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An Improved Algorithm to Solve Transportation Problems for Optimal Solution

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Abstract

In this paper, we have developed an algorithm to obtain initial basic feasible solution of transportation problems where the object is to minimize the transportation cost. The proposed method is compared with well-known existing methods including Least-Cost Method and North-West Corner Method and is found to yield better results. Feasible solution from the proposed method leads to solution closest to the optimal solution; and in some numerical examples same as the optimal solution.

Key words: Transportation problem, Initial Basic Feasible Solution, Optimal Solution

1. INTRODUCTION

Transportation Problem is the special class of Linear Programming Problem in the field of Applied Mathematics and also in Operation Research. Linear programming is an important mathematical tool that deals with the use of limited resources in an optimal approach [3]. The word programming means predict to minimize cost, use of resources, the time or maximize profit etc. Such problems are called optimal problems.

The type of linear programming problem which may be solved using a simplified version of the simplex technique called is known as transportation problem. The main objective in such problems is to minimize the total cost involved in transporting a commodity (single product) from one or more centers called origins (sources, supply or capacity of center) to distinct places called destination (sinks, demand or requirement of center) [4, 5].

The basic initial feasible solution methods to solve transportation problems are

1. **Northwest Corner Method (NWCM)** – This is the first technique to solve transportation problems. It is a simple procedure to minimize the cost. Initially in northwest corner method, we compute the northwest cell in transportation table and cross out rows or columns with zero supply or demand this procedure continues till minimum cost is obtained. [2,7]

2. **Least Cost Method (LCM)** – In this method, we compute the lowest cost in transportation table and cross out row or cross out columns with zero supply and demand. This process is continued till minimum cost is obtained.[1-3]

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3. **Vogel's Approximation Method (VAM)** - An effective technique to find initial basic feasible solution is Vogel's Approximation Method or Unit Cost Penalty Method. In this method, initially we take the penalty cost lowest and next lowest difference in each row and column. Further taking maximum penalty in column and minimum penalty in row then cross out row or column with zero supply or demand. This process is repeated till minimum cost is attained. [2-5]

In recent years, researchers have made significant contribution towards developing efficient algorithms for finding initial basic feasible solution of transportation problems. Three different schemes for finding initial basic feasible solution were introduced by Hosseini [1].An improved least cost algorithm was developed and comparative study with LCM and VAM was carried out by sharifuddin et al. [5]. Mishra [2] presented a study about existing methods to solve transportation problems. Das et al. [3] presented logical Vogel algorithm and also discussed the limitation of Vogel's method.

In this paper, we discuss a new approach to solve the transportation problems. The objective in these problems is to minimize the cost incurred in transporting commodities from sources to destinations. We discuss the transportation model together with its mathematical formulation in section 2. Section 3 presents the algorithm of our proposed method. In section 4, we provide solutions of 5 problems by implementing the proposed method. For understanding of readers, one problem is solved at length. Finally, a comparative study with existing methods including NWCM and LCM is carried out. It is found that proposed method gives more accurate results.

2. TRANSPORTATION MODEL

The general and usual form of the TP is available by the following Model:

Destination Source	D_1	D_2	D_3	... D_J.......	D_n	SUPPLY a_i
S_1	x_{11} c_{11}	x_{12} c_{12}	x_{13} c_{13}	.	.	a_1
S_2	x_{21} c_{21}	 c_{22}	 c_{23}	.	.	a_2
. .....S_I.....				...x_{ij}...		a_i
S_m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{m3} c_{m3}		x_{mn} c_{mn}	a_m
DEMAND b_j	b_1	b_2	b_3	b_j	b_n	

Figure 1.1.2 Transportation Problem Model

Suppose there are n destinations and m sources. Let a_i be the number of supply units presented at sources i and let b_j be the number of demand units required at destination j , c_{ij} represents the cost of transporting one unit of commodity from source i to destination j. If $x_{ij}(x_{ij} \geq 0)$ is the number of units transported from source i to destination j , then the equal linear programming representation will be determine non-negative value of x_{ij} satisfying both the availability constraints

$$\sum_{j=1}^{n} x_{ij=a_i}\ (i = 1,2,3 \ldots \ldots . m) \ldots\ldots (a)$$

as well as the requirement constraints

$$\sum_{i=1}^{m} x_{ij=b_j}\ (j = 1,2,3 \ldots \ldots . n) \ldots\ldots (b)$$

and minimizing the total cost of transportation

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

It is also assumed that total availabilities $\sum a_i$ satisfy the total requirement $\sum b_j$ that is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ .$$

This is known as balanced problem.

3. PROPOSED ALGORITHM

Step1: Calculate in 1st approximation the penalty by taking the difference between lowest and next lowest cost in each row or columns.

Mathematically

$|c_{12} - c_{11}|$ Considering c_{12} is smallest and c_{11} is next smallest cost in row and $|c_{21} - c_{11}|$ c_{21} is smallest and c_{11} is next smallest cost in column.

Step2: Taking largest difference in column and smallest difference in row $Max(b_j)$& $Min(a_i)$ assuming a_i minimum cost in row and $b_j$maximum cost in column. Reduce the matrix (row or Column) with zero supply or demand.

Step3: Calculate in 2nd approximation the smallest cost in the transportation table $x_{ij} = \min(a_i b_j)$, assuming a_i minimum cost in row or b_j minimum cost in column. Reduce the matrix with zero supply or demand.

