The Fitting of a SARIMA model to Monthly Naira-Euro Exchange Rates

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Abstract

The time plot of the series NEER shows an overall positive trend with a peak in early 2010 after a deep depression in late 2008. No seasonality is discernible by this visual inspection. Seasonal (i.e. 12-month) differencing yields SDNEER which exhibits an overall slightly positive trend and a high peak in late 2009. Still the visual inspection of the time plot in Figure 2 hardly makes any seasonality obvious. A non-seasonal differencing of SDNEER yields DSDNEER with an overall horizontal trend. Though the time plot in Figure 3 does not give an impression of any regular seasonality, the correlogram of Figure 4 reveals seasonality of period 12 months. Besides there is indication that the product of two moving average components both of order one is involved: one component is seasonal and the other non-seasonal. In addition to that a significant spike of the partial autocorrelation function at lag 12 suggests the involvement of a seasonal autoregressive component of order one. Therefore a $(0, 1, 1)x(1, 1, 1)_{12}$ SARIMA model is proposed and fitted. It has been shown to be adequate

Keywords: : Naira-Euro Exchange Rates, SARIMA models, Nigeria

1. Introduction

Modelling of Nigerian foreign exchange rates has engaged the attention of some researchers in recent times. A few of such publications include Olowe(2009), Etuk(2012), etc. Not much, if at all, has been done in respect of modelling Nigerian naira-European euro exchange rates. This work is aimed at fitting a Seasonal Autoregressive Integrated Moving Average (SARIMA) Model to the monthly naira-euro exchange rates.

Many economic and financial time series are known to exhibit some seasonality. In spite of their volatility, foreign exchange rates tend to be seasonal. For instance, Etuk(2012) observed that monthly Nigerian naira-US dollar exchange rates follow a 12-month seasonal pattern, fitted an $(0, 1, 1)x(1, 1, 1)_{12}$ SARIMA model to the series and on its basis computed the 2012 forecasts.

SARIMA models are an adaptation of autoregressive integrated moving average (ARIMA) models to specifically fit seasonal time series. That is, their construction takes into consideration the underlying seasonal nature of the series to be modeled. Many authors have written on SARIMA models extensively. A few amongst them are Box and Jenkins (1976) who proposed them, Priestley(1981), Madsen(2008), Gerolimetto(2010) and Suhartono(2011). A knowledge of the theoretical properties of the models is the basis of their identification and estimation.

2. Materials and Methods

The data for this write-up are 96 monthly averages of Naira-Euro exchange rates from 2004 to 2011 retrievable from the Data and Statistics publication of the Central Bank of Nigeria website www.cenbank.org

2.1 SARIMA Models

A time series $\{X_t\}$ is said to follow an autoregressive moving average of orders p and q denoted by ARMA(p, q) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \quad (1)$$

where $\{\varepsilon_t\}$ is a white noise process and the α's and the β's constants such that (1) is both stationary and invertible.

Suppose (1) is put as

$$A(L)X_t = B(L)\varepsilon_t \quad (2)$$

where $A(L) = 1 - \alpha_1 L - ... - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + ... + \beta_q L^q$ and $L^k X_t = X_{t-k}$. Let the d^{th} difference of X_t be denoted by $\nabla^d X_t$ where $\nabla = 1-L$. A replacement of X_t in (1) by $\nabla^d X_t$ yields an autoregressive integrated moving average model of orders p, d and q (denoted by ARIMA(p, d, q)) in $\{X_t\}$.

Suppose that $\{X_t\}$ is observed to be seasonal of period s. Let D be the degree of seasonal (i.e. s-point) differencing, $\{X_t\}$is said to follow a $(p, d, q)x(P, D, Q)_s$ SARIMA model if

$$A(L)\Phi(L^s)\nabla^d\nabla^D_s X_t = B(L)\Theta(L^s)\varepsilon_t \quad (3)$$

where $\Phi(L)$ and $\Theta(L)$ are polynomials in P and Q respectively with coefficients such that the model is stationary and invertible respectively.

