 3]$ be equipped with the usual metric $d(x, y) = |x - y|$ Define $f, g: [0, 3] \rightarrow [0, 3]$ by

$$f(x) = \begin{cases} x & \text{if } x \in [0, 1), \\ 3 & \text{if } x \in [1, 3]. \end{cases}$$

And

$$g(x) = \begin{cases} 3 - x & \text{if } x \in [0, 1), \\ 3 & \text{if } x \in [1, 3]. \end{cases}$$

Then for any $x \in [1, 3]$, x is a coincidence point and $fgx = gfx$, showing that f, g are weakly compatible maps on $[0, 3]$.

Remark 1.8:- If self maps A and B of a Menger space $(X, F, *)$ are compatible then they are weakly compatible.

Lemma 1.9. Let $(X, M, *)$ be a Menger space. Then for all $x, y \in X$, $M(x, y, \cdot)$ is a non-decreasing function.

Lemma 1.10. Let $(X, M, *)$ be a Menger space. If there exists $k \in (0, 1)$ such that for all $x, y \in X$

$$M_{x,y}(t) \geq M_{x,y}(kt) \quad \forall t > 0$$

then $x = y$.

Lemma 1.11. Let $\{x_n\}$ be a sequence in a Menger space $(X, M, *)$. If there exists a number $k \in (0, 1)$ such that

$$M_{x_{n+2}, x_{n+1}}(kt) \geq M_{x_{n+1}, x_n}(t) \quad \forall t > 0 \text{ and } n \in \mathbb{N}.$$

Then $\{x_n\}$ is a Cauchy sequence in X .

Lemma 1.12. The only t -norm $*$ satisfying $r * r \geq r$ for all $r \in [0, 1]$ is the minimum t -norm, that is

$$a * b = \min \{a, b\} \text{ for all } a, b \in [0, 1].$$

Example 1.13. Let (X, d) be a metric space. Define $a * b = \min \{a, b\}$ and

$M_{x,y}(t) = \frac{t}{t+d(x,y)}$, for all $x, y \in X$ and all $t > 0$. Then $(X, M, *)$ is a Menger space. It is called the Menger space induced by d .

2. MAIN RESULT

Now we prove the following results:

In the rest of the paper we assume that a Menger space $(X, M, *)$ satisfies the following:

(1) $M_{x,y}(t) \rightarrow 1$ as $t \rightarrow \infty$ for all $x, y \in X$

(2) If $\{x_n\}$ and $\{y_n\}$ are sequence in X such that $x_n \rightarrow x$ and $y_n \rightarrow y$ then

$$M_{x_n, y_n}(t) \rightarrow M_{x,y}(t) \text{ as } n \rightarrow \infty$$

Theorem 2.1: Let $(X, M, *)$ be a Menger space and A, B, S and T be self maps on X satisfying the following conditions:

1. $A(X) \subset T(X), B(X) \subset S(X), S(X)$ and $T(X)$ are closed

$$2. M_{(Ax, By)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(Sx, Ty)}(t), M_{(Ty, By)}(t), M_{(Sx, Ax)}(t), \\ \max \left\{ M_{(Sx, By)}(t), M_{(Ty, Ax)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Sx, By)} \left(\frac{2}{k} - 1 \right) t, M_{(Ty, Ax)}(t) \right\} \end{array} \right) \right\} \quad (2.1.1)$$

For all $x, y \in X$ $t > 0$ and for some $0 < k < 1$. where $\varphi: [0,1] \rightarrow [0,1]$, is a continuous function and $\varphi(t) > t$

If $0 < t < 1$. Suppose the pairs (A, S) and (B, T) satisfy common E.A. property and (A, S) and (B, T) are weakly compatible. Then A, B, S , and T have a unique common fixed point in X .

Proof: Since (A, S) and (B, T) satisfy common E.A. property, there exist sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\lim_{n \rightarrow \infty} M_{Ay_n, z}(t) = \lim_{n \rightarrow \infty} M_{Bx_n, z}(t) = \lim_{n \rightarrow \infty} M_{Sy_n, z}(t) = \lim_{n \rightarrow \infty} M_{Tx_n, z}(t) = 1$$

For some $z \in X$ and for all $t > 0$

So that

$$\lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Sy_n = z.$$

Since $S(X)$ and $T(X)$ are closed subsets of X , there exist $u, v \in X$ such that $Su = z$ and $Tv = z$ We show that $Au = z$. Put $x = u$ and $y = x_n$ in (2.1.1)

$$M_{(Au, Bx_n)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(Su, Tx_n)}(t), M_{(Tx_n, Bx_n)}(t), M_{(Su, Au)}(t), \\ \max \left\{ M_{(Su, Bx_n)}(t), M_{(Tx_n, Au)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Su, Bx_n)} \left(\frac{2}{k} - 1 \right) t, M_{(Tx_n, Au)}(t) \right\} \end{array} \right) \right\}$$

On letting $n \rightarrow \infty$ we get

$$M_{(Au, z)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(z, z)}(t), M_{(z, z)}(t), M_{(z, Au)}(t), \\ \max \left\{ M_{(z, z)}(t), M_{(z, Au)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(z, z)} \left(\frac{2}{k} - 1 \right) t, M_{(z, Au)}(t) \right\} \end{array} \right) \right\}$$

$$= \varphi \left(M_{(z, Au)}(t) \right) > M_{(z, Au)}(t) \text{ if } Au \neq z, \text{ a contradiction.}$$

Therefore $Au = z$.

