EFFECT OF MAGNETIC FIELD AND SLIP VELOCITY ON THIRD GRADE BLOOD FLOW AND HEAT TRANSFER THROUGH A STENOSED ARTERY.

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ABSTRACT

In this study, we considered the effect of magnetic field and slip velocity on blood flow and heat transfer through a stenosed artery using a third grade fluid model. The solution techniques employed are based on Galerkin weighted residual and Newton Raphson methods. Analytical expression for the flow velocity, temperature profile, volume flow rate, wall shear stress and resistance to flow were obtained and the results are presented graphically. The numerical simulation carried out reviewed that higher value of slip velocity significantly increased the flow velocity, flow rate, and wall shear stress but reduced the flow resistance and heat transfer rate while flow velocity, flow rate and shear stress gradually decreased with increased value of the magnetic field parameter but increased the flow resistance and heat transfer rate. Other parameter that enhance the flow velocity are the pressure gradient, shear thinning and Reynold number while that of heat transfer rate are the shear thinning, third grade parameter and Eckert number. Finally, it is reviewed from the results that the effect of slip velocity is more noticeable compare to that of magnetic field effect.

Keywords: Stenosed artery, Slip Velocity, Magnetic Field, Pressure Gradient, Eckert number, Reynold number, Shear thinning.

Nomenclatures

w - Fluid velocity

$\overline{w}$ - Dimensionless fluid velocity

t - Time component

$\bar{t}$ - Dimensionless time component

r - Radial distance

y - Dimensionless radial distance

z - Axial distance

w_s - Slip velocity

V_0 - Dimensionless Slip velocity

T - Temperature profile

$\bar{\theta}$ - Dimensionless temperature profile

T_w - Pipe temperature

T_m - Fluid temperature

R_0 - Radius of the normal artery

R(z) - Radius of the artery in a stenotic region

β_0 - Magnetic Field Strength

ψ - Resistance to flow

K - Thermal conductivity

Q - Volumetric flow rate

τ_s - Wall Shear Stress

Ƹ - Maximum height of the stenosis

L - Length of the stenosis

RE - Reynold number

M - Magnetic field parameter

G - Pressure gradient

Ω - Shear thinning

E_n - Eckert number

Λ - Third grade parameter

INTRODUCITON

1.1 Blod Flow

Blood flow through normal as well as stenosed artery has been given serious concerned by many researchers (1-5) in recent time because of its importance in human anatomy and physiology. Blood pumped into the arteries are transported into the capillary tube where the exchange of gases as well as nutrient take place and then transported back through the vein into the heart. That is, the heart moves blood efficiently through the branching networks of arteries, capillaries and veins and the lungs cycle here quite effectively through the branching pulmonary passage thereby keeping the cell of our bodies alive and functioning. The whole of this process is called circulating system of blood which help in providing the essential nourishment needed for supporting life Derrickson and Tortora [6].

In order to understand the effect of stenosis in the lumen of an artery many researchers [7,8,9] investigated the flow of blood through stenosed arteries treating blood as Newtonian fluid. However, experimental studies review that in a stenotic region, the shear rate of blood is low and therefore the Non-Newtonian flow behavior of blood is quite prominent. Sapna [10] considered blood as power law fluid model and studied the effect of Non-Newtonian behavior of blood flow. Amit and Shrivastav [11] modeled blood as Hershel-Bulkley fluid to represent the Non-Newtonian character of blood in small blood vessel. The effect of Non-Newtonian nature of blood in small blood vessels has been taken into account by Rekha and Usha [12]. They modeled blood as a Casson fluid.

Some of the other researchers that represent the Non-Newtonian character of blood are Alshare et.al [13], Hatami, *et al* [14], Haleh *et al* [15], Akbari *et al* [16], Hatami *et al* [17].

1.2 Heat Transfer

Heat transfer is energy in transit and the energy transfer by the heat flow cannot be measured directly but it concept has physical meaning because of the fact that it is related to measurable quantity called temperature. In a system where there is temperature difference heat transfer from a region of higher temperature to low temperature. When fluid flow over a solid body or inside pipe, temperature of the fluid and the solid surface take place as a result of the motion of the fluid relative the surface. The heat transfers between the living tissue and the blood network that passes through it depends on the properties of the blood and the surrounding tissue, the geometry of the blood vessel and the blood flow through it.

Heat transfers in the blood vessel have been studied by many researchers. Sharma *et al* [18] used finite difference method to obtained dynamic response of heat transfer in blood flow through artery under stenotic condition. Their study shows the effect of heating and cooling on the temperature distribution inside artery during hyperthermia and cryosurgery. The effect of heat transfer on the motion of blood in a diseased artery has been modelled under optically thin fluid assumption by Ogulu and Abbey [19]. Their study shows that, in addition to the constriction of the blood vessel and the effect of a magnetic field, the heat transfer also affects the blood flow in Cardiovascular system. Some other researcher that studies heat transfer in blood flow are Srinivas *et al* [20], Mekheimer *et al* [21], Kumar and Katiyar [22], Khanafer *et al* [23].

1.3 Magnetic Field Effect

A magnetic field can be viewed as the magnetic effect of electric current and magnetic materials. Since blood is considered as an electrically conducting fluid, subjecting it to a magnetic field would result in producing an electromagnetic force and an electric current flows. This is to say that, the erythrocyte (RBCs) in the blood is a highly specialized cell with small negative charge on it, so presence of magnetic field can influence the motion of the red blood cells and the whole blood as well. That is, if an externally magnetic field is applied to a moving and electrically conducting fluid, it will induce electric as well as magnetic fields. The interaction between the induce current and the applied magnetic field produces a body force per unit volume called Lorent's force which tend to retard the moving blood.

Many researchers have considered externally applied magnetic field in their studies. The idea of electromagnetic field in medical research was firstly given by Kolin [24] and later Korchevskii and Marochunik [25] who discussed the possibility of regulating the movement of blood in the human system by applying magnetic field. Thereafter Vardanyan [26] studied the effect of magnetic field on blood flow theoretically and his work was later corroborated by Sud, *et al* [27] by considering different model. It was observed by these authors that the effect of constant magnetic field slow down the speed of blood. In a more recent development, some of the other researchers that investigated blood flow through a stenosed arteries under the influence of externally applied magnetic field are Aiman and Bourham [3], Chitra and Karlth, Keijan [2], Akbari *et al* [16] Hatami et al [14], Alimohamadi and Imani [28].

1.4 Slip Velocity Effect

Slip velocity is an important factor in blood flow modelling, since it employment on the constricted region can increase the flow velocity on one hand and decrease the resistance on the other hand. It presence can also lead to increase the volume flow rate and shear stress.To this end, Devajyoti and Uday (29) investigated the pulsatile flow of blood as two-fluid model with the suspension of all the erythrocyte in the core peripheral of plasma as a Newtonian fluid. Their results show that increased in slip velocity increased the flow velocity and flow rate but decreased the effective viscosity. Theoretical investigation concerning the influence of externally imposed periodic body acceleration on the flow of blood through a time dependent stenosed arterial segment by taking into account the slip velocity at the wall of the artery has been investigated by Sinsh *et al* (30). Srikanth, *et al* (31) investigated blood flow through an overlapping clogged tapered artery in the presence of catheter. They considered slip velocity at the arterial wall since cholesterol deposition is resulting in the stenosis formation. They solved analytically equation governing the fluid under the assumption of mild stenosis. Their results were presented graphically and from the graphs, it was observed that the slip velocity and divergence tapered artery facilitate the fluid flow. Arun (32) and, Geeta and Siddique (33) Studies blood flow with the employment of velocity slip at the constricted artery in their studies.

In the present analysis we have considered the effect of magnetic field and slip velocity on blood flow and heat transferthrough constricted artery using a third grade fluid model.

2.0 Mathematical Models

The momentum equation describing the steady fluid flow and the energy equation describing the steady heat transfer as obtained by Mohammed [36] are respectively given as

$$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial \hat{P}}{\rho\partial z}-\frac{\sigma\beta_0^2 w}{\rho}=0 \quad (2.1)$$

$$\frac{\mu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2+\frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4+\frac{K}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=0 \quad (2.2)$$

From the LHS of equation (2.1), the first term can be written as

$$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \quad (2.3)$$

while that of equation (2.2) from the LHS, the third term can be written as

$$\frac{K}{\rho C_\rho}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=\frac{K}{r\rho C_\rho}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad (2.4)$$

When we substituted (2.3) into (2.1) and (2.4) into (2.2), we respectively obtained

$$\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial \hat{P}}{\rho\partial z}-\frac{\sigma\beta_0^2 w}{\rho}=0 \quad (2.5)$$

and

$$\frac{\nu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2+\frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4+\frac{K}{r\rho c_p}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0 \quad (2.6)$$

Since we employed slip velocity in the stenosed artery as shown in the figure below, the corresponding slip conditions to equations (2.5) and (2.6) are respectively given as

$$\left.\begin{array}{lll} w=w_s & \text{at} & r=R(z) \\ \frac{\partial w}{\partial r}=0 & \text{at} & r=0 \end{array}\right\} \quad (2.7)$$

and

$$\left.\begin{array}{lll} T=T_w & \text{at} & r=R(z) \\ \frac{\partial T}{\partial r}=0 & \text{at} & r=0 \end{array}\right\} \quad (2.8)$$

