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A Modified Algorithm of Improved Explicit Euler's Method To Solve Initial Value Problems.

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Abstract:

The aim of this paper is to develop an improved approximation method for the calculation of numerical solution of initial value problems $y_o(x_o) = y_o$ in Ordinary differential equations. In this study, we extend in the intent of Euler, to propose a newly second order Euler method. The proposed method applied on some IVP and compared with the Improving the Modified Euler Method and other existing methods and it explains that the results of proposed method are logical, accurate and convergent of order two. In this study a linear and non-linear $f(x) = 0$ exactly stable expressed one step numerical integration algorithm is developed for solving non-linear and linear initial value problems (IVPs) in Ordinary differential equations. The accuracy and stability properties of the method are inquired and expressed to yield at least second order and A-stable. Through the simple improvement we established and able to find very much improve performance by improved Modified Euler method. The results acquired by the numerical experiments indicate the effectiveness of the proposed method in solving problems.

Keywords: ODEs, initial value problems, stability, accuracy, convergent, Numerical Values, Euler method, modified and Improved.

Introduction:

In Mathematics, one of the most significant methods are used in differential equations for solving modeling problems of physical and engineering sciences. Recently, these have been developed in Medicine, Biology, Anthropology, Economics and Business and many more [1]. A general form of the nth order ordinary differential equation is described as:

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_o(x) = f(x) \quad (1.1)$$

Also unfortunately, many could not be solved exactly. A nonlinear ODE is one which has either non-linearity in its dependent variables or contains product of dependent vra Nevertheless, these types of problems, which we face when we are using the analytical techniques. To resolve such issues that arise due to different kind of errors, we use numerical methods, precisely numerical integration with initial value problems but spread of error in such numerical schemes can somehow be controlled [2]. It used forward Euler method to make it Improved Modified Euler method of order two. IME method also gives better results as compared to the Euler method and Modified Euler method [3]. The result of this work was reducing the size of iterations. It may be noted that this method is very power and efficient as to find analytical and numerical solutions of ordinary differential equations (ODEs) [4]. The new method implemented on the some standard (IVPs). The comparison between Modified Euler method and other existing Euler method has been discussed. In this study, a newly improvement that had contributed to a third (3^{rd}) order proposed. It was concluded that the Modified Euler method is better, efficient and accurate.[5]. In this study show Euler method and fourth order Range-kutta method (RK-Four) for solving initial value problems (IVPs) in ordinary differential equations. We compare numerical solutions with the exact solution to verify the accuracy. So many numerical problems are given to show the reliability [7]. In this study, a new numerical technique is proposed. This study shows the numerical comparison between Adomian decomposition method and Runge - Kutta method. The numerical results are most accurate by imple-menting this new method [8]. This method results to be of second order and stable. Numerical experiments are also discussed. In this algorithm, the author has presented at low cost used by the method as it is shown in the table via CPU values [9]. It was shown that the modified improved modified Euler method is of order two. Better performance has been achieved [10]. Even then there be a big count of ordinary differential equations whose solution could not be found in closed form so we have to use numerical methods to find approximately solutions of an ODE with the dictated initial condition. Many mathematicians had analyzed differential equations and added to the areas including Newton, Leibniz, the Bernoulli family and Euler.

Methodology and Problem Statement:

Euler method: The Euler method is the most uncomplicated, not only of completely one-step methods, but also of completely methods for the estimate solution of initial value problems.

$$y_{n+1} = y_n + hk_1 \quad \text{where, } k_1 = f(x_n, y_n) \tag{1.2}$$

Therefore, the above method expressed as follows:

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y(x_n + \frac{h}{2})) \tag{1.3}$$

And the Trapezoidal rule as:

$$y_{n+1} = y_n + \frac{h}{2}(f(x_n, y_n) + f(x_n + 1, y_n + 1)) \tag{1.4}$$

By putting the forward Euler step for the missing y-values to find the Modified Euler (ME) method.

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)) \tag{1.5}$$

This method can be written as:

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$$y_{n+1} = y_n + hk_2$$
$$where, k_1 = f(x_n, y_n) and k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \quad (1.6)$$

The Improved Euler method (IEM):

$$y_{n+1} = y_n + \frac{h}{2}(f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))) \quad (1.7)$$

Above equation can be written as:

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$
$$where, k_1 = f(x_n, y_n) and k_2 = f(x_n + h, y_n + hk_1) \quad (1.8)$$

Improved Modified Euler method:

In this work, the author Improved the Modified Euler method by putting the forward Euler method, instead of yn in the inner-function for the rating of the Modified Euler method[3]. This improvement conduct to a new method called IME method. It is given as:

$$y_{n+1} - y_n = hf(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n + h, y_n + hk_1))$$
$$where, k_1 = f(x_n, y_n) and k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n + h, y_n + hk_1)) \quad (1.9)$$

