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The Dynamics of Oil Price Shock and Its Frequency Jump in A Developing Country

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Abstract

In many natural resources price, the intrinsic stochastic element driving the pricing process is the central characteristic. Understanding this underlying stochastic process is of cheap importance, particularly for crude oil due to its essential role in the world economy. Modeling petroleum prices assumes the market price of crude oil follows a continuous stochastic process that assumes smooth changes, either in a single factor or multi-factor Gaussian frame work. This changes (or jumps) however is not necessarily smooth but jumps unexpectedly rapidly. This paper determines the dynamics of these rapid jumps in the prices of crude oil, both in terms of spit prices and future prices. Furthermore, we obtain the future price of crude oil given the price dynamics.
Keywords: Oil prices, Jump diffusion, Price volatility, Stochastic dynamics.

1. Introduction

Since the 1980 great recession, the price of crude oil in the New York Mercantile Exchange, one of the markets where international crude oil is being traded, has been on the increase with fluctuations fuelled basically by the speculations, forces of demand and supply, climate change and price of substitutes. The price of crude oil rose to its peak in the 1990s when the Gulf war broke out and Middle East suppliers used it as a tool for frustrating America.

Ever since, the price of crude has been on the increase with emerging economics like India and China helping to drive up the price as their demand for fossil fuel surpasses the supply in the market presently.

In the recent advent of the last global economic recession, industries in the developed nations such as America started closing shop and fast growing economies like China started faltering and this caused a drop in the demand for crude oil. On the other hand, oil producing nations like Nigeria in their bid to reap bountifully from the hike in price kept flooding the market with crude oil.

Crude oil is the main stay of the Nigeria economy, the price of oil plays a vital role in shaping the economic well being of the country. The price of oil has witnessed significant fluctuations since 1974. For instance, it oscillate between $17 per barrel and $26 at different times in 2002 and about $53 per barrel by October 2004 (Philip and Akintaye,2006). Between 1986 and 2010, oil prices increased more than 6 folds from $23 per barrel in January 2000 to a peak at an all time high at $146 per barrel in July 2008 before crashing to $42 per barrel by December of 2008. The dynamical increase in the oil price will also increase the frequency as in the figure below. For the year 2009, oil price average $61.73 per barrel (Hassan and Zahid,2011). The price of oil has continued to trend upward as a result of the political crisis in the Middle East, particularly, the revolutions in some Arab Countries including Tunisia, Egypt, Libya, Yemen and Syria as well as the Iranian nuclear crisis which led to a ban of the import of Iranian oil by U.S.A and European countries and threats of repercussion from Iran.

The Nigeria economy is exposed to oil price shocks since oil contributes over 90% of the total revenue. This shock is so severe that the Nigeria budget is even tied to a particular price of crude oil and the budget was adjusted in some occasions when there is a sudden change in crude oil price such as the reduction of budget due to a fall in oil prices, during the last global financial crisis.

This is even worsened due to the fact that despite the four refineries, Nigeria is still exposed to oil price shocks due to massive importation of refined petroleum products. As an oil exporter and importer of refined products, Nigeria is thus vulnerable to oil price volatility.

“Price volatility “refers to the degree to which prices rise or fall over a period of time. In an efficient market, prices reflect known existing and anticipated future circumstances of supply and demand and factors that could affect them. Changes in market prices tend to reflect changes in what markets collectively known or anticipate.

When market prices tend to change a lot over relatively a short time, the market is said to have high volatility. When relatively stable prices prevail, the market is said to have low volatility. In energy markets, assets represent huge investments, typically hundreds of millions if not billions of dollars. The ability of those investments to earn a return depends upon the ability to produce fuels or power and sell it at a viable price.

Some amount of price volatility is an inevitable consequence of a market-based economy. Since companies invest