2.2 Model Estimation

The determination of the orders must be done before parameter estimation in the model (3). The correlogram which refers to the autocorrelation and the partial autocorrelation plot is the basis for order determination. Seasonality is easily revealed by a spike at the seasonal lag.

A $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA model is the product of two moving average components both of order one: one nonseasonal and the other 12-point seasonal. Such a model is given by

$$X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_{12}\varepsilon_{t-12} + \beta_{13}\varepsilon_{t-13} \quad (4)$$

and has an autocorrelation function (ACF) with significant values only at lags 1, 11, 12 and 13, with the values at lags 11 and 13 equal. (Box and Jenkins (1976), Suhartono(2011)).

ARMA model estimation is traditionally based on the maximum likelihood criterion, the least squares criterion or the maximum entropy criterion. Under each criterion mentioned above the estimation results from the application of nonlinear techniques of optimization since the ARMA model involves items of a white noise process. An initial estimate of the solution is usually made and employed. The process then involves iterations with each iteration expected to be an improvement on its predecessor until the system converges to an optimal solution depending on the level of accuracy desired. However for pure autoregressive or pure moving average models there exist linear optimization techniques (See for example Box and Jenkins(1976), Priestley(1981), Oyetunji(1985)). There are attempts to propose linear optimization techniques for mixed ARMA estimation (See for example Etuk(1987, 1998)).

After the model must have been estimated it is tested for goodness-of-fit. To do this, its residuals are analysed. Under the hypothesis of model adequacy, the residuals should be uncorrelated with zero mean and follow a Gaussian distribution. Eviews software which is based on the least squares criterion was used for the fitting of the model.

3. Results and Discussion

A visual inspection of the time plot of NEER in Figure 1 reveals an overall positive secular trend with no observed regular seasonality. However a multiplicative model is suggestive given the tendency of the seasonal component to increase with time. A 12-month differencing of NEER yields the seasonal difference SDNEER which has an overall slightly positive trend with no clear seasonality but still suggestive of a multiplicative model (see Figure 2). Nonseasonal differencing of SDNEER yields DSDNEER which exhibits an overall horizontal trend and no observable regular seasonality (see Figure 3). Clearly D = d = 1. However the ACF of DSDNEER in Figure 4 has significant spikes at lags 1, 12 and 13. Therefore s=12. Moreover, this indicates the involvement of a component of the form (4). That is q = 1 = Q. Its partial autocorrelation function (PACF) has a significant spike at lag 12. This suggests the involvement of a seasonal autoregressive component of order one. That is, P = 1. The model proposed is therefore of order $(0, 1, 1)x(1, 1, 1)_{12}$. Estimation as summarized in Table 1 yields

$$DSDNEER_t + 0.3612DSDNEER_{t-12} = \varepsilon_t + 0.1875\varepsilon_{t-1} - 0.7847\varepsilon_{t-12} - 0.2660\varepsilon_{t-13} \quad (5)$$

(±0-1204) (±0.1143) (±0.1062) (±0.1281)

The model (5) was subjected to some diagnostic tests for goodness-of-fit. Only the first lag moving average coefficient is not statistically significant, being less than twice its standard error. As much as 60% of the variation in DSDNEER is accounted for by the model. The fitted model agrees closely with the data (See Figure 5). The histogram of the residuals in Figure 6 is normal with zero as the axis of symmetry. The correlogram of the residuals in Figure 7 is such that all the autocorrelations are non-significant. All of these attest to the adequacy of the model.

4. Conclusion

The foregoing discussion shows that NEER follows a $(0, 1, 1)x(1, 1, 1)_{12}$ SARIMA model. That means that it follows the subset ARMA model in DSDNEER given by equation (5). This model has been shown to adequately explain the variation in the monthly Naira-Euro exchange rates.

References

Box, G. E. P. & Jenkins, G. M. (1976). *Time Series Analysis, Forecasting and Control,* San Francisco: Holden-Day.

