

An Epq Model Having Weibull Distribution Deterioration With Exponential Demand and Production With Shortages Under Permissible Delay In Payments

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Abstract

In the fundamental production inventory model, in order to solve the economic production quantity (EPQ) we always fix both the demand quantity and the production quantity per day. But, in the real situation, production is usually dependend on demand. This paper derives a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of decay and the condition of permissible delay in payments. Usually no interest is charged if the outstanding amount is settled within the permitted fixed settlement period. Therefore, it makes economic sense for the customer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. In this model shortages are permitted and fully backordered . The purpose of this paper is to investigate a computing schema for the EPQ. The model is illustrated with a numerical example.

Keywords Economic production quantity, permissible delay, weibull distribution, deterioration.

1. Introduction

For solving the EPQ for each cycle, we always fix both the demand quantity and production quantity per day in the crisp model. But, in the real situation, both of them probably will have some little disturbances per day. In recent years, many researchers have studied inventory models for deteriorating items such as electronic components, food items, drugs and fashion goods. Deterioration is defined as decay, change or spoilage that prevent the items from being used for its original purpose. There are many items in which appreciable deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the model. Therefore, many authors have considered Economic order quantity models for deteriorating items.Acting as the driving force of the whole inventory system, demand is a key factor that should be taken into consideration in an inventory study. There are mainly two categories demands in the present studies, one is deterministic demand and the other is stochastic demand. Some noteworthy work on deterministic demand are : Chung and Lin [1,2] have considered constant demand, Giri and Chakrabarty[3] and Teng and Chang[4] have considered time-dependent demand where as Giri and Chaudhuri[6], Bhattacharya[7] and Wu.et.al[8] have worked on inventory level-dependent demand and Wee and Law [9] have considered price-dependent demand . Among them, ramp type demand is a special type of time-dependent demand. Hill [10] was the first to introduce the ramp type demand to the inventory study. Then Mandal and Pal [11] introduced the ramp type demand to the inventory study of the deteriorating items. Deng.et.al [12] and Shah and Jaiswal[13] have extensively studied this type of demand.

In the classical inventory model depletion of inventory is caused by a constant demand rate alone. But subsequently, it was noticed that depletion of inventory may take place due to deterioration also. In the early stage of the study, most of the deteriorating rates in the models are constant, Padmanabhana and Vratb [14], and Bhunia and Maiti [15] worked on constant deterioration rate. In recent research, more and more studies have begun to consider the relationship between time and deteriorating rate. Wee[18], and Mahapatra[19], considered deterioration rate as linear increasing function of time. Chakrabarty.et.al[20] have considered three-parameter Weibull distribution. In this connection, studies of many researchers like Ghare and Schrader [21], Goyel et al. [22] are very important. Misra [23] developed a two parameter Weibull distribution deterioration for an inventory model. In recent research, the extensive use of trade credit as an alternative has been addressed by Goyal[4] who developed an economic order quantity(EOQ) model under the conditions of permissible delay in payments. Chung[5] then developed an alternative approach to the problem. Chand and Ward [6] analyzed Goyal's problem under assumptions of the classical economic order quantity model, obtaining different results. Next Aggarwal and Jaggi's [7] model to shortages.There are several interesting and relevant papers related to the delay of payments such as Chu et al.[9], Chung [10], Hwang and Shinn[11], Jamal et al[12], Sarker et al[13,14] , Shah[15], Shinn[16], Khouja and Mehrez[17] and there references. However , these studies were developed under the assumption that the items obtained from an outside supplier and the entire lot size is delivered at the same time. In fact, when an item can be produced in-house, the replenishment rate is also the production rate and is hence finite . Hence we amen Goyal's model by considering the replenishment rate is finite, the difference between purchasing price and selling cost and taking into consideration decay and shortage.

In the present paper, efforts have been made to analyze an EPQ model that have weibull distribution deterioration assuming demand rate to be exponential. Here production is demand dependent.

2. Assumptions and Notations

2.2 Assumptions:

- The demand is taken as exponential ,R (t)= ae^{bt}.
- Rate of production varies with demand i.e. $P = kR(t)$ where k is constant.
- Replenishment is instantaneous.
- Lead-time (i.e. the length between making of a decision to replenish an item and its actual addition to stock) is assumed to be zero. The assumption is made so that the period of shortage is not affected.
- The rate of deterioration at any time t>0 follow the two parameter Weibull distribution: $\theta(t) = \gamma\beta t^{\beta-1}$, where $\gamma(0 < \gamma < 1)$ is the scale parameter and $\beta(> 0)$ is the shape parameter. The implication of the Weibull rate (two parameter) is that the items in inventory start deteriorating the instant they are received into inventory.
- Shortages are allowed and are fully backlogged.

2.2 Notations:

- Replenishment rate is finite and it is demand dependent.
- Lead time is zero.
- T the cycle time.
- I(t) inventory level at time t.
- C_1 is the holding cost per unit time.
- C_2 is the shortage cost per unit time.
- C_3 is the unit purchase cost .
- C_4 is the fixed ordering cost of inventory.
- θ deterioration rate of finished items .
- M trade credit period.
- $I_1(t)$ the inventory level that changes with time t during production period.
- $I_2(t)$ the inventory level that changes with time t during non-production period.
- $I_3(t)$ the inventory level that changes with time t during shortage period.