based on expectations about prices, high price volatility creates uncertainty and risk, and risk premiums rise to compensate. Volatility prices can also affect labour markets, increasing temporary layoffs or promoting surge hiring.
Exponential Levy models generalize the classical Black and Scholes setup by allowing crude oil prices to jump while preserving the independence and stationary of returns. There are ample reasons for introducing jumps in financial modeling. First of all, asset prices do jump and some risks simply cannot be handled within continuous-path models.
The diffusion process is widely used to described the evolution of asset returns over time. In option pricing it allows the use of Black and Scholes type formulae to value European option on stocks, foreign currencies, interest rates, commodities and futures. Under this process, instantaneous asset returns are normally distributed. However, the distributions observed in the market exhibit non-zero skewness and higher kurtosis than the normal distribution which produces pricing errors when the Black and Scholes formula is used.
A way of obtaining distributions consistent with market data is to assume crude oil price follows a Jump-diffusion process.
Merton (1976) developed a model in which the arrival of normal information is modeled as a diffusion process, while the arrival of abnormal information is modeled as Poisson process.
The jump diffusion process can potentially described crude oil prices mere accurately at the cost of making the market incomplete, since jumps in the stock price cannot be hedged using traded securities. If the market is incomplete, the payoffs of the option cannot be replicated and the option cannot be priced.
Finally jump process correspond to genuinely incomplete markets, whereas all continuous-path models are either complete or completable with a small number of additional assets.
The price evolution of a risky assets are usually modeled as the trajectory of a diffusion process defined on some underlying probability space, with the geometric Brownian motion used as canonical reference model. Brick (1987) had shown that geometric Brownian motion can indeed be justified as the rational expectations equilibrium in a market with homogeneous agent.
Following Black and Scholes (1972, 1973), a significant plateau has been reach by many authors in model of price dynamics.
Stein and Stein (1991), Heston (1993), Hull and White (1987) among others followed the approach to pricing options on stock with Stochastic volatility which starts by specifying the joint process for the oil price and its volatility. Their models are typically calibrated to the prices of a few options or estimated from the time series of options.
Some of these studies suggest rising oil prices reduced output and increased inflation in the 1970s and early 1980s and falling oil prices boosted output and lowered inflation particularly, in the U.S in the mid-to late 1980s. Bohi (1989), Bernanke et al (1997) analyzed the possibility that the 1974 economic recession in the United States may have the consequence of the Federal Reserve's policy response to the inflation triggered by an oil price shock.
The studies found out that changes in domestic output arose due to the Federal Reserve's policy of monetary tightening induced inflation sparked off by crude oil price changes. However, most empirical studies carried out have focused on the oil importing countries, particularly the developed countries. Few studies exist yet on the effect of oil price volatility on key macro-economic variables for oil exporting country as Nigeria.
In general, developing countries like Nigeria as opposed to industrialized countries, have only limited financial tools to implement financial policy, so it is worth looking at how the monetary authority of such a resource rich country responds to an international oil price shock.
The price evolution of a risky assets are usually modeled as the trajectory of a diffusion process defined on some underlying probability space, with the geometric Brownian motion the best used as the canonical reference model. Brick(1987) had shown that geometric Brownian motion can indeed be justified as the rational expectations equilibrium in a market with homogenous agent.
The objective of this paper however is to determine the dynamics of impacts of crude oil price shock and its frequency jump.

2. Mathematical Setup

Let P_t denote price at time t, which is said to follows a geometric Brownian motion (GBM) process with trend μ and volatility parameter σ if

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_t, \tag{1}$$

where $dW = \xi_t\sqrt{dt}$ the increment of a Wienner process is continuous time (Dixit and Pindyck,1994). ξ_t has mean equal to zero and unity as standard deviation.
Let $y_t = \log(P_t) - \log(P_{t-1})$ denote the natural logarithm of the ratio of price in period t to the price in period $t-1$.

Then y_t is normally distributed with variance σ^2 and mean $\alpha = \mu - \frac{\sigma^2}{2}$ so that the pure diffusion model now gives

$$y_t = \mu + \sigma W_t. \quad (2)$$

Merton(1976) described $(Y-1)$ as an impulse function that produces a finite jump. We hereby introduce jumps into the model in the Merton(1976) style, so that when an event occurs, the jump component is modeled as a Poisson driven process N_t, where

$$dN_t = \begin{cases} 0, with\ probabilty\ \ 1-\lambda dt \\ Y-1\ with\ probability\ \ \lambda dt \end{cases}. \quad (3)$$

Then equation (1) results to a stochastic process for the random variable P_t, thus;

$$dP_t = P_t\{\mu dt + \sigma dW + dN_t\}. \quad (4)$$

Where dW_t has the same properties as in equation(1) and dN_t is the independent Poisson process described in equation(3).