Etuk, E. H. (1987). On the Selection of Autoregressive Moving Average Models. *An Unpublished Ph. D. Thesis, Department of Statistics, University of Ibadan, Nigeria.*

Etuk, E. H. (1998). An Autoregressive Integrated Moving average (ARIMA) Simulation Model: A Case Study. *Discovery and Innovation,* 10(1&2), 23-26.

Etuk, E. H. (2012). Forecasting Nigerian Naira-US Dollar Exchange Rates by A Seasonal Arima Model. *American Journal of Scientific Research,* 59, 71-78

Gerolimetto,M.(2010).ARIMA and SARIMA Models. www.dst.unive.it/~marherita/TSLecturenotes6.pdf

Madsen, H. (2008). *Time Series Analysis,* London: Chapman & Hall.

Olowe, R. A. (2009). Modelling Nigeria/Dollar Exchange Rate Volatility: Application of Garch and Assymetric Models. *International Review of Business Research Papers,* 5(3), 377-398.

Oyetunji, O. B. (1985). Inverse Autocorrelations and Moving Average Time Series Modelling. *Journal of Official Statistics,* 1, 315-322.

Priestley, M. B. (1981). *Spectral Analysis and Time Series,* London: Academic Press.

Suhartono (2011). Time Series Forecasting by using Autoregressive Integrated Moving Average : Subset, Multiplicative or Additive Model. *Journal of Mathematics and Statistics*, 7(1), 20-27

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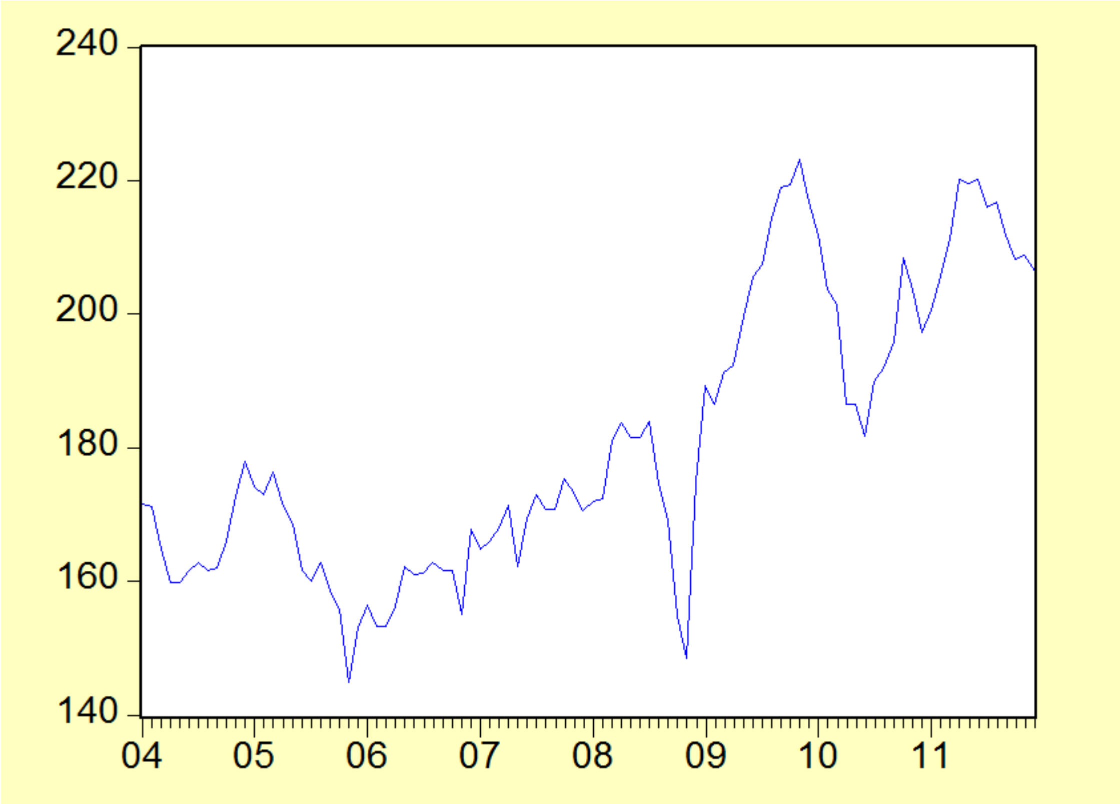