- $I_4(t)$ the inventory level that changes with time t during the period when shortages are fully backlogged

3. Mathematical Modeling and Analysis

Here we assume production starts at t=0 at the rate K and the stock attains a level Q at $t= t_1$. The production stops at $t= t_1$ and the inventory gradually depletes to zero at $t=t_2$ mainly to meet the demands and partly for deterioration .Now shortages occur and accumulate to the level S at time $t=t_3$.The production starts again at a rate K at $t=t_3$ and the backlog is cleared at time t=T when the stock is again zero. The cycle then repeats itself after time T.
The model is represented by the following diagram:

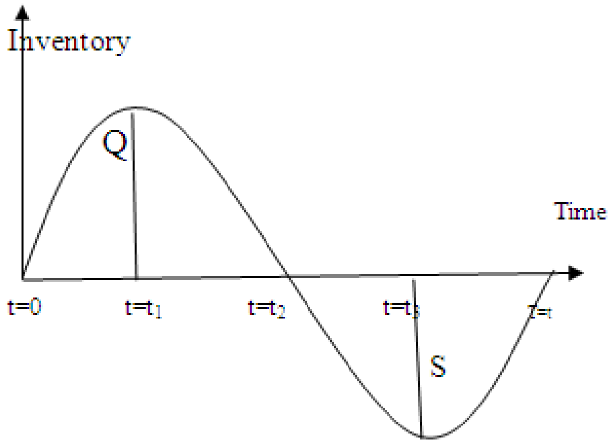

Figure: Graphical representation of inventory situation

Let I(t) be the inventory level at any time $t(0\leq t\leq T)$ and demand rate R(t) is assumed to be deterministic and is increasing exponentially with time. Further let $R(t)= a e^{bt}$, $0\leq b<1$, $a>0$.

The differential equations describing instantaneous state of I(t) in the interval[0,T] are

$$\frac{dI_1(t)}{dt} + \beta\gamma t^{\beta-1} I_1(t) = ae^{bt}(k-1) \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \beta\gamma t^{\beta-1} I_2(t) = -ae^{bt} \qquad (2)$$

$$\frac{dI_3(t)}{dt} = -ae^{bt} \qquad (3)$$

$$\frac{dI_4(t)}{dt} = -(k-1)ae^{bt} \qquad (4)$$

with the initial conditions $I_1(t_0)=0$,$I_1(t_1)=Q$,$I_2(t_1)=Q$, $I_2(t_2)=0$,$I_3(t_2)=0$, $I_3(t_3)=S$, $I_4(t_3)=S$, $I_4(T)=0$.

Now solving the above differential equations we get

$$I_1(t) = a(k-1)\{t - \gamma t^{\beta+1} + \frac{bt^2}{2} - \frac{b\gamma t^{\beta+2}}{2} + \frac{\gamma t^{\beta+1}}{\beta+1} + \frac{b\gamma t^{\beta+2}}{\beta+2}\} \quad 0 \leq t \leq t_1 \qquad (5)$$

$$I_2(t) = -a(t + \frac{bt^2}{2} + \frac{\gamma t^{\beta+1}}{\beta+1} + \frac{b\gamma t^{\beta+2}}{\beta+2} - \gamma t^{\beta+1} - \frac{b\gamma t^{\beta+1}}{2})$$
$$+a(t_2 + \frac{bt_2^2}{2} + \frac{\gamma t_2^{\beta+1}}{\beta+1} + \frac{b\gamma t_2^{\beta+2}}{\beta+2} - \gamma t_2 t^{\beta} - \frac{b\gamma t^{\beta} t_2^2}{2}) \quad t_1 \leq t \leq t_2 \qquad (6)$$

$$I_3(t) = \frac{a(e^{bt_2} - e^{bt})}{b} \qquad t_2 \le t \le t_3 \tag{7}$$

$$I_4(t) = \frac{(\beta-1)a(e^{bt_3} - e^{bt})}{b} + S \qquad t_3 \le t \le T \tag{8}$$

Holding cost over the period $[0,T] = \int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_2(t)dt$