That is, yn in ME method (1.5) was replaced $y_n + hk_1 where, k_1 = f(x_x, y_n)$. Nevertheless, it had been showed that the performance of IME method is very poor as compared to Modified Euler method. Therefore, thus the new improvement of the Euler method is proposed, which is also known as MIME method [10]. This was achieved by using $y_n + h/2 f(x_n, y_n)$ to replace yn in IME method (1.9) to develop,

$$y_{+1} - y_n = hf(x_n + \frac{h}{2}, y + \frac{h}{2} f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1))$$
$$where, k_1 = f(x_n, y_n) and k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \quad (1.10)$$

Known as Modified Improved Modified Euler method. We extend the Euler method by substituting $y_n + h/4k_1 where, k_1 = f(x_n, y_n)$ instead by yn in Improved Modified Euler method (1.9), to propose the following equation:

$$y_{n+1} - y_n = hf(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n + \frac{h}{2}, y_n + \frac{h}{4} k_1))$$
$$where k_1 = f(x_n, y_n) and k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n + \frac{h}{2}, y_n + \frac{h}{4} k_1)) \quad (1.11)$$

Called proposed method, Modified improved Explicit Euler method. This method is very much better performance for the numerical solutions of Ordinary differential equations with initial value problems (IVPs) stable, convergent and accurate this method is also called second order.

STABILITY FUNCTION:

Stability polynomial function analysis of one step method is generally accomplished applying the linear model problem

$$y' = \lambda y, y(x_0) = y_o, x_o \leq x \quad (1.12)$$

Where λ is complex. And y(x) can be written as:

$$y(x) = \eta e^{\lambda(x - x_o)} \quad (1.13)$$

So that

$$y_{n+1} = y_n + h\lambda(y_n + \frac{h}{2}.\lambda(y_n + \frac{h}{4}\lambda y_n))$$
$$If n = 0$$
$$y_1 = y_o + h\lambda(y_o + \frac{h}{2}\lambda y_o + \frac{h^2}{8}\lambda^2 y_o) \quad (1.14)$$
$$y_1 = y_o + h\lambda y_o + \frac{h^2}{2}\lambda^2 y_o + \frac{h^3}{8}\lambda^3 y_o$$
$$y_1 = (1 + \lambda h + \frac{h^2}{2}\lambda^2 + \frac{h^3}{8}\lambda^3) y_o$$

Suppose that $z = h\lambda$ then, the universal form of the stability polynomial function of new proposed method is:

$$y_1 = 1 + z + \frac{z^2}{2} + \frac{z^3}{8}$$

Where $y_1 = \phi(z)$

$$\phi(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{8} \tag{1.15}$$

The stability polynomial function of the newly method presents that it is a second order proposed method.

Results and Discussions:

In this work, the absolute errors of numerical values of y(x) are computed for the initial value problems in the examples which are presented in table1-3. These calculations were accomplished applying varying step-size of h=0.2, 0.0785 and 0.2 respectively. By applying the proposed method on problems, we get the solutions of IVPs.

These are all the following three problems

$$1. y'(x) = x + y, y(0) = 1 and [0,2]$$

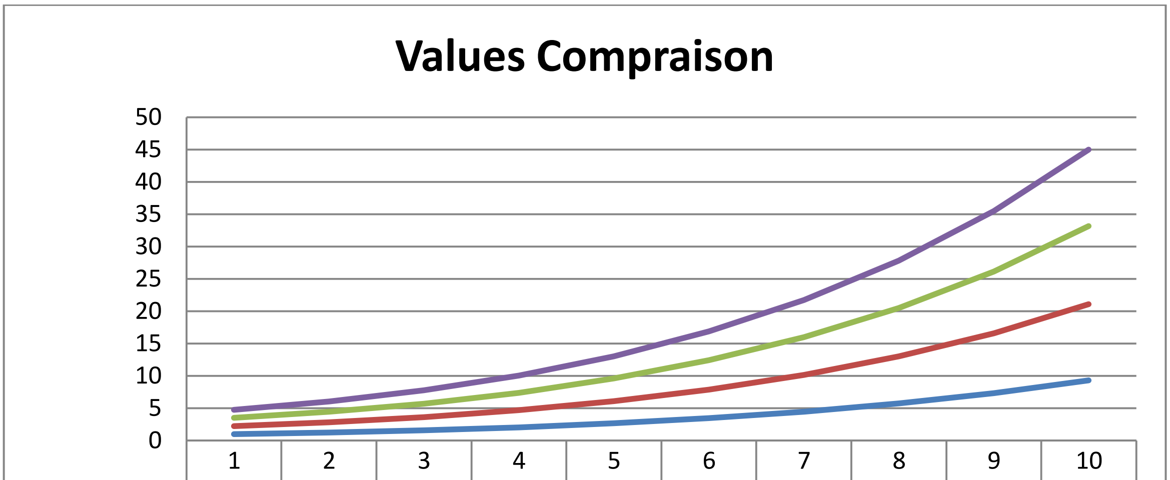
$$2. y'(x) = 1 + (y(x))^2, y(0) = 1 and [0, \frac{\pi}{4}]$$

$$3. y'(x) = y(x), y(0) = 1 and [0,1]$$

The following graphs and tables explain the results and absolute error of numerical problems.