FIGURE 1: NEER

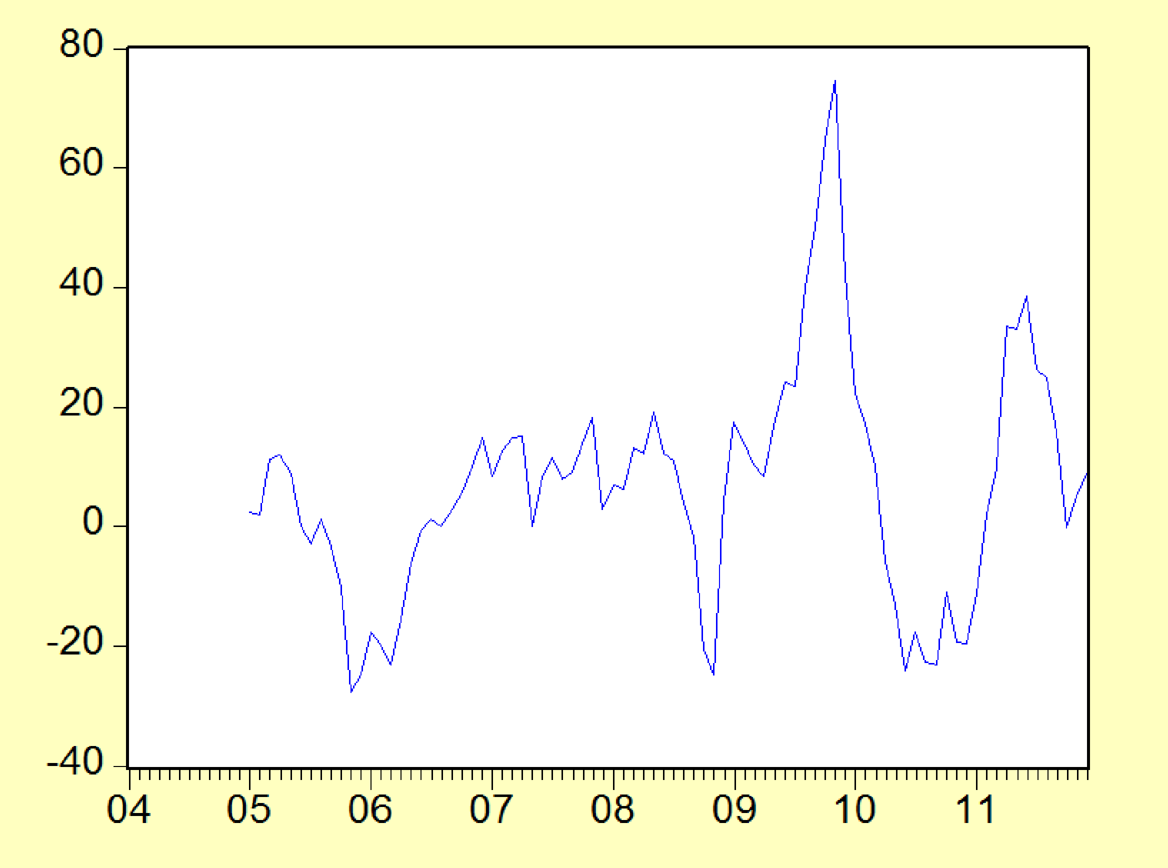

FIGURE 2: SDNEER

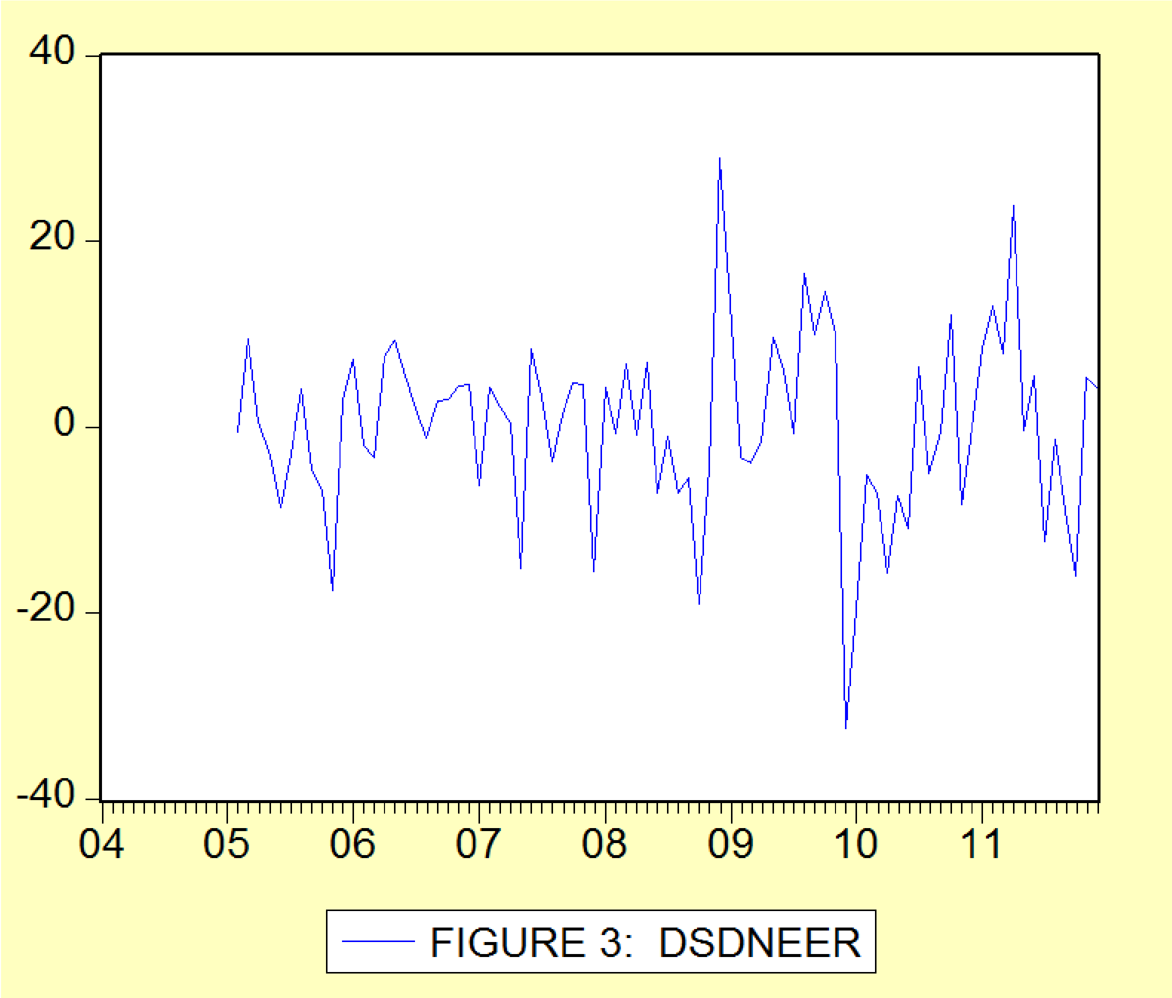