To non-dimensionalize equations (2.5), (2.6), (2.7) and (2.8), we introduce the following parameters and variables

$$\left.\begin{array}{ll} \overline{w}=\frac{w}{d/t_0}, & y=r/R_0 \\ \bar{t}=\frac{t}{t_0}, & V_0=\frac{w_s t_0}{d} \end{array}\right\} \quad \bar{\theta}=\frac{T-T_w}{T_m-T_w} \quad (2.9)$$

When equation (2.9) is substituted into (2.5) and (2.6), after simplifying we obtained respectively

$$\frac{1}{RE}\cdot\frac{\partial}{\partial y}\left(y\frac{\partial\overline{\omega}}{\partial y}\right)+\Omega\left(6\left(\frac{\partial\overline{\omega}}{\partial y}\right)^2\frac{\partial^2\overline{\omega}}{\partial y^2}+\frac{2}{y}\left(\frac{\partial\overline{\omega}}{\partial y}\right)^3\right)+G-M\overline{\omega}=0 \quad (2.10)$$

as the dimensionless momentum equation.

where

$$\left.\begin{array}{l} RE = \frac{R_0^2}{Vt_0}, G = -\frac{t_0^2}{d\rho}\frac{\partial \hat{p}}{\partial z}, \\ \Omega = \frac{\beta_3 d^2}{t_0 \rho R_0^4}, \quad M = \frac{t_0 \sigma \beta_0^2}{\rho} \end{array}\right\} \qquad (2.11)$$

with the corresponding dimensionless slip conditions simplified as

$$\left.\begin{array}{lll} \overline{w} = V_0 & \text{at} & y = \frac{R(z)}{R_0} = R_b \\ \frac{\partial \overline{w}}{\partial y} = 0 & \text{at} & y = 0 \end{array}\right\} \qquad (2.12)$$

and

$$E_n\left(\frac{\partial \overline{\omega}}{\partial y}\right)^2 + \phi\left(\frac{\partial \overline{\omega}}{\partial y}\right)^4 + \Lambda \frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial \overline{\theta}}{\partial y}\right) = 0 \qquad (2.13)$$

as the dimensionless energy equation.

where

$$\left.\begin{array}{l} E_n = \frac{Vd^2}{t_0(T_m - T_w)R_0^2 C_\rho}, \quad \Phi = \frac{2\beta_3 d^4}{t_0^3 (T_m - T_w) R_0^4 \rho C_\rho} \\ \text{and} \quad \Lambda = \frac{Kt_0}{R_0^2 \rho C_\rho} \end{array}\right\} \qquad (2.14)$$

with the corresponding dimensionless slip conditions simplified as

$$\left.\begin{array}{lll} \overline{\theta} = 0 & \text{at} & y = R_b \\ \frac{\partial \overline{\theta}}{\partial y} = 0 & \text{at} & y = 0 \end{array}\right\} \qquad (2.15)$$

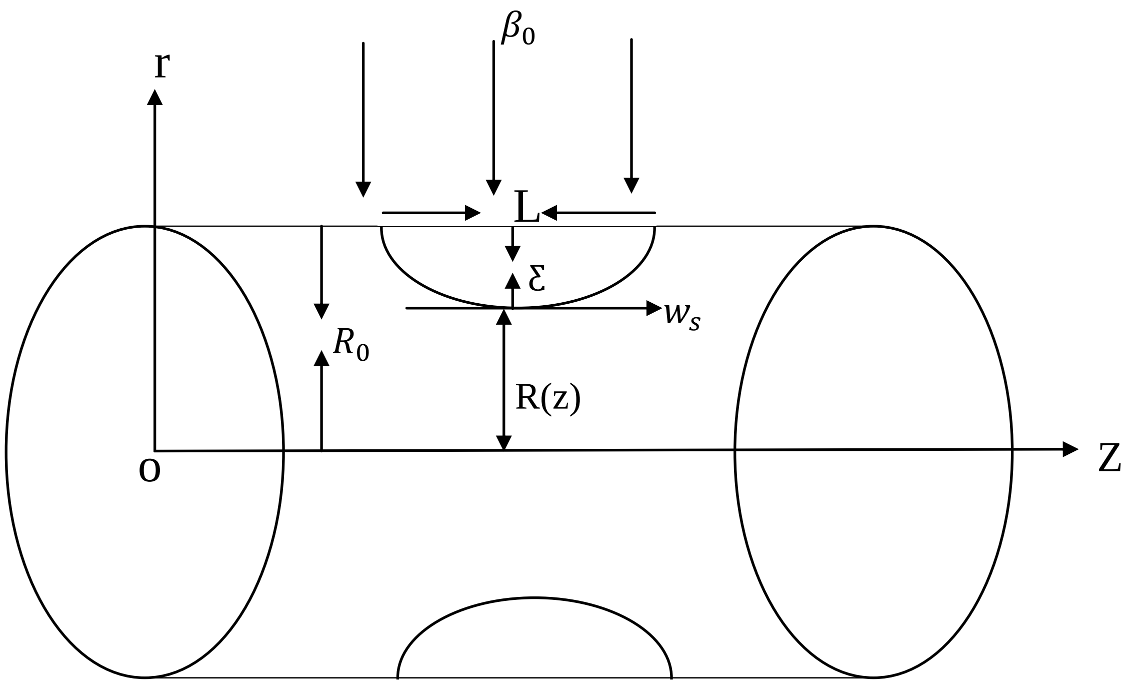

Figure1.Geometry of the stenosis

and has been described by Young [34] and Biswas [35]

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\varepsilon}{2R_0}\left[1 + \frac{cos\pi z}{L}\right], & \text{for} \quad |z| \leq L \\ R_0, & \text{for} \quad |z| > L \end{cases} \qquad (2.16)$$

3.0 Methods of Solution

We have used Galerkin weighted residual method for getting the required solutions of the problems. By this method , to obtain the velocity profile of the steady fluid flow, we assume a trial function of the form

$$\overline{w}(y) = a_0 + a_1 y + a_2 y^2 \qquad (3.1)$$

Subjecting (3.1) to the slip conditions (2.8) and after simplification yields

$$a_1 = 0 \text{ and } \quad a_2 = \frac{1}{Rb^2}(V_0 - a_0) \tag{3.2}$$

Putting (3.2) into (3.1) and simplified to obtain

$$\bar{w}(y) = \frac{V_0 y^2}{Rb^2} + a_0\left(1 - \frac{y^2}{Rb^2}\right) \tag{3.3}$$

Now,

$$\text{Let } \bar{r} = \frac{y}{Rb} \tag{3.4}$$

Putting (3.4) into (3.3) to obtain

$$w(\bar{r}) = V_0\bar{r}^2 + a_0(1 - \bar{r}^2) \tag{3.5}$$

For convenience sake, we drop the bar and write (3.5) as

$$w(r) = V_0 r^2 + a_0(1 - r^2) + a_2 r^2(1 - r^2) \tag{3.6}$$

From (3.6), we have the followings

$$\frac{\partial w}{\partial r} = 2V_0 r - 2a_0 r + 2a_2 r - 2a_2 r^3 - 2a_2 r^3 \tag{3.7}$$

$$\frac{\partial^2 w}{\partial r^2} = 2V_0 - 2a_0 + 2a_2 - 12a_2 r^2 \tag{3.8}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = 4V_0 + 4a_2 - 4a_0 - 16a_2 r^2 \tag{3.9}$$

$$\left(\frac{\partial w}{\partial r}\right)^2 = 16a_2^2 r^6 - 16V_0 a_2 r^4 + 16a_0 a_2 r^4 - 16a_2^2 r^4 + 4V_0^2 r^2 - 8v_0 a_0 r^2 + 8V_0 a_2 r^2 + 4a_0^2 r^2 - 8a_0 a_2 r^2 + 4a_2^2 r^2 \tag{3.10}$$

$$\frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3 =$$

$$-128a_2^3 r^8 + 192V_0 a_2^2 r^6 - 192a_0 a_2^2 r^6 + 192a_2^3 r^6 - 96V_0^2 a_2 r^4 + 192V_0 a_0 a_2 r^4 - 192V_0 a_2^2 r^4 - 96a_0^2 a_2 r^4 + 192a_0 a_2^2 r^4 - 96a_2^3 V^4 + 16V_0^3 r^2 - 48V_0^2 a_0 r^2 + 48V_0^2 a^2 r^4 +$$

$48V_0a_0^2r^4 - 96V_0a_0a_2r^2 + 4V_0a_2^2r^2 - 16a_0^3r^2 + 48a_0^2a_2r^2 - 48a_0a_2^2r^2 + 16a_2^3r^2$

(3.11)

$6\left(\frac{\partial^2 w}{\partial r^2}\right)\left(\frac{\partial w}{\partial r}\right)^2 = 48V_0^2r^2 - 144V_0^2a_0r^2 + 144V_0^2a_2r^2 - 480V_0^2a_2r_2 + 144V_0a_0^2r^2 -$

$40V_0a_0a_2r^2 + 960V_0a_0a_2r^4 + 144V_0a_2^2r^2 - 144V_0a_2^2r^4 + 1344V_0a_2^2r^6 + 144a_0^2a_2r^2 -$