Step4: compute 1st& 2nd approximation alternative especially to get a desired minimum cost.

Step5: Minimizing total cost of transportation

$\boldsymbol{Z} = x_{11}c_{11} + x_{12}c_{12} \ldots \ldots .. \quad x_{mn}c_{mn}$ Where $\sum_{j=1}^{n} x_{ij=a_i}$,$\sum_{i=1}^{m} x_{ij=b_j}$

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4. NUMERICAL ILLUSTRATION

In this paper, consider four different-size cost minimizing transportation problems, selected form literature. We also use these examples to perform a comparative study of proposed algorithm with northwest corner and least cost methods. We solve example 2 step-by-step .continuous

Eaxmple-1

Destination Source	D1	D2	D3	D4	D5	Supply
S1	4	1	2	4	4	60
S2	2	3	2	2	2	35
S3	3	5	2	4	4	40
Demand	22	45	20	18	30	$\sum$ 135

273

Example-2

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Demand	6	10	15	$\sum$ 31

143

Example-3

Destination Source	D1	D2	D3	D4	Supply
S1	7	5	9	11	30
S2	4	3	8	6	25
S3	3	8	10	5	20
S4	2	6	7	3	15
Demand	30	30	20	10	$\sum$ 90

415

Example-4

Destination Source	D1	D2	D3	D4	Supply
S1	3	1	7	4	300
S2	2	6	5	9	400
S3	8	3	3	2	500
Demand	250	350	400	200	$\sum$ 1200

2850

Example-5

Destination Source	D1	D2	D3	Supply
S1	6	4	1	50
S2	3	8	7	40
S3	4	4	2	60
Demand	20	95	35	$\sum$ 150

555

Table: 1

4.1 Example illustration

We present here the step-wise solution of one of these problems for better understanding of the reader. Considering this, step by step allocations in various cost cells are explained below only for Ex-2 from Table 1

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Demand	6	10	15	$\sum 31$

Table2: Solve step by step problem 2

STEP:1 Calculate the penalty by taking the difference between lowest and next lowest cost in each row or columns Table 2.1.

Destination Source	D1	D2	D3	Supply	Row penalty
S1	6	8	4	14	2
S2	4	9	8	12	4
S3	1	5 2	6	0	1
Demand	6	5	15	$\sum 31$	
Column penalty	3	6	2		

Table: 2.1

STEP2: Using Table 2.1, Taking smallest difference in row is min{2,4,1}=1 and largest difference in columns max{3,6,2}=6,x32=min{5,10}=5 then delete the S3 row because its zero supply. |D2-S3=|10-5|=5 is reaming demand

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
Demand	6	5	15	$\sum 26$

Table: 2.2

STEP3: Now we computing approximation the smallest cost in the transportation Table 2.2 is 4.

Destination Source	D1	D2	D3	Supply
S1	6	8	14 4	0
S2	4	9	8	12
Demand	6	5	1	$\sum 12$

Table 2.2 is 4

X13= min{14, 15}=14 then delete the S1 row because its zero supply |D3-S1|=|15-14|=1 is reaming demand.

Destination Source	D1	D2	D3	Supply	Row penalty
S2	4	9	1 8	12-1	4
Demand	6	5	0	$\sum 11$	
Colum penalty	2	4	7		

Table: 2.3

STEP4: Similarly repeat the above steps, Using Table 2.3 taking the smallest and next smallest penalty in each row{4} and each column {2,4,7} now taking minimum penalty in row is 4 and maximum penalty in column is 7 x23=min{12,1}=1 then delete D3 column.

Destination Source	D1	D2	Supply
S2	6 4	9	11-6=5
Demand	0	5	$\sum 5$

Table: 2.4

Now we compute minimum cost in transportation Table2.4 is 4, x21=min{6,11}=6 then delete the D1colums because its zero demand |11-6|=5 is remand supply.

Destination Source	D2	Supply
S2	9 5	5-5=0
Demand	0	$\sum$ 0

Table:2.5

In last cell same penalty row and column, x22=min{5,5}=5 then delete the whole matrix because its zero supply and demand.

STEP5: Using Table2 Total allocates the point for obtained minimize cost

Destination Source	D1	D2	D3	Supply
S1	6	1 8	14 4	14
S2	6 4	5 9	1 8	12
S3	1	5 2	6	5
Demand	6	10	15	$\sum$ 31

Z= 4*14+4*6+9*5+8*1+2*5=143

5. RESULTS AND DISCUSSION

We test the performance of proposed method in comparison to NWCM and LCM by considering five different-size examples. Table-3 shows that the result obtained using proposed method in examples1, 2, 4 and 5 is same as the optimal result, while in example3 it is close to the optimal solution. It can be clearly seen that the proposed method gives more effective results in contrast to the existing methods – NWCM and LCM.

No: of example	Type of problem	Result of NWCM	Result of LCM	Result of proposed Method	Optimal Solution
Example-1	3*5	363	278	273	273
Example-2	3*3	228	163	143	143
Example-3	4*4	540	435	415	410
Example-4	3*4	4400	2900	2850	2850
Example-5	3*3	730	555	555	555

Table 3

6. CONCLUSION

In this paper, we have developed an improved algorithm for obtaining the initial basic feasible solution of transportation problems. The proposed algorithm is also tested for optimality. A comparison of proposed algorithm is made with Least Cost Method and North West Corner Method by considering 5 numerical examples. It is observed that the proposed algorithm yields more reliable results in contrast to the conventional methods.

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