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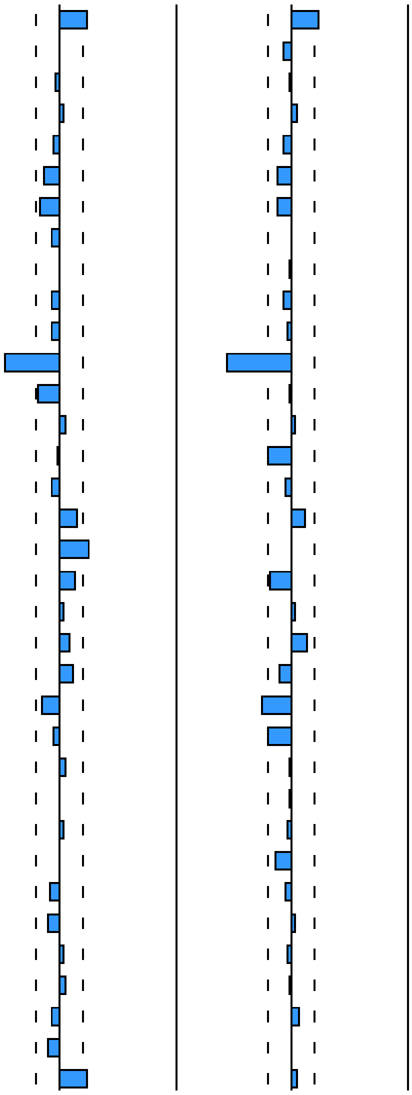
FIGURE 3: DSDNEER