$480a_0^2a_2r^4 - 144a_0a_2^2r^2 + 960a_0a_2^2r^4 - 1344a_0a_2^2r^6 - 480a_2^3r^4 + 1344a_2^3r^6 -$

$48a_2^3r^2 + 1152a_2^3r^8$ (3.12)

The residue for equation (2.6) can be written as

$$R_1(a_0, a_2, r) = G + \frac{1}{RE}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \Omega\left(6\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - Mw \qquad (3.13)$$

Using the transformation (3.4) and substituting (3.6), (3.9), (3.11), and (3.12) into (3.13) to obtain

$R_1(a_0, a_2.r) = G + \frac{1}{RE}(4V_0 + 4a_2 - 4a_0 - 16a_2r^2) + \Omega(48V_0^2r^2 - 144V_0^2a_0r^2 +$

$144V_0^2a_2r^2 - 480V_0^2a_2r^4 + 144V_0a_0^2r^2 - 40V_0a_0a_2r^2 + 960V_0a_0a_2r^2 + 144V_0a_2^2r^2 -$

$144V_0a_2^2r^4 + 1344V_0a_2^2r^6 + 144a_0^2a_2r^2 - 480a_0^2a_2r^4 - 144a_0a_2^2r^2 + 960a_0a_2^2r^4 -$

$1344a_0a_2^2r^6 - 480a_2^3r^4 + 1344a_2^3r^6 - 48a_0^3r^2 + 48a_2^3r^2 - 1152a_2^3r^8 - 128a_2^3r^8 +$

$192V_0a_2^2r^6 - 192a_0a_2^2r^6 + 192a_2^3r^6 - 96V_0^2a_2r^4 + 192V_0a_0a_2r^4 - 192V_0a_2^2r^4 -$

$96a_0^2a_2r^4 + 192a_0a_2^2r^4 - 96a_2^3r^4 + 16V_0^3r^2 - 48V_0^2a_0r^2 + 48V_0^2a_2r^2 + 48V_0a_0^2r^2 -$

$96V_0a_0a_2r^2 + 48V_0a_2^2r^2 - 16a_0^3r^2 + 48a_0^2a_2r^2 - 48a_0a_2^2r^2 + 16a_2^3r^2) - M(V_0r^2 + a_0 -$

$a_0r^2 + a_2r^2 - a_2r^4)$ (3.14)

By differentiating (3.6) with respect to a_0 *and* a_2, we obtain the weight functions respectively as

$$w_1(r) = (1 - r^2) \qquad (3.15)$$

and

$$w_2(r) = r^2(1 - r^2) \qquad (3.16)$$

Now taking into account the orthogonality of the residue $R_1(a_0, a_2, r)$with respect to the weight functions $w_1(r)$ and $w_2(r)$, we get the following systems;

$$\int_0^1 w_1(r)\, R_1(a_0, a_2, r)dr = o \qquad (3.17)$$

$$\int_0^1 w_2(r) R_1(a_0, a_2, r)dr = 0 \qquad (3.18)$$

When equations (3.14) and (3.15) are substituted into (3.17), we integrate and simplified to obtain

$$\frac{2G}{3} + \frac{4}{RE}\left(\frac{2V_0}{3} - \frac{2a_0}{3} - \frac{16a_2}{5}\right) + 16\Omega\left[\frac{2V_0^2}{5} - \frac{8V_0^2 a_0}{5} + \frac{16V_0^2 a_2}{5} + \frac{8V_0 a_0^2}{5} + \frac{362V_0 a_2^2}{105} - \frac{646V_0 a_0 a_2}{105} - \frac{118a_0^2 a_2}{105} - \frac{532a_2^2 a_0}{105} + \frac{20480a_2^3}{3465} - \frac{8a_0^3}{15}\right] + M\left(\frac{-2V_0}{15} - \frac{8a_0}{15} - \frac{8a_2}{105}\right) = 0 \qquad (3.19)$$

Equation (3.19) can be re-written as

$$327520\Omega a_2^3 - 29568\Omega a_0^3 + 191136\Omega V_0 a_2^2 + 88704\Omega V_0 a_0^2 - 280896\Omega a_2^2 a_0 - 62304\Omega a_0^2 a_2 - 341088\Omega V_0 a_0 a_2 - \left(1848M + 88704\Omega V_0^2 + \frac{9240}{RE}\right) a_0 + \left(-264M + 25344\Omega V_0^2 - \frac{44352}{RE}\right) a_2 = -2310G - \frac{9240V_0}{RE} - 22176\Omega V_0^2 + 462MV_0 \qquad (3.20)$$

Similarly,when equations (3.14) and (3.16) are substituted into (3.18), we integrate and simplified to obtain

$$\frac{2G}{15} + \frac{4}{RE}\left(\frac{2V_0}{15} - \frac{2a_0}{15} + \frac{32a_2}{105}\right) + 16\Omega\left(\frac{6V_0^2}{35} - \frac{24V_0^2 a_0}{35} - \frac{16V_0^2 a_2}{35} + \frac{24V_0 a_0^2}{35} + \frac{194V_0 a_2^2}{105} + \frac{58V_0 a_0 a_2}{35} - \frac{16a_0^2 a_2}{35} - \frac{56a_2^2 a_0}{165} - \frac{1408a_2^3}{15015} - \frac{8a_0^3}{35}\right) + M\left(-\frac{2V_0}{35} - \frac{8a_0}{105} - \frac{8a_0}{315}\right) = 0 \qquad (3.21)$$

Equation (3.21) can be re-written as

$-164736\Omega a_0^3 - 6720\Omega a_2^3 + 4944208\Omega V_0 a_0^2 + 1331616\Omega V_0 a_2^2 - 244608\Omega a_0 a_2^2 -$

$329472\Omega a_2 a_0^2 + 1194336\Omega V_0 a_0 a_2 - \left(3432M + 494208\Omega V_0^2 + \frac{24024}{RE}\right) a_0 - \left(1144M +\right.$

$\left.329472\Omega V_0^2 - \frac{54912 a_2}{RE}\right) a_2$

$= -6006G - \frac{24024 V_0}{RE} - 123552\Omega V_0^2 + 2574 M V_0$ (3.22)

By substituting the appropriate values of the parameters $\Omega, V_{0,}\ M, G, and\ RE$ into (3.20) and (3.22), after simplificationwe obtain respectively

$-85.13333333a_0^3 - 14.77633478a_2^3 + 64.00000000a_0^2 + 21.3333333a_2^2 -$

$73.14285714a_0^2 a_2 - 85.13333333a_2^2 a_0 + 36.57142858a_0 a_2 - 19.14962963a_0 -$

$4.005502645a_2 = -3.062407407$ (3.23)

and

$-36.57142857a_0^3 - 15.00366300a_2^3 + 27.42857142a_0^2 + 13.57575758a_2^2 -$

$73.14285714a_0^2 a_2 - 54.30303030a_2^2 a_0 + 36.57142858a_0 a_2 - 7.476402115a_0 -$

$5.003597883a_2 = -0.9145767196$ (3.24)

Solving the system of non-linear equations (3.23) and (3.24) using Newton Raphson's method, we obtained the values of a_0 and $a_{2\,,}$and when substituted into (3.6) and simplified, we obtained the velocity profile as

$\mathrm{w(r)} = 0.5754503946 - 0.3754503946r^2 - 0.004124970522r^2(1 - r^2)$ (3.25)

By simulating the appropriate values of the parameters in (3.20) and (3.22) and follow the procedures above, we obtain the corresponding values of a_0, a_2 and velocity profile w(r). The results are shown in the table1 below.

Also, to obtained the temperature profile of the heat transfer using Gerlakin's weighted residual method, we assume a trial function of the form

$$\overline{\theta}(y) = C_0 + C_1 y + C_2 y^2 \quad (3.26)$$

Subjecting (3.26) to the slip conditions (2.11) and after simplification we obtain

$$C_1 = 0 \text{ and } \quad C_2 = -\frac{C_0}{R_b^2} \quad (3.27)$$

Putting (3.27) into (3.26) and simplified to obtain

$$\overline{\theta}(y) = C_0\left(1 - \frac{y^2}{R_b^2}\right) \quad (3.28)$$

Hence, the trial function can be written in the form

$$\overline{\theta}(y) = a_3\left(1 - \frac{y^2}{R_b^2}\right) + a_4\frac{y^2}{R_b^2}\left(1 - \frac{y^2}{R_b^2}\right) \quad (3.29)$$