The terms dW_t and dN_t together make up up the instantaneous component of the unanticipated return. If a jump occurs at time t, then its size is $dN_t = Y_t - 1$, $(Y_t - 1$,is a random variables, which described the percentage change in the asset return);

Now, assume P_t follows instead the Ornstein-Uhlenbeck process with jump

$$dP(t) = \alpha dt + \gamma P_1(t^-)\int_R z\,\bar{N}(dt,dz); \quad P(0) = p \in R. \quad (5)$$

Define for a given function $\theta(z)$,

$$G(t) = \exp\left(\int_0^t \int_R \theta(z)\,\bar{N}(dt,dz) - \int_{|z|<R} e^{0(z)} - 1 - \theta(z)\right\}v(dv)\cdot t), \quad (6)$$

then by Itô's formula, we have

$$dG(t) = G(t^-)\int_R e^{\theta(z)} - 1\}\bar{N}(dt,dz). \quad (7)$$

Hence, if we put

$$\bar{P}(t) = P(0)dG(t) + \alpha G(t)\int_0^t G^{-1}(s)ds, \quad (8)$$

we have

$$d\bar{P}(t) = P(0)dG(t) + \alpha G(t)G^{-1}(t)dt + \int_0^t G^{-1}(s)ds \cdot dG(t)$$

$$= \alpha dt + P(0)G^{-1}\int_R \{e^{\theta(z)} - 1\}\bar{N}(dt,dz)$$

$$+\alpha \cdot \int_0^t G^{-1}(s)ds \cdot [G(t^-)\int_R \{e^{\theta(z)} - 1\}\bar{N}(dt,dz)]\alpha dt$$

$$+\left[P(0)G(t^-) + \alpha G(t^-)\int_0^t G^{-1}(s)ds\right]\int_R \{e^{\theta(z)} - 1\}\bar{N}(dt,dz)$$

$$= \alpha dt + \bar{P}(t^-)\int_R \{e^{\theta(z)} - 1\}\bar{N}(dt,dz) \quad (9)$$

so $P(t) := \bar{P}(t)\ solve\ dP(t) = \alpha dt + \gamma P(t^-)\int_R z\,\bar{N}(dt,dz); \quad P(0) = p \in R$,if we choose $\theta(z)$ such that

$$e^{\theta(z)} - 1 = \gamma z \qquad a.s.v \quad (10)$$

or

$$\theta(z) = In(1+\gamma z)\ a.s.v. \quad (11)$$

It is natural to assume that the terms dW_t and dN_t are independent, since the first component reflects ordinary

movements in price while the second component reflects unusual changes in oil price (Wilmot and Mason,2011).
With probability λ jumps occurs and with probability $1-\lambda$ no jump occurs at time t. The sizes of jump are assumed to be lognormally distributed with θ as the drift of the logarithm of the jump size and δ as the volatility of the logarithm of the jump size so that $In(Y_t) \sim N(\theta, \delta^n)$
$dN_t = 0$ implies no jump occurred at time t.
The mixed jump-diffusion process is now expressed (Wilmot and Mason,2011) as

$$y_t = \mu + \sigma z_t + In(Y_t)J_t \quad (12)$$

The compound Poisson process with jump size intensity λ and jump size distribution μ is a stochastic process $(P_t)_{t\geq 0}$ defined by

$$P_t = \sum_{i=1}^{N_t} Y_i. \quad (13)$$

Where $\{Y_i\}_{i\geq 1}$ is a sequence of independent random variable with law μ and N is a Poisson process with intensity λ independent from $\{Y_i\}_{i\geq 1}$.
In other words, a compound Poisson process is a piecewise constant process with jump times of a standard Poisson process and whose jump sizes are $i.i.d.$ random variables with a given law.
Let $(P_t)_{t\geq 0}$ be a compound Poisson process with jump intensity λ and jump size distribution μ . Then P is a piecewise constant Lévy process and its characteristic function is given by

$$E[e^{ivP_t}] = \exp\{t\lambda \int_{\mathbb{R}} (e^{ivP} - 1)\, \mu(dP) \quad (14)$$

The Merton (1976) model is one of the first applications of jump processes in financial modeling. In this model, to take into account price discontinuities, one adds Gaussian jumps to the log-price, thus:

$$S_t = S_0 e^{rt+P_t},\ P_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i\ , Y_i \sim N(\mu, \delta^2)\ . \quad (15)$$

The advantage of this choice of jump size distribution is to have a series representation for the density of the log-price (as well as for the prices of European options) as;

$$P_t(y) = e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(\lambda t)^k \exp\{-\frac{(y-\gamma t-k\mu)^2}{2(\sigma^2 t+k\delta^2)}}{K!\sqrt{2\pi(\sigma^2 t+k\delta^2)}} \quad (16)$$

3. THE PRICE SHOCK'S DYNAMIC AND ITS FREQUENCY

Since the economy of a developing (Nigeria for example) is exposed to crude oil price shocks. It is important for policy makers to be aware of the economic impacts to the domestic economy from the dynamics and the volatile crude oil price. In this sequel we state

THEOREM 1

Suppose

$$dP(t) = \alpha dP + \sigma dB(t) + \int_R \gamma(z)\bar{N}(dt,dz),\ P(0) = p \in đ \quad (17)$$

Where α, σ are constants, γ : đ→đ, is the dynamics of the oil price process,
given,
$Y(t) = \exp(P(t))$, lognormal due to the assumption that log return relatives are normally distributed.
Then the dynamics of the future price will jump is given as;

$$dY(t) = Y(t^-)[\beta dt + \theta dB(t) + \lambda \int z\bar{N}(dt,dz)], \quad (18)$$

for a given constants β, θ and λ.