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.244	0.244	5.1180	0.024
		2	-0.008	-0.071	5.1231	0.077
		3	-0.040	-0.022	5.2649	0.153
		4	0.031	0.050	5.3511	0.253
		5	-0.048	-0.076	5.5618	0.351
		6	-0.146	-0.123	7.5274	0.275
		7	-0.176	-0.118	10.395	0.167
		8	-0.058	-0.003	10.714	0.218
		9	-0.006	-0.009	10.717	0.296
		10	-0.066	-0.077	11.142	0.347
		11	-0.059	-0.031	11.485	0.404
		12	-0.498	-0.567	36.132	0.000
		13	-0.192	-0.016	39.861	0.000
		14	0.055	0.030	40.175	0.000
		15	-0.021	-0.206	40.223	0.000
		16	-0.062	-0.045	40.625	0.001
		17	0.159	0.125	43.316	0.000
		18	0.257	-0.001	50.509	0.000
		19	0.133	-0.190	52.449	0.000
		20	0.033	0.027	52.569	0.000
		21	0.089	0.136	53.464	0.000
		22	0.120	-0.105	55.129	0.000
		23	-0.162	-0.264	58.215	0.000
		24	-0.052	-0.206	58.533	0.000
		25	0.046	-0.019	58.792	0.000
		26	0.006	-0.012	58.797	0.000
		27	0.033	-0.038	58.936	0.000
		28	0.007	-0.132	58.942	0.001
		29	-0.096	-0.047	60.157	0.001
		30	-0.100	0.029	61.499	0.001
		31	0.025	-0.032	61.582	0.001
		32	0.046	-0.012	61.870	0.001
		33	-0.058	0.069	62.351	0.002
		34	-0.107	-0.001	64.014	0.001
		35	0.249	0.049	73.161	0.000