By using the transformation (3.4) and dropping bar, equation (3.29) can be written as

$$\theta(r) = a_3(1 - r^2) + a_4 r^2(1 - r^2) \quad (3.30)$$

From (3.6) and (3.30) we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = -4a_3 + 4a_4 - 16a_4 r^2 \quad (3.31)$$

$$\left(\frac{\partial w}{\partial r}\right)^4 = 64v_0^2 a_2 r^2 - 64v_0^3 a_0 r^4 - 64v_0^3 a_2 r^4 - 128v_0^3 a_2 r^6 + 96v_0^2 a_2^2 r^4 + 96v_0^2 a_0^2 r^4 - 384v_0^2 a_2^2 r^6 + 384v_0^2 a_2^2 r^8 - 512v_0 a_2^3 r^{10} - 384v_0 a_2^3 r^6 - 64v_0 a_0^3 r^4 + 768v_0 a_2^3 r^8 + 64v_0 a_2^3 r^4 - 64a_0^3 a_2 r^4 + 128a_0^3 a_2 r^6 + 96a_0^2 a_2^2 r^4 - 384a_0^2 a_2^2 r^6 + 384a_0^2 a_2^2 r^8 + 512a_0 a_2^3 r^{10} + 384a_0 a_2^3 r^6 - 768a_0 a_2^3 r^8 - 64a_0 a_2^3 r^4 + 16a_2^3 r^4 - 128a_2^3 r^6 + 16a_0^4 r^4 - 512a_2^4 r^{10} + 384a_2^4 r^8 + 16v_0^4 r^4 + 256a_2^4 r^{12} + 192v_0 a_0^2 a_2 r^4 - 384v_0 a_0^2 a_2 r^6 - 192v_0 a_0 a_2^2 r^4 + 768v_0 a_0 a_2^2 r^6 - 768v_0 a_0 a_2^2 r^8 - 192v_0^2 a_0 a_2 r^4 + 384v_0^2 a_0 a_2 r^6 \quad (3.32)$$

The residue for equation (2.9) is given as

$$R_3(r,a_3,a_4) = E_n\left(\frac{\partial w}{\partial r}\right)^2 + \phi\left(\frac{\partial w}{\partial r}\right)^4 + \Lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\overline{\theta}}{\partial r}\right) = 0 \quad (3.33)$$

Substituting (3.10), (3.31) and (3.32) into (3.33) to obtain

$$R_3(r,a_3,a_4) = E_n(4v_0^2r^2 - 8a_0v_0r^2 + 8v_0a_2r^2 - 16v_0a_2r^2 + 4a_0^2r^2 - 8a_0a_2r^2 + 16a_0a_2r^4 + 4a_2^2r^2 - 16a_2^2r^4 + 16a_2^2r^6) + \phi(64v_0^2a_2r^2 - 64v_0^3a_0r^4 - 64v_0^3a_2r^4 - 128v_0^3a_2r^6 + 96v_0^2a_2^2r^4 + 96v_0^2a_0^2r^4 - 384v_0^2a_2^2r^6 + 384v_0^2a_2^2r^8 - 512v_0a_2^3r^{10} - 384v_0a_2^3r^6 - 64v_0a_0^3r^4 + 768v_0a_2^3r^8 + 64v_0a_2^3r^4 - 64a_0^3a_2r^4 + 128a_0^3a_2r^6 + 96a_0^2a_2^2r^4 - 384a_0^2a_2^2r^6 + 384a_0^2a_2^2r^8 + 512a_0a_2^3r^{10} + 384a_0a_2^3r^6 - 768a_0a_2^3r^8 - 64a_0a_2^3r^4 + 16a_2^3r^4 - 128a_2^3r^6 + 16a_0^4r^4 - 512a_2^4r^{10} + 384a_2^4r^8 + 16v_0^4r^4 + 256a_2^4r^{12} + 192v_0a_0^2a_2r^4 - 384v_0a_0^2a_2r^6 - 192v_0a_0a_2^2r^4 + 768v_0a_0a_2^2r^6 - 768v_0a_0a_2^2r^8 - 192v_0^2a_0a_2r^4) - \Lambda(4a_3 - 4a_4 + 16a_4r^2) \quad (3.34)$$

By taking the derivative of (3.30) with respect to $a_3$and a_4, we obtained the weight functions as obtained in (3.15) and (3.16) respectively.

The following systemsare obtainedby taking into account the orthogonality of the residue $R_3(r,a_3,a_4)$ with respect to the weight functions given in (3.15) and (3.16)

$$\int_0^1 R_3(r,a_3,a_4)\,w_1(r)dr = 0 \quad (3.35)$$

$$\int_0^1 R_3(r,a_3,a_4)\,w_2(r)dr = 0 \quad (3.36)$$

Substituting (3.15) and (3.34) into (3.35), we integrate and simplified to obtained

$$4E_n\left(\frac{2v_0^2}{15}-\frac{4a_0v_0}{15}+\frac{4v_0a_2}{105}+\frac{2a_0^2}{15}-\frac{4a_0a_2}{105}+\frac{24a_2^2}{135}\right)+16\phi\left(\frac{16v_0^3a_2}{315}-\frac{8v_0^3a_0}{35}+\frac{228v_0^2a_2^2}{3465}+\frac{12v_0^2a_0^2}{35}-\frac{504v_0a_2^3}{45045}-\frac{8v_0a_2^3}{35}-\frac{8a_0^3a_2}{315}+\frac{504a_2^3a_0}{45045}+\frac{228a_0^2a_2^2}{3465}+\frac{2a_0^4}{35}+\frac{3690a_2^4}{675675}+\frac{8v_0a_0^2a_2}{105}-\frac{152v_0a_0a_2^2}{1155}-\frac{24v_0^2a_0a_2}{35}+\frac{192v_0^4}{35}+\frac{2a_0^4}{35}\right)+4\Lambda\left(-\frac{11a_3}{3}-\frac{4a_4}{5}\right)=0 \quad (3.37)$$

Equation (3.37) can be re-written as

$$617760\phi a_0^4+49440\phi a_2^4-2471040\phi v_0a_2^3-120960\phi a_2^3a_0+274560\phi a_0^3a_2+120960\phi a_0a_2^3-823680\phi v_0a_0^2a_2-1422726\phi v_0a_0a_2^2+711360\phi a_0^2a_2^2+(360360E_nv_0+2471040\phi v_0^3)a_0^2+(102960E_nv_0+2196480\phi v_0^3)a_2^2-(720720E_nv_0+2471040\phi v_0^3)a_0+(102960E_nv_0+2196480\phi v_0^3)a_2-(102960E_n+7413120\phi v_0^2)a_0a_2-9909900\Lambda a_3-2162160\Lambda a_4$$

$$=-360360E_nv_0^2-3706560\phi v_0^4 \quad (3.38)$$

Similarly, by Substituting (3.16) and (3.34) into (3.36), we integrate and simplified to obtained

$$4E_n\left(\frac{2v_0^2}{35}-\frac{4a_0v_0}{35}-\frac{4v_0a_2}{315}+\frac{2a_0^2}{35}-\frac{4a_0a_2}{315}+\frac{52a_2^2}{385}\right)+16\phi\left(\frac{64v_0^3a_2}{693}-\frac{8v_0^3a_0}{63}+\frac{372v_0^2a_2^2}{9009}+\frac{12v_0^2a_0^2}{63}-\frac{1992v_0a_2^3}{135135}-\frac{8v_0a_0^3}{63}+\frac{8a_0^3a_2}{231}+\frac{1992a_0a_2^3}{135135}+\frac{124a_0^2a_2^2}{3003}+\frac{32v_0^4}{63}+\frac{2a_0^4}{63}+\frac{14178a_2^4}{15015}-\frac{8v_0a_2a_0^2}{77}-\frac{24v_0^2a_0a_2}{63}-\frac{248v_0a_0a_2^2}{3003}\right)-4\Lambda\left(\frac{-2a_3}{15}-\frac{26a_4}{15}\right)=0 \quad (3.39)$$

Equation (3.39) can be re-written as

$$68640\phi a_0^4+2041632\phi a_2^4-274560\phi v_0a_0^3-31872\phi v_0a_2^3+74880\phi a_0^3a_2+31872\phi a_0a_2^3-224640\phi v_0a_2a_0^2-178560\phi v_0a_0a_2^2+89280\phi a_0^2a_2^2+(30888E_n+411840\phi v_0^2)a_0^2+(73008E_n+89280\phi v_0^2)a_2^2-(61776E_nv_0+274560\phi v_0^3)a_0^2+(-6864E_nv_0+199680\phi v_0^3)a_2+(61776E_n-823680\phi v_0^2)a_0a_2-72072\Lambda a_3-669240\Lambda a_4=-30888E_nv_0^2-68640\phi v_0^4 \quad (3.40)$$

Solving (3.38) and (3.40) using Newton Raphson's method to obtain

$$a_3 = \frac{1}{204204\Lambda}(194480\phi v_0^4 - 777920v_0^3 a_0 - 141440\phi v_0^3 a_2 + 1166880\phi v_0^2 a_0^2 + 424320\phi v_0^2 a_0 a_2 + 236640\phi v_0^2 a_2^2 - 777920\phi v_0 a_0^3 - 424320\phi v_0 a_0^2 a_2 - 473280\phi v_0 a_0 a_2^2 - 60928\phi v_0 a_2^3 + 194480\phi a_0^4 + 141440\phi a_2^3 a_2 + 236640\phi a_0^2 a_2^2 + 60928\phi a_0 a_2^3 + 17008\phi a_2^4 + 51051E_n v_0^2 - 102102E_n v_0 a_0 + 4862E_n v_0 a_2 + 51051E_n a_0^2 - 4862E_n a_0 a_2 + 11271E_n a_2^2) \tag{3.41}$$

$$a_4 = \frac{1}{29172\Lambda}(38896\phi v_0^4 - 155584v_0^3 a_0 - 56576\phi v_0^3 a_2 + 233376\phi v_0^2 a_0^2 + 169728\phi v_0^2 a_0 a_2 + 53856\phi v_0^2 a_2^2 - 155584\phi v_0 a_0^3 - 169728\phi v_0 a_0^2 a_2 - 107712\phi v_0 a_0 a_2^2 - 23936\phi v_0 a_2^3 + 38896\phi a_0^4 + 56576\phi a_0^3 a_2 + 53856\phi a_0^2 a_2^2 + 23936\phi a_0 a_2^3 + 4144\phi a_2^4 + 7293E_n v_0^2 - 14586E_n v_0 a_0 - 4862E_n v_0 a_2 + 7293E_n a_0^2 + 4862E_n a_0 a_2 + 1105E_n a_2^2) \tag{3.42}$$

Substituting the appropriate values of the parameters ϕ, Λ, E_n, v_0 and the constants a_0 and a_2 into (3.41) and (3.42), we obtain the values of a_3 and a_4 , and when substituted into (3.30) and simplified, we obtained the temperature profile as

$$\theta(r) = 0.0083489100 - 0.0083129149r^2 + 0.0083129149r^2(1 - r^2) \tag{3.43}$$

By simulating the appropriate values of the parameters ϕ, Λ, E_n, v_0 and the constants a_0 and a_2 into (3.41) and (3.42), we obtain the corresponding values of a_3, a_4 and temperature profile $\theta(r)$. The results are shown in the table2 below.