In a similar way we can show that $Bv = z$.

Thus u is a coincidence point of A and S ,

v is a coincidence point of B and T .

Since (A, S) is weakly compatible $ASu = SAu$ so that $Az = Sz$ and (B, T) is weakly compatible $BTv = TBv$ so that $Bz = Tz$

Now we show that z is a common fixed point of A and S

$$M_{(Az, z)}(t) = M_{(Az, Bv)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(Sz, Tv)}(t), M_{(Tv, Bv)}(t), M_{(Sz, Az)}(t), \\ \max \left\{ M_{(Sz, Bv)}(t), M_{(Tv, Az)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Sz, Bv)} \left(\frac{2}{k} - 1 \right) t, M_{(Tv, Az)}(t) \right\} \end{array} \right) \right\}$$

On letting $n \rightarrow \infty$ we get

$$= \varphi \left\{ \min \left(\begin{array}{l} M_{(Az, z)}(t), M_{(z, z)}(t), M_{(Az, Az)}(t), \\ \max \left\{ M_{(z, z)}(t), M_{(z, Az)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(z, z)} \left(\frac{2}{k} - 1 \right) t, M_{(z, Az)}(t) \right\} \end{array} \right) \right\}$$

$$= \varphi \left(M_{(Az, z)}(t) \right) > M_{(Az, z)}(t) \text{ if } Az \neq z, \text{ a contradiction.}$$

Therefore $Az = z$.

Hence $Az = Sz = z$.

Similarly $Bz = Tz = z$.

Hence $Az = Sz = Tz = Bz = z$.

Thus z is a common fixed point for A, B, S and T .

Uniqueness:

If x and y are fixed points of A, B, S and T , then

$$\begin{aligned}
 M_{(x,y)}(t) &= M_{(Ax,By)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(Sx,Ty)}(t), M_{(Ty,By)}(t), M_{(Sx,Ax)}(t), \\ \max \left\{ M_{(Sx,By)}(t), M_{(Ty,Ax)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Sx,By)} \left(\frac{2}{k} - 1 \right) t, M_{(Ty,Ax)}(t) \right\} \end{array} \right) \right\} \\
 &= \varphi \left\{ \min \left(\begin{array}{l} M_{(x,y)}(t), M_{(y,y)}(t), M_{(x,x)}(t), \\ \max \left\{ M_{(x,y)}(t), M_{(y,x)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(x,y)} \left(\frac{2}{k} - 1 \right) t, M_{(y,x)}(t) \right\} \end{array} \right) \right\} \\
 &= \varphi \left(M_{(x,y)}(t) \right) > M_{(x,y)}(t) \text{ if } x \neq y, \text{ a contradiction.}
 \end{aligned}$$

Hence $x = y$.

Consequently A, B, S , and T have unique common fixed point.

Theorem 2.2 Let (X, M^*) be a Menger space and A, B, S and T be self maps on X satisfying the following conditions:

1. $A(X) \subset T(X), B(X) \subset S(X), S(X)$ and $T(X)$ are closed

$$2. M_{(Ax,By)}(t) \geq \varphi \left\{ \min \left(\begin{array}{l} M_{(Sx,Ty)}(t), \\ \min \left\{ M_{(Ty,By)} \left(\frac{2}{k} - 1 \right) t, M_{(Sx,Ax)}(t) \right\}, \\ \min \left\{ M_{(Ty,By)}(t), M_{(Sx,Ax)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Sx,By)}(t), M_{(Ty,Ax)} \left(\frac{2}{k} - 1 \right) t \right\}, \\ \max \left\{ M_{(Sx,By)} \left(\frac{2}{k} - 1 \right) t, M_{(Ty,Ax)}(t) \right\} \end{array} \right) \right\} \quad (2.2.1)$$

for all $x, y \in X$ $t > 0$ and for some $0 < k < 1$. where $\varphi: [0,1] \rightarrow [0,1]$, is a continuous function and $\varphi(t) > t$

If $0 < t < 1$. Suppose the pairs (A, S) and (B, T) satisfy common E.A. property and (A, S) and (B, T) are weakly compatible. Then A, B, S , and T have a unique common fixed point in X .

Proof: The proof is similar to that of Theorem 2.1

Corollary 2.3: (X, M_*) be a Menger space and A, B, S and T be self maps on X satisfying the following conditions:

1. $A(X) \subset T(X), B(X) \subset S(X), S(X)$ and $T(X)$ are closed
2. $M_{(Ax, By)}(t) \geq$

$$\varphi \left\{ \min \left(\sup_{t_1+t_2=\frac{2}{k}(t)} M_{(Sx, Ty)}(t), \min \{ M_{(Sx, Ax)}(t_1), M_{(Ty, By)}(t_2) \}, \sup_{t_3+t_4=\frac{2}{k}(t)} \max \{ M_{(Sx, By)}(t_3), M_{(Ty, Ax)}(t_4) \} \right) \right\} \quad (2.3.1)$$

for all $x, y \in X$ $t > 0$ and for some $0 < k < 1$. where $\varphi: [0,1] \rightarrow [0,1]$, is a continuous and increasing function and $\varphi(t) > t$

If $0 < t < 1$. Suppose the pairs (A, S) and (B, T) satisfy common E.A. property and (A, S) and (B, T) are weakly compatible. Then A, B, S , and T have a unique common fixed point in X .

Proof: Since φ is increasing we observe that $(2.3.1) > (2.2.1)$. consequently from Theorem 2.2 the result follows.

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