FIGURE 4: CORRELOGRAM OF DSDNEER

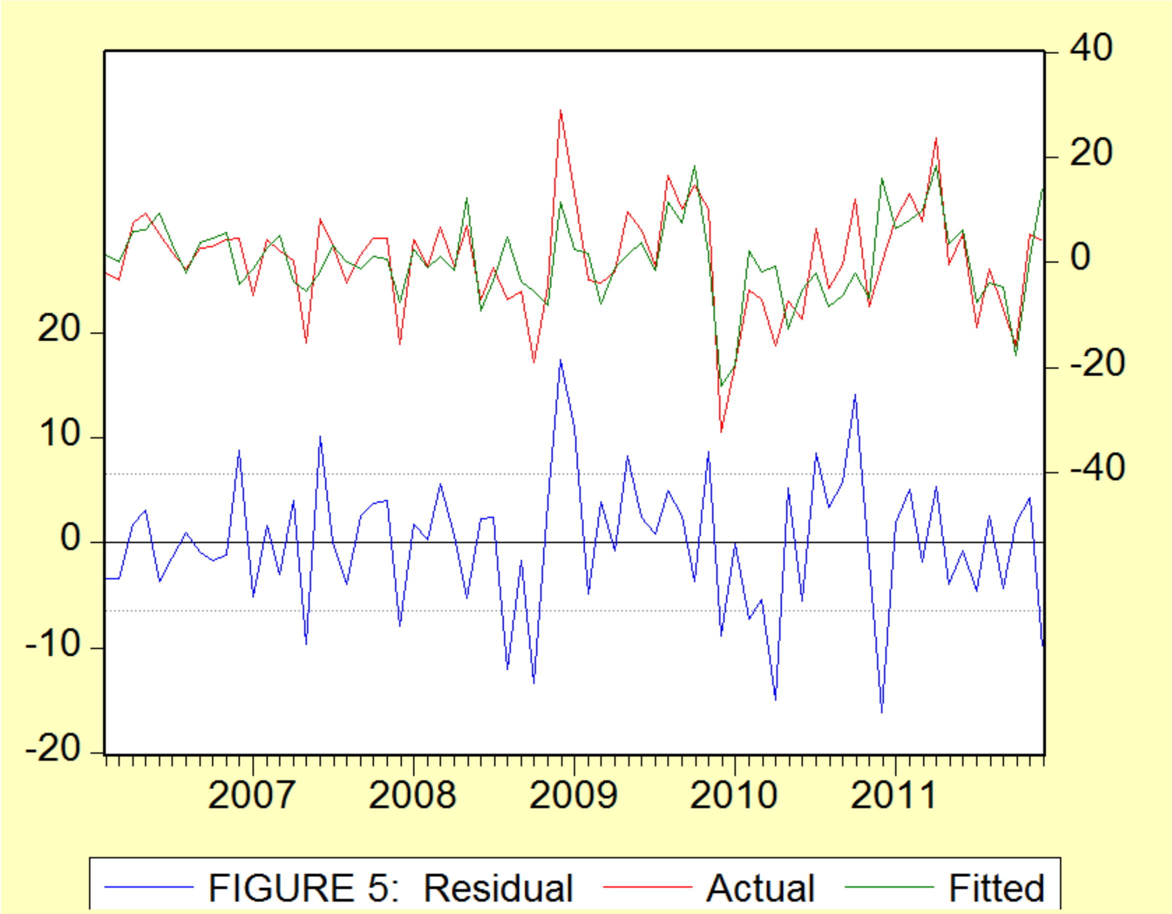
TABLE 1: MODEL ESTIMATION

Dependent Variable: DSDNEER
Method: Least Squares
Date: 12/27/12 Time: 16:38
Sample(adjusted): 2006:02 2011:12
Included observations: 71 after adjusting endpoints
Convergence achieved after 19 iterations
Backcast: 2005:01 2006:01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	-0.361165	0.120376	-3.000296	0.0038
MA(1)	0.187528	0.114308	1.640548	0.1056
MA(12)	-0.784656	0.106216	-7.387333	0.0000
MA(13)	-0.265967	0.128131	-2.075749	0.0418

R-squared	0.598093	Mean dependent var	0.379577
Adjusted R-squared	0.580097	S.D. dependent var	10.04295
S.E. of regression	6.507821	Akaike info criterion	6.638575
Sum squared resid	2837.566	Schwarz criterion	6.766050
Log likelihood	-231.6694	F-statistic	33.23510
Durbin-Watson stat	2.105409	Prob(F-statistic)	0.000000

Inverted AR Roots	.89+.24i	.89 -.24i	.65+.65i	.65 -.65i
	.24+.89i	.24 -.89i	-.24 -.89i	-.24+.89i
	-.65+.65i	-.65+.65i	-.89+.24i	-.89 -.24i
Inverted MA Roots	.99	.86+.49i	.86 -.49i	.50+.85i
	.50 -.85i	.01+.98i	.01 -.98i	-.34
	-.48 -.85i	-.48+.85i	-.83+.49i	-.83 -.49i
	-.96			