Volume Flow Rate

The volume flow rate denoted by Q is given by

$$Q = 2\pi \int_0^{R(z)} rw(r)dr \tag{3.44}$$

Putting (4.7) into (3.44) and evaluate to obtain

$$Q = 12\left[3V_0\left(R(z)\right)^4 + a_0\left(6\left(R(z)\right)^2 - 3\left(R(z)\right)^4\right) + a_2\left(3\left(R(z)\right)^4 - 2\left(R(z)\right)^6\right)\right] \quad (3.45)$$

Shear Stress

The shear stress denoted by $\tau_S$is given as

$$\tau_s = \mu\frac{\partial w}{\partial r}\mid_{r=R(z)} + 2\beta_3\left(\frac{\partial w}{\partial r}\right)^3\mid_{r=R(z)} \quad (3.46)$$

Simplified (3.46) to obtain

$$\tau_s = 2\mu R(Z)\left(V_0 - a_0 + a_2 - 2R\left((Z)\right)^2 a_2\right) + 16R(Z)\beta_3(V_0 - a_0 + a_2 - 2(R(Z))^2 a_2) \quad (3.47)$$

Resistance to Flow

The resistance to flow can be denoted as ψ and is given by

$$\psi = \frac{-\frac{\partial \hat{P}}{\partial z}}{12\left[3V_0\left(R(z)\right)^4 + a_0\left(6\left(R(z)\right)^2 - 3\left(R(z)\right)^4\right) + a_2\left(3\left(R(z)\right)^4 - 2\left(R(z)\right)^6\right)\right]} \quad (3.48)$$

Table 1: Values of the parameters used in the numerical results and the corresponding velocity profile.

Figures	G	V_0	RE	Ω	M	w(r)
	1.5	0.25	0.9	10	0.35	$0.5746 - 0.3755r^2 - 0.0041r^2(1-r^2)$
2	2.0	0.25	0.9	10	0.35	$0.7056 - 0.5056r^2 - 0.0473r^2(1-r^2)$
	2.5	0.25	0.9	10	0.35	$0.7489 - 0.5489r^2 - 0.0670r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3188 - 0.1188r^2 - 0.0169r^2(1-r^2)$
3	1.5	0.25	0.9	10	0.65	$0.2850 - 0.0850r^2 - 0.0009r^2(1-r^2)$
	1.5	0.25	0.9	10	0.95	$0.2517 - 0.0517r^2 - 0.0058r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.5769 - 0.3269r^2 - 0.1192r^2(1-r^2)$
4	1.5	0.35	0.9	10	0.35	$0.6744 - 0.3244r^2 - 0.1179r^2(1-r^2)$
	1.5	0.45	0.9	10	0.35	$0.7718 - 0.3218r^2 - 0.1167r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3385 - 0.1385r^2 - 0.0483r^2(1- r^2)$
5	1.5	0.25	0.9	20	0.35	$0.3539 - 0.1539r^2 - 0.0526r^2(1-r^2)$
	1.5	0.25	0.9	30	0.35	$0.3823 - 0.1823r^2 - 0.0558r^2(1-r^2)$
	1.5	0.25	0.3	10	0.35	$0.2669 - 0.0669r^2 - 0.0028r^2(1-r^2)$
6	1.5	0.25	0.6	10	0.35	$0.3118 - 0.1118r^2 - 0.0179r^2(1-r^2)$
	1.5	0.25	0.9	10	0.35	$0.3338 - 0.1375r^2 - 0.0333r^2(1-r^2)$

Table 2: Values of the parameters used in the numerical results and the corresponding temperature profile.

Figs	Φ	E_n	Λ	V_0	$\theta(r)$
	1.25	1.5	1.35	0.25	$-0.0043r^2 + 0.0043 + 0.0038r^2(1- r^2)$
7	1.50	1.5	1.35	0.25	$-0.0055r^2 + 0.0055 + 0.0052r^2(1- r^2)$
	1.75	1.5	1.35	0.25	$-0.0083r^2 + 0.0083 + 0.0083r^2(1- r^2)$
	1.25	1.5	1.35	0.25	$-0.0126r^2 + 0.0126 + 0.0132r^2(1- r^2)$
8	1.25	1.8	1.35	0.25	$-0.0136r^2 + 0.0136 + 0.0141r^2(1- r^2)$
	1.25	2.1	1.35	0.25	$-0.0146r^2 + 0.0146 + 0.0149r^2(1- r^2)$
	1.25	1.5	1.35	0.25	$-1.8958r^2 + 1.8958+ 2.3970r^2(1- r^2)$
9	1.25	1.5	1.65	0.25	$-1.5511r^2 + 1.5511+ 1.9562r^2(1- r^2)$
	1.25	1.5	1.95	0.25	$-1.3834r^2 + 1.3834 + 1.7448r^2(1- r^2)$
	1.25	1.5	1.35	0.25	$-0.0027r^2 + 0.0027+ 0.0024r^2(1- r^2)$
10	1.25	1.5	1.35	0.35	$-0.0017r^2 + 0.0017 + 0.0015r^2(1- r^2)$
	1.25	1.5	1.35	0.45	$-0.0008r^2 + 0.0008 + 0.0008r^2(1- r^2)$

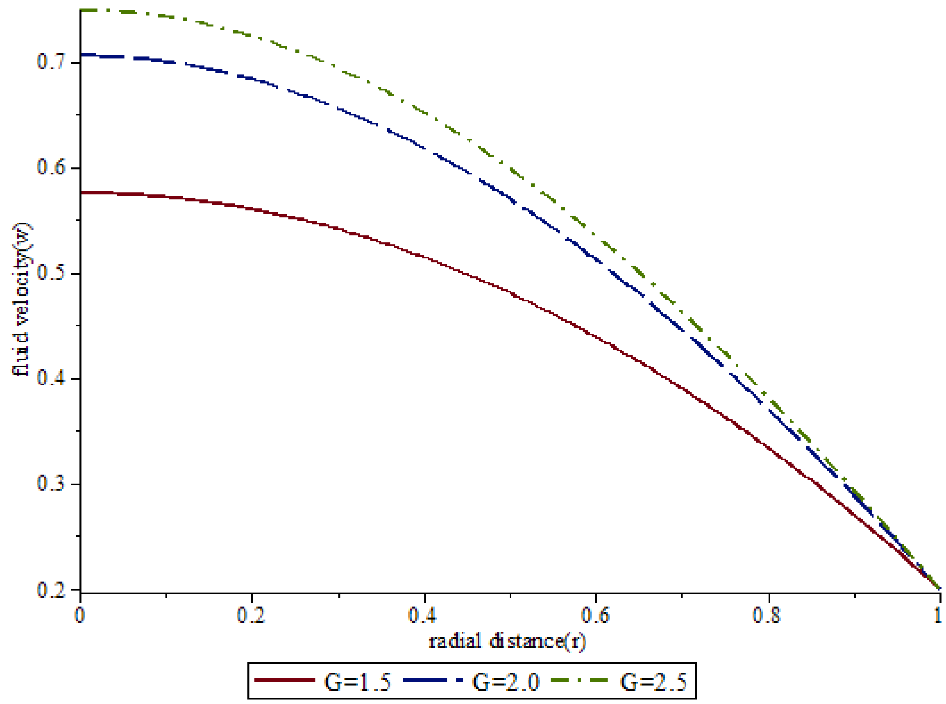

Figure2: Variation of Velocity Profile of Blood Flow for various values of the Pressure Gradient in the radial direction.