$$= C_1 a(k-1)\left[\frac{t_1^2}{2} - \frac{\gamma t_1^{\beta+2}}{\beta+2} + \frac{bt_1^3}{6} - \frac{b\gamma t_1^{\beta+3}}{2(\beta+3)} + \frac{\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\gamma t_1^{\beta+3}}{(\beta+2)(\beta+3)}\right]$$
$$-aC_1\left[\frac{(t_2^2 - t_1^2)}{2} + \frac{b(t_2^3 - t_1^3)}{6} + \frac{\gamma(t_2^{\beta+2} - t_1^{\beta+2})}{(\beta+1)(\beta+2)} + \frac{b\gamma(t_2^{\beta+2} - t_1^{\beta+3})}{(\beta+2)(\beta+3)} - \frac{\gamma(t_2^{\beta+2} - t_1^{\beta+2})}{\beta+2}\right.$$
$$\left.-\frac{b\gamma(t_2^{\beta+3} - t_1^{\beta+3})}{2(\beta+3)}\right] + aC_1\left[t_2(t_2 - t_1) + \frac{bt_2^2(t_2 - t_1)}{2} + \frac{\gamma t_2^{\beta+1}(t_2 - t_1)}{\beta+1}\right.$$
$$\left.+\frac{b\gamma t_2^{\beta+2}(t_2 - t_1)}{(\beta+2)} - \frac{\gamma t_2(t_2^{\beta+1} - t_1^{\beta+1})}{\beta+1} - \frac{b\gamma t_2^2(t_2^{\beta+1} - t_1^{\beta+1})}{2(\beta+1)}\right] \tag{9}$$

Shortage Cost(S.C) over the period [0,T]

$$= C_2 \int_{t_2}^{T} I(t)dt$$

$$= C_2 \int_{t_2}^{t_3} I(t)dt + C_2 \int_{t_3}^{T} I(t)dt$$

$$= C_2 \frac{a}{b^2}(e^{bt_2} - e^{bt_3}) + \frac{C_2 a}{b} e^{bt_2}(t_3 - t_2) + \frac{C_2(k-1)ae^{bt_3}(T - t_3)}{b}$$
$$- \frac{C_2(k-1)a(e^{bT} - e^{bt_3})}{b^2} + C_2 S(T - t_3) \tag{10}$$

$$\text{Deteriorating Cost} = C_3 \int_0^{t_1} (P-D)dt - C_3 \int_{t_1}^{t_2} D\,dt$$

$$= aC_3 \frac{(k-1)(e^{bt_1} - 1)}{b} - \frac{aC_3(e^{bt_2} - e^{bt_1})}{b} \tag{11}$$

Set up Cost = C_4 (12)

Case I: $M \le t_2$

In this situation since the length of period with positive stock is larger than the credit period, the buyer can use the sale revenue to earn interest at an annual rate I_e in $(0,T_2)$ .The interest earned IE_1, is

$$I.E_1 = C I_e \int_0^{t_2} a e^{bt} dt \tag{13}$$
$$= \frac{C I_e a(e^{bt_2} - 1)}{b}$$

However beyond the credit period, the unsold stock is supposed to be financed with an annual rate I_c and the interest payable I.P . is given by

$$I.P = C I_c \int_M^{t_2} a e^{bt} dt = \frac{C I_c a(e^{bt_2} - e^{bM})}{b} \quad (14)$$

Total average Cost (TVC)= (Holding Cost+Deteoration Cost+ Shortage Cost+Setup Cost+Interest payable-Interest earned)/T

$$TVC=(C_4+H.C.+S.C.+I.P-I.E_1)/T \quad (15)$$

Case II: $M > T_2$

Since $M > T_2$, the buyer pays no interest but earns interest at an annual rate I_e during the period (0,M). Interest earned in this case , denoted by $I.E_2$, is given by

$$I.E_2 = C I_e \int_0^{t_2} a e^{bt} dt + C I_e \int_{t_2}^{M} a e^{bt} dt$$

$$= \frac{C I_e a(e^{bM} - 1)}{b} \quad (16)$$

Total average Cost (TVC)= (Holding Cost+Deteoration Cost+ Shortage Cost+Setup Cost-Interest earned)/T

$$TVC=(C_4+H.C.+D.C.+S.C.-I.E_2)/T \quad (17)$$

Using the initial conditions the total average cost becomes function of t_1 and T. Hence we find the global optimal solution of total average cost by using LINGO 12.

4. Numerical

Case (i): When permissible delay period is less than the cycle time i.e $0<M \leq T$ then interest is earnerd for the period [0,M] but beyond the permissible delay period [M,T] interest I_c is charged on the outstanding amount. So total variable cost is calculated considering both interest charged and interest earned.
Let C_1=\$2 per unit; C_3= \$.02 per unit ; C_4= \$100 per order ; b=.2; beta=.6; I_c=12% , I_E=10% ; M=.3year ; a=3 units, gamma= .03; t_0=.7; t_1=7.
Optimal Value \$ 24.7525.

Case (ii): When permissible delay period is greater than the cycle time then interest charged I_c =0. So the total variable cost is calculated considering only the interest earned.
Let C_1=\$2 per unit; C_3= \$.02 per unit ; C_4= \$100 per order ; b=.4; beta=1.6; I_E=10% ; M=.3year ; a=3 units, gamma= .03; t_0=.7; t_1=7.

Optimal Value \$ 34.952.

5. Conclusion

In the present paper an EPQ model of deteriorating items having weibull distribution deterioration has been studied with permissible delay in payments. The paper , however considers only one break in the delay period.A natural extension of the model would be to study the case of N breaks in the permissible delay period , i.e. to assume $M=M_i$, if $q_{i-1} \leq q \leq q_i$,i=1,2,…..,N, Where $M_1 < M_2<\ldots<M_N$, and $q_0=0<q_1<\ldots<q_N =\infty$.

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