PROOF

Choose $f \in C^2$(đ) and put $Y(t) = f(P(t))$. Then by the Itȯs formula

$$dY(t) = f'(P(t))[\alpha dt + \sigma dB(t)] + \frac{1}{2} \sigma^2 f''(P(t)dt + \int_{|z|<R}\{f(P(t^-) + \gamma(z)) - f(P(t^-)) - \gamma(z)f'(P(t^-))\}v(dz)dt + \int_R\{f(P(t^-) + \gamma(z) - f(P(t^-))\}\bar{N}(dt,dz). \quad (19)$$

In particular, if $f(p) = \exp(p)$ this gives$dY = Y(t)[\alpha dt + \sigma dB(t)] + \frac{1}{2}\sigma^2 Y(t)dt$

$$+\int_{|z|<R}\{\exp(P(t^-) + \gamma(z)) - \exp(P(t^-)) - \gamma(z)\exp(P(t^-)\}v(dz)dt$$

$$+\int_R \{\exp(P(t^-) + \gamma(z) - \exp(P(t^-))\}\ \bar{N}(dt,dz)$$

$$= Y(t^-)[(\alpha + \frac{1}{2}\sigma^2 ++ \sigma dB(t) + \int_R\{e^{\gamma(z)} - 1\}\bar{N}(dt,dz)]. \quad (20)$$

We see that $Y(t)$ solve the equation

$$dY(t) = Y(t^{-})[\beta dt + \theta dB(t) + \lambda\int_R z\bar{N}(dt,dz)]$$

if and only if $\sigma + \frac{1}{2}\sigma^2 + \int_{|z|<R}\{e^{\gamma(z)} - 1 - \gamma(z)\}v(dz) = \alpha \qquad \sigma = 0$

and

$$e^{\gamma(z)} - 1 = \lambda z\ (i.e\ \gamma(z) = In(1+\lambda z)a.e, v. \tag{21}$$

Theorem 2: Let the dynamics of the oil price be given as;

$$dP(t) = (m - P(t)dt + \sigma dB(t) + \gamma\int_R z\bar{N}(dt,dz); P(0) = p \in R \quad , \tag{22}$$

wehere, $(m,\gamma\ constants)(the\ mean - reverting\ Lévy$-Ornstein-Uhlenbeck process).

Then the future oil price is given as;

$$P(t) = m + (P_0 - m)e^{-t} + \sigma\int_0^t e^{(s-t)}dB(s) + \gamma\int_0^t\int_R ze^{(s-t)}\bar{N}(dt,dz). \tag{23}$$

PROOF

We first make some general remarks;

Suppose $dP_i(t) = \alpha_i(t,w)dt + \sigma_i(t,w)dB(t) + \int_R \gamma_i(t,z,w)\bar{N}(dt,dz)\ for\ i = 1,2$

Define $Y(t) = P_1(t)\cdot P_2(t). Then, by\ the\ Itó\ formula\ with\ f(p_1,p_2) = p_1\cdot p_2,$

$$dY(t) = P_2(t)[\alpha_1 dt + \sigma_1 dB(t)] + P_1(t)[\alpha_2 dt + \sigma_2 dB(t)] + \frac{1}{2}\cdot 2\sigma_1\sigma_2 dt$$

$$+\int_{|z|<R}\{(P_1(t^-) + \gamma_1(t,z))(P_2(t^-) + \gamma_2(t,z)) - P_1(t^-)P_2(t^-))$$

$$-P_2(t^-)\gamma_1(t,z) - P_1(t^-)\gamma_2(t,z)\}v(dz)dt$$

$$+\int_R\{(P_1(t^-) + \gamma_1(t,z)\ P_2(t^-) + \gamma_2(t,z)) - P_1(t^-)P_2(t^-)\}\bar{N}(dt,dz)$$

$$= P_2(t)[\alpha_1 dt + \sigma_1 dB(t)] + P_1(t)[\alpha_2 dt + \sigma_2 dB(t)] + \sigma_1\sigma_2 dt + \int_{|z|<R}\gamma_1(t,z)\gamma_2(t,z)\}v(dz)dt$$

$$+\int_R\gamma_1(t,z)\gamma_2(t,z) + P_1(t^-)\gamma_2(t,z) + P_2(t^-)\gamma_1(t,z)\}\bar{N}(dt,dz) \tag{24}$$