1. Table no.1 Numerical Results of problem No 1.

No.Iteration	Exact	New	IME	MIME	Error new	Error IME	Error MIME
1	1	1.243	1.248	1.244	0.243	0.248	0.244
2	1.242805516	1.583903	1.596352	1.586568	0.341097484	0.353546484	0.343762484
3	1.583649395	2.044345563	2.067534848	2.049586096	0.460696168	0.483885453	0.465936701
4	2.044237601	2.650745932	2.689062654	2.659794209	0.606508332	0.644825053	0.615556609
5	2.651081857	3.435360783	3.494612688	3.449868524	0.784278927	0.843530831	0.798786667
6	3.436563657	4.437575517	4.525405931	4.459739336	1.00101186	1.088842274	1.023175679
7	4.440233845	5.705479706	5.831896859	5.738201469	1.26524586	1.391663014	1.297967623
8	5.710399934	7.297790721	7.475841756	7.344882195	1.587390787	1.765441822	1.634482261
9	7.306064849	9.28620247	9.532830309	9.352646042	1.980137621	2.22676546	2.046581193
10	9.299294929	11.75825322	12.0953843	11.85053346	2.458958287	2.796089369	2.551238534



	1	2	3	4	5	6	7	8	9	10
MIME	1.244	1.58657	2.04959	2.65979	3.44987	4.45974	5.7382	7.34488	9.35265	11.8505
IME	1.248	1.59635	2.06753	2.68906	3.49461	4.52541	5.8319	7.47584	9.53283	12.0954
New	1.243	1.5839	2.04435	2.65075	3.43536	4.43758	5.70548	7.29779	9.2862	11.7583
Exact	1	1.24281	1.58365	2.04424	2.65108	3.43656	4.44023	5.7104	7.30606	9.29929

Graphical Representation of Numerical Values of Example No 1.

Figure1. Graphical Representation of Absolute Error of Problem No 1.

Table No.2 Numerical Results of Problem No 2.

No. Iteration	Exact	New	IME	MIME	Error new	Error IME	Error MIME
1	0	1.081702059	1.082064862	1.081822993	1.081702059	1.082064862	1.081822993
2	0.0984914	1.170079748	1.170865994	1.170341799	1.071579748	1.072365994	1.071841799
3	0.19891237	1.265679422	1.266960011	1.266106177	1.066779422	1.068060011	1.067206177
4	0.30334668	1.369092306	1.370950146	1.369711338	1.065792306	1.067650146	1.066411338
5	0.41421356	1.480958203	1.483490262	1.481801749	1.066758203	1.069290262	1.067601749
6	0.53451114	1.601969525	1.605289225	1.603075277	1.067469525	1.070789225	1.068575277
7	0.66817864	1.732875646	1.737115678	1.734287678	1.064675646	1.068915678	1.066087678
8	0.82067879	1.874487631	1.879803252	1.876257492	1.053787631	1.059103252	1.055557492
9	1	2.027683372	2.034256265	2.029871353	1.027683372	1.034256265	1.029871353
10	1.21850353	2.19341314	2.201455947	2.19608977	0.97491314	0.982955947	0.97758977

Figure2. Graphical Representation of Absolute Error of Problem 2.

Table No.3 Numerical Results of Problem No 3.

No. Iteration	Exact	New	IME	MIME	Error New	Error IME	Error MIME
1	1	1.221	1.224	1.222	0.221	0.224	0.222
2	1.105170918	1.490841	1.498176	1.493284	0.385670082	0.393005082	0.388113082
3	1.221402758	1.820316861	1.833767424	1.824793048	0.598914103	0.612364666	0.60339029
4	1.349858808	2.222606887	2.244531327	2.229897105	0.87274808	0.894672519	0.880038297
5	1.491824698	2.713803009	2.747306344	2.724934262	1.221978312	1.255481647	1.233109564
6	1.648721271	3.313553474	3.362702965	3.329869668	1.664832204	1.713981695	1.681148397
7	1.8221188	4.045848792	4.11594843	4.069100734	2.223729992	2.293829629	2.246981934
8	2.013752707	4.939981375	5.037920878	4.972441097	2.926228668	3.02416817	2.95868839
9	2.225540928	6.031717259	6.166415154	6.076323021	3.806176331	3.940874226	3.850782092
10	2.459603111	7.364726774	7.547692149	7.425266732	4.905123663	5.088089038	4.96566362

Figure3. Graphical Representation of Absolute Error of Problem No 3.

Conclusion:

In this study, the method has formed at improving as compared to Modified Euler method. The comparison between the proposed method and other existing Euler methods. Modified Improved Explicit Euler method owns wider area of absolute stability. We have also gone on to prove that proposed method is also of order two**.** The absence of requirement of hardware and software stained the level from our numerical results.

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