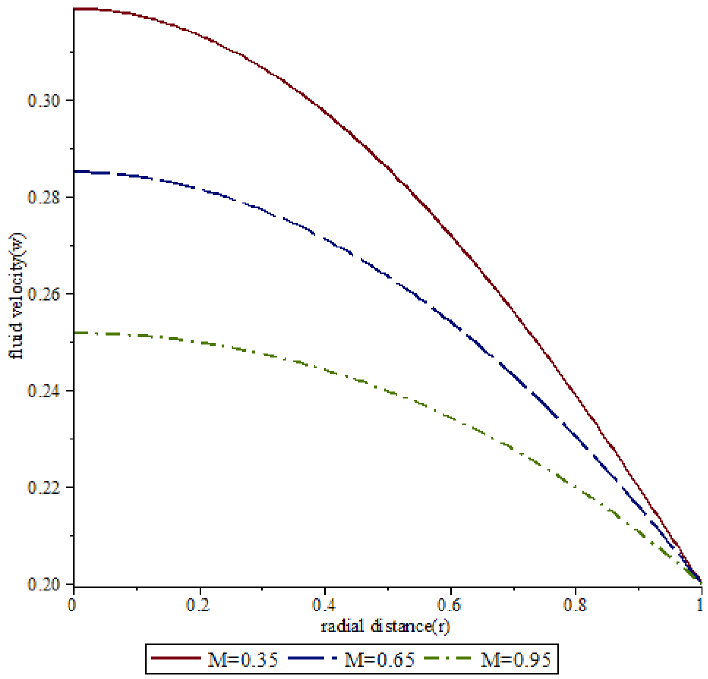

Figure3: Variation of Velocity Profile of Blood Flow for various values of Magnitude Field Parameter in the radial direction.

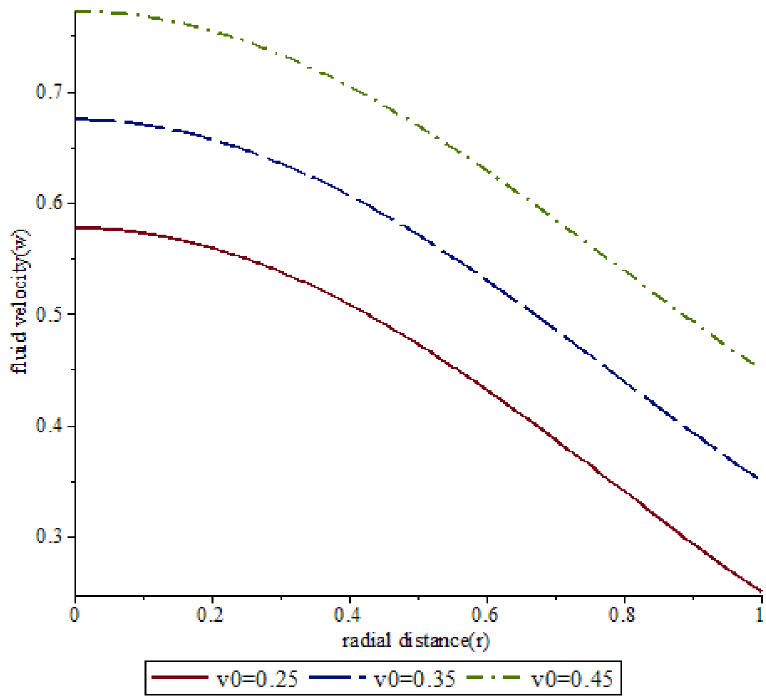

Figure4: Variation of Velocity Profile of Blood Flow for various values of the Slip velocity in the radial direction.

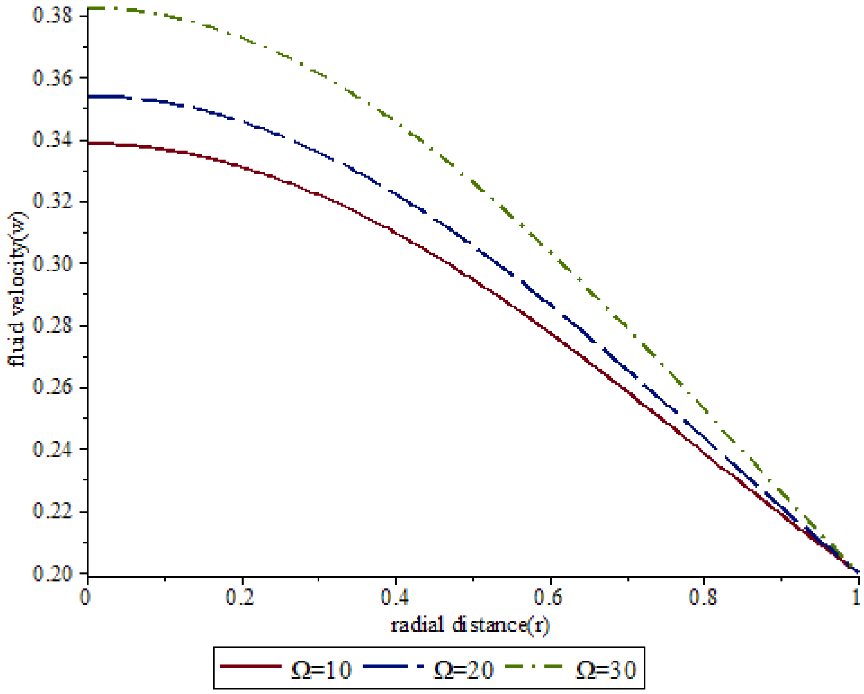

Figure5: Variation of Velocity Profile of Blood Flow for various values of the Shear Thinning in the radial direction.

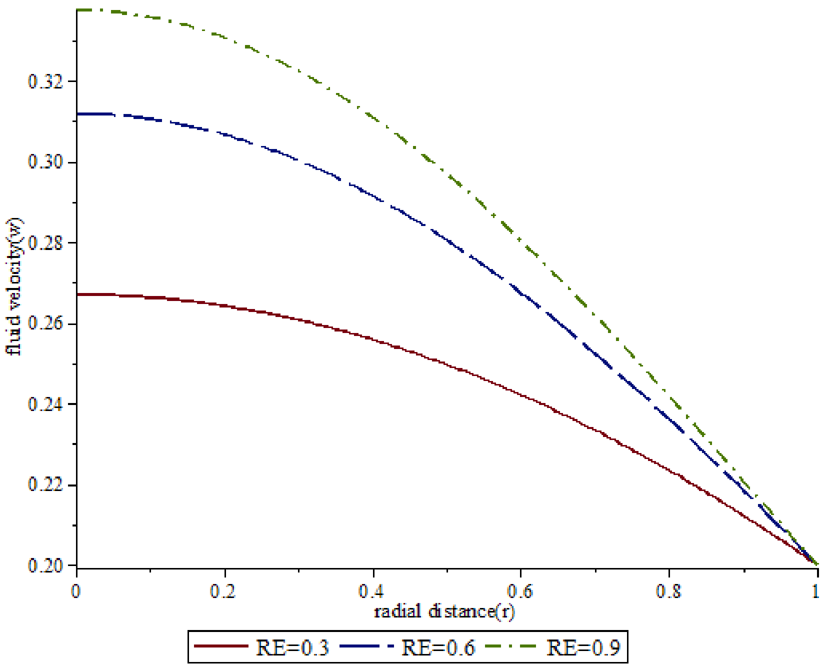

Figure6: Variation of Velocity Profile of Blood Flow for various values of the Reynold number in the radial direction.

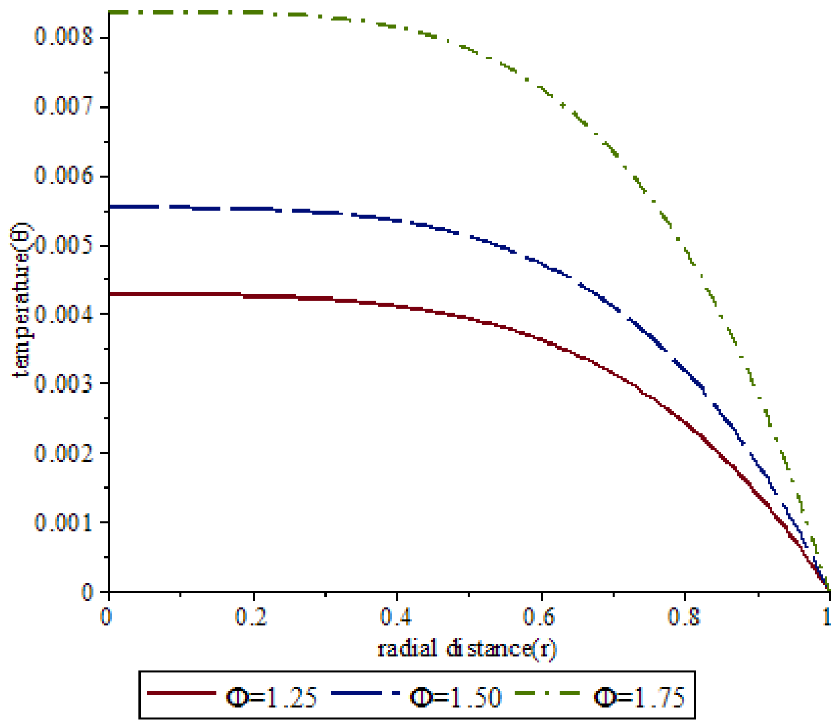

Figure7: Variation of Temperature Profile of Heat Transfer for various values of the Shear Thinning in the radial direction.