In particular, if $dP(t) = \alpha dt + \sigma dB(t) + \int_R \gamma(t,z)\bar{N}(dt,dz)$, so that;

$$d(e^{\lambda t}P(t) - P(t)\lambda e^{\lambda t}dt + e^{\lambda t}[\alpha dt + \sigma dB(t)] + \int_R e^{\lambda t}\gamma(t,z)\bar{N}(dt,dz)$$

$$= e^{\lambda t}dP(t) + \lambda P(t)e^{\lambda t}dt. \tag{25}$$

Now consider the equation

$$dP(t) = (m - P(t)dt + \sigma dB(t) + \gamma\int_R z\bar{N}(dt,dz),$$

$$where\ we\ assume\ that\ \gamma z > -1\ for\ a.a\ z(v). It\ can\ be\ written$$

$$d(e^t P(t)) = me^t dt + \sigma e^t dB(t) + \gamma e^t\int_R z\bar{N}(dt,dz), \tag{26}$$

with solution

$$P(t) = P(0)e^{-t} + m\int_0^t e^{(s-t)}ds + \sigma\int_0^t e^{(s-t)}dB(s) + \gamma\int_0^t\int_R ze^{(s-t)}\bar{N}(dt,dz) \tag{27}$$

Or

$$P(t) = m + (P_0 - m)e^{-t} + \sigma\int_0^t e^{(s-t)}dB(s) + \gamma\int_0^t\int_R ze^{(s-t)}\bar{N}(dt,dz) \tag{28}$$

as required.

4. Conclusion

This paper determines the dynamics of the oil price as well as obtains the future oil price given the dynamics. In getting (23) we have made us of the fact that

$F(t) := \exp\left\{-\int_0^t\int_R \theta(z)\bar{N}(dt,dz) + \int_{|Z|<R}\left(e^{0(z)} - 1 - \theta(z)\right)v(dz)\cdot t\right\}$,for suitable $\theta(z)$.

Equation (23) is the future price of crude oil characterized by jumps. Oil prices have important micro-economics effects with commodity risk having a potentially significant impact on a firm's profit. A better understanding of the stochastic process driving the price of oil and its derivates is very necessary. Also, knowledge of the underlying dynamical behavior of the underlying asset with aid in crude oil forecasting, investment decisions and other pricing of oil-linked financial instruments.

Oil price shocks are often followed by economic downturns (Hamilton, 2011). It is therefore paramount for policy makers to know the crude oil price dynamics following the events in the market for oil.

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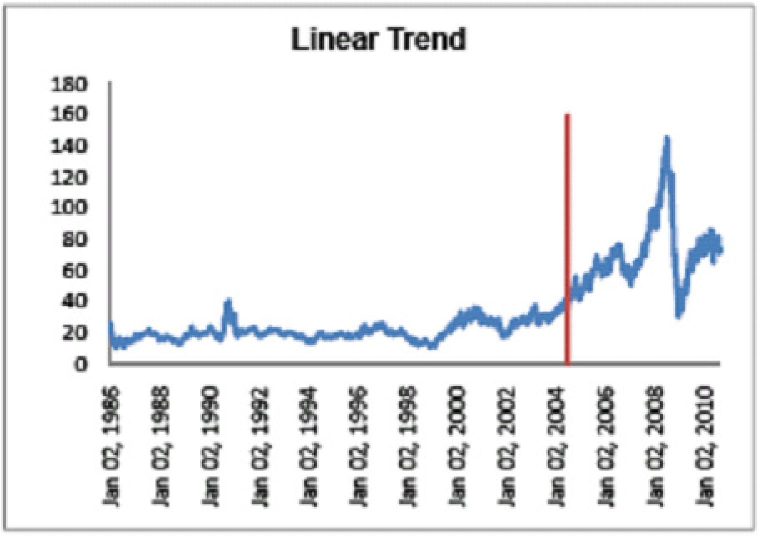

Figure 1. The oil price fluctuation and the price hike in the year 2004 to 2008.The figure above shows that the dynamical increase in oil price will also increase the frequency.