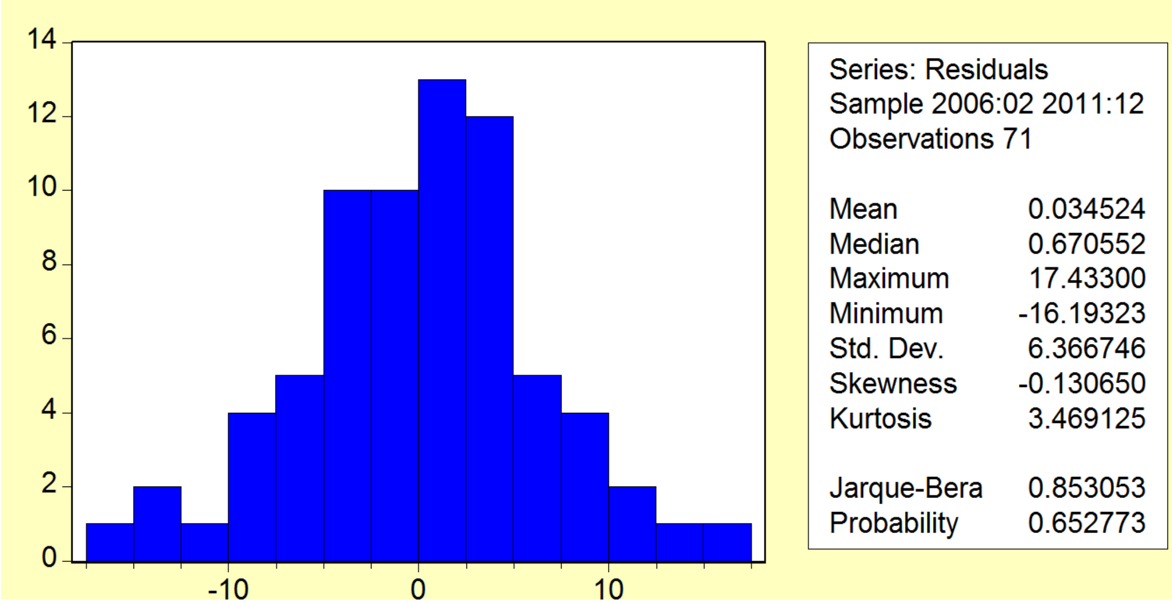


FIGURE 6: HISTOGRAM OF THE RESIDUALS

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.072	-0.072	0.3861	
		2	0.002	-0.003	0.3863	
		3	-0.103	-0.103	1.1893	
		4	-0.005	-0.021	1.1916	
		5	-0.131	-0.136	2.5424	0.111
		6	0.062	0.032	2.8485	0.241
		7	-0.179	-0.185	5.4418	0.142
		8	0.031	-0.021	5.5219	0.238
		9	0.020	0.014	5.5544	0.352
		10	-0.077	-0.137	6.0565	0.417
		11	-0.044	-0.059	6.2211	0.514
		12	-0.065	-0.140	6.5960	0.581
		13	-0.090	-0.133	7.3244	0.603
		14	0.084	0.005	7.9670	0.632
		15	-0.046	-0.129	8.1668	0.698
		16	-0.105	-0.181	9.2049	0.685
		17	0.114	0.005	10.455	0.656
		18	0.143	0.074	12.443	0.571
		19	0.044	0.006	12.634	0.631
		20	0.066	0.010	13.073	0.667
		21	0.076	0.110	13.673	0.690
		22	0.088	0.137	14.484	0.697
		23	-0.079	-0.086	15.159	0.712
		24	-0.164	-0.137	18.138	0.578
		25	-0.033	0.004	18.262	0.632
		26	0.047	0.041	18.522	0.675
		27	0.011	0.019	18.537	0.728
		28	0.054	0.052	18.890	0.758
		29	-0.039	0.020	19.078	0.793
		30	-0.140	-0.098	21.572	0.712
		31	0.082	0.081	22.449	0.714
		32	-0.005	0.040	22.453	0.760

FIGURE 7: CORRELOGRAM OF THE RESIDUALS