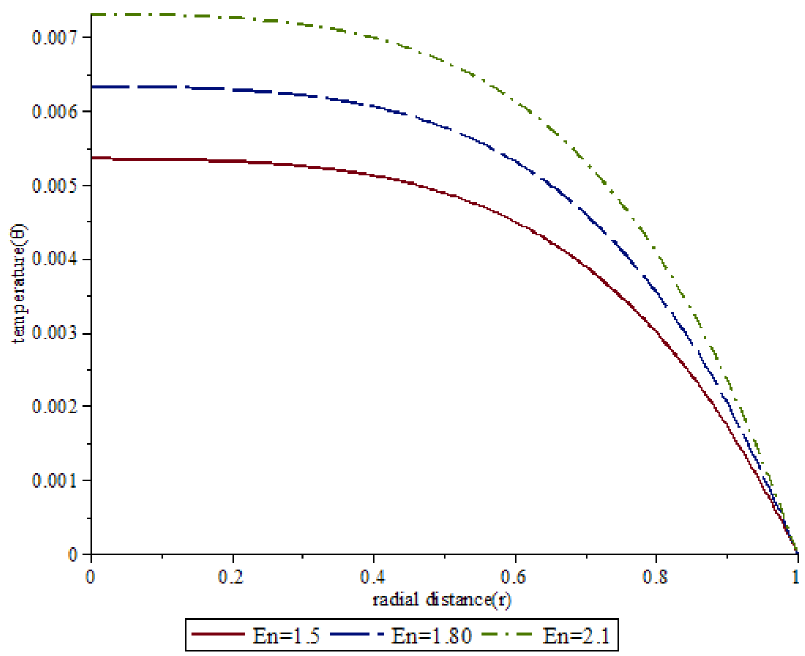

Figure8: Variation of Temperature Profile of Heat Transfer for various values of the Eckert number in the radial direction.

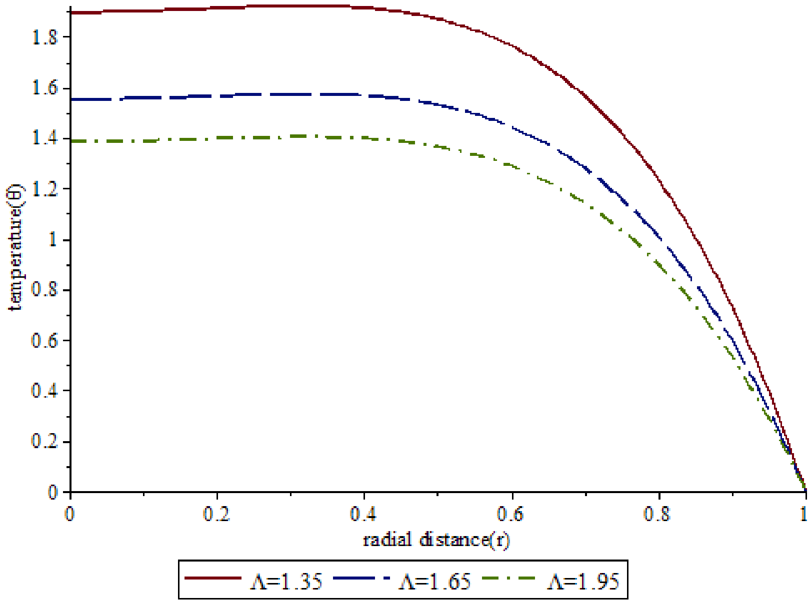

Figure9: Variation of Temperature Profile of Heat Transfer for various values of the Third Grade Parameter in the radial direction.

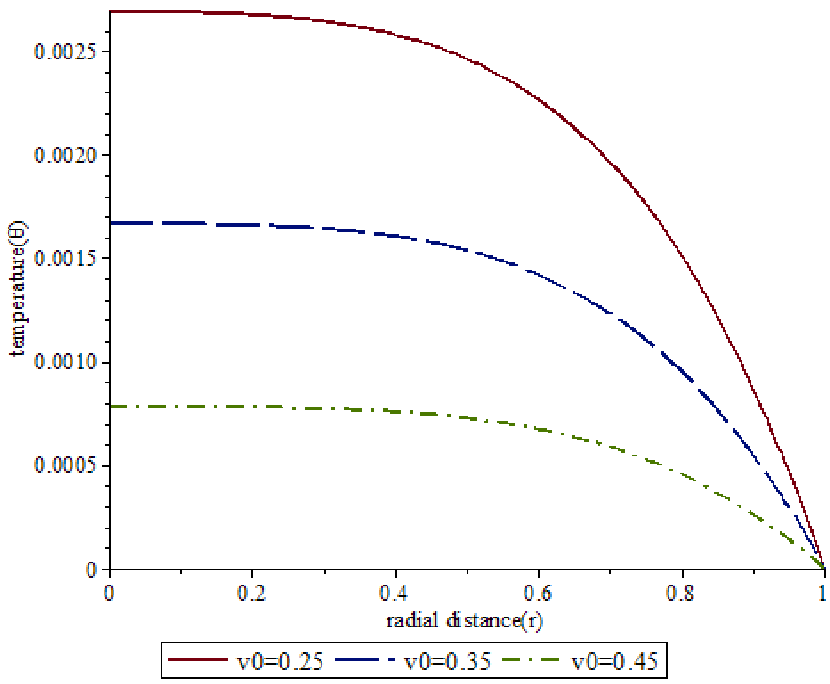

Figure 10: Variation of Temperature Profile of Heat Transfer for various values of the Slip Velocity in the radial direction.

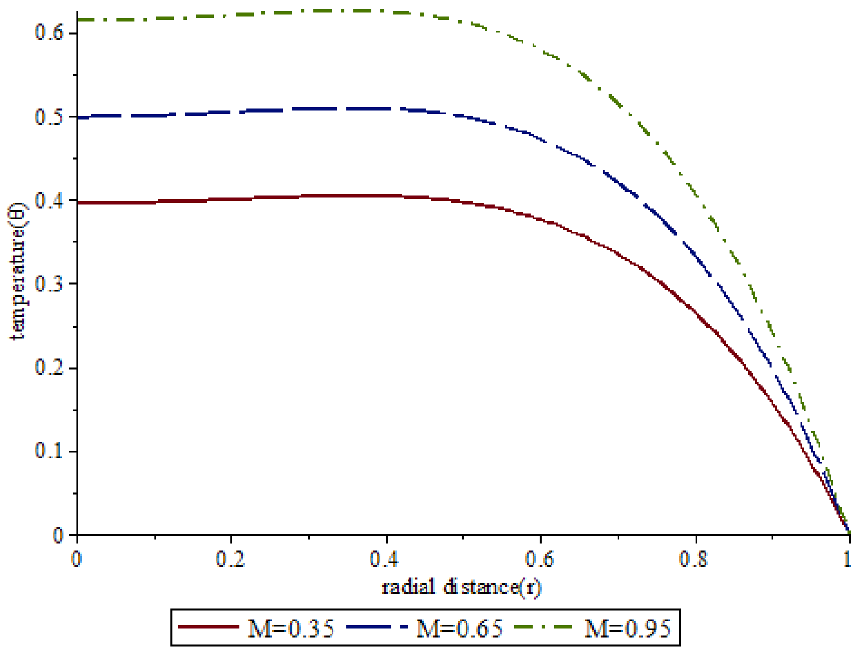

Figure 11: Variation of Temperature Profile of Heat Transfer for various values of the Magnetic Field Parameter in the radial direction.

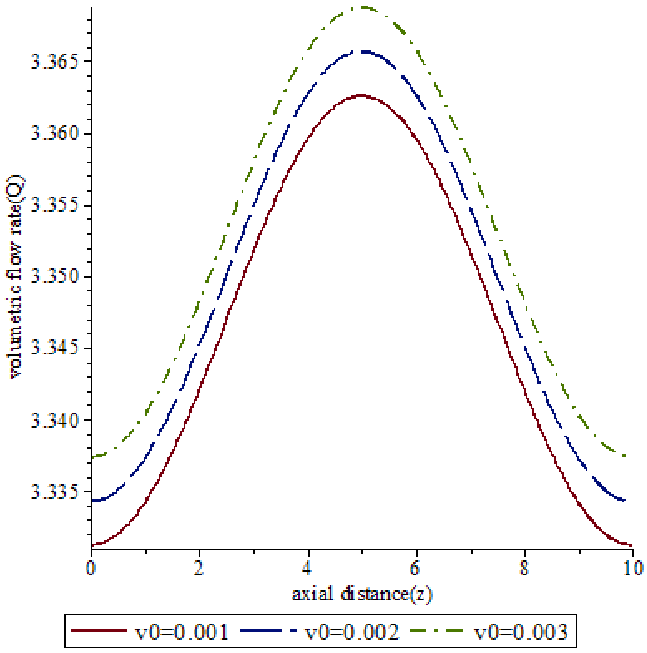

Figure 12: Variation of Volumetric Flow Rate of Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction.

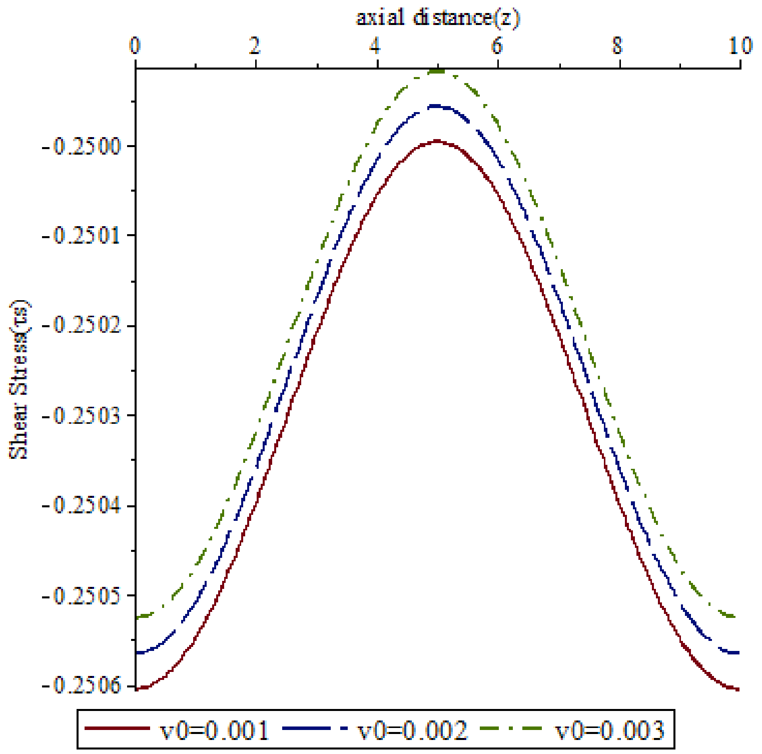

Figure 13: Variation of Shear Stress of Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction.

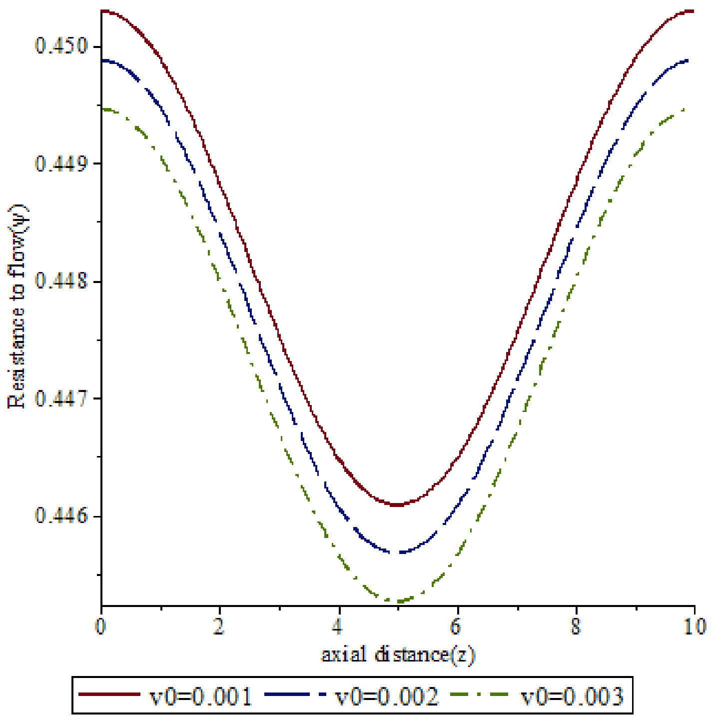

Figure 14: Variation of Resistance to Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction.

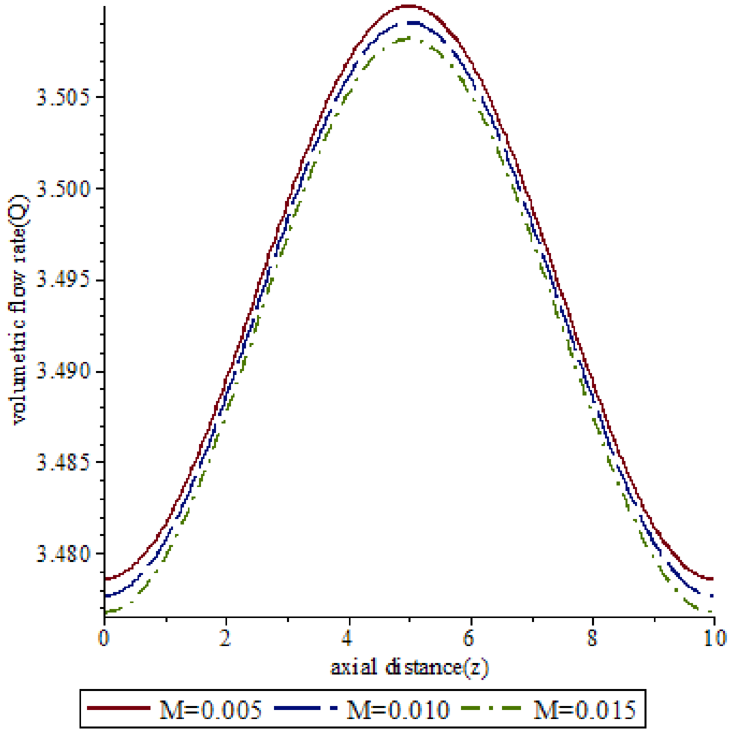

Figure 15: Variation of Volumetric Flow Rate of Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

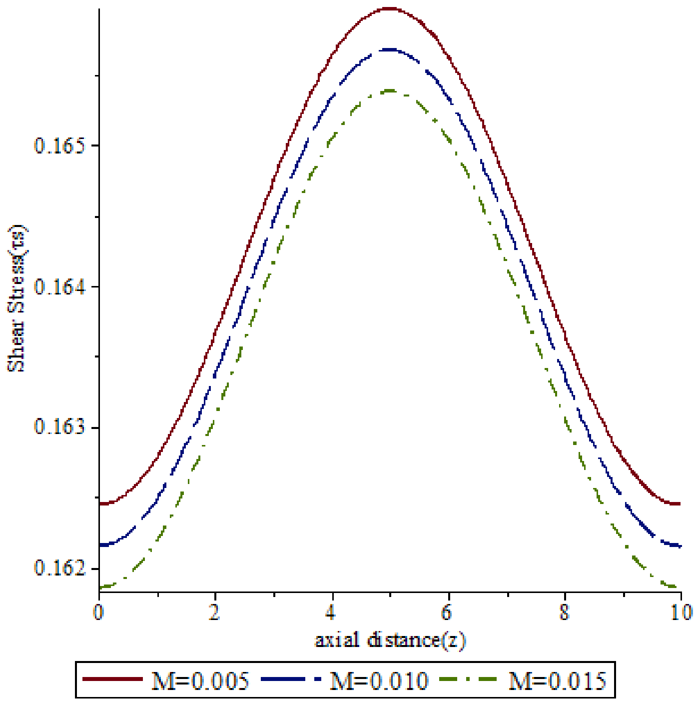

Figure 16: Variation of Shear Stress of Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

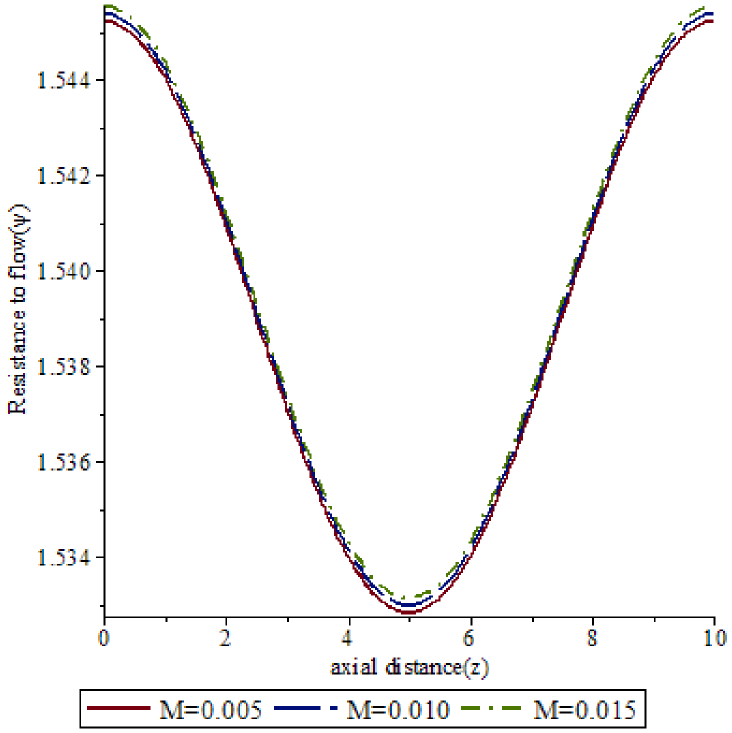

Figure 17: Variation of Resistance to Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

4.0 Discussion of Results

The expression for the various flow parameters and heat transfer rate has been found by solving the momentum and the energy equations using Galerkin weighted residual technique. Computer software package has been used as a tool for getting solutions of flow velocity, temperature profile, volumetric flow rate, resistance to flow and wall shear stress for steady flow of blood and heat transfer through stenosed artery.

Figures 2,3,4, 5 and 6 shows the variation of velocity along the radial distance for different values of the pressure gradient, magnetic field parameter, slip velocity, shear thinning, and Reynold number. we observe from figure 4 that velocity profile increases significantly with increases values of the slip velocity. This is because the slip velocity at the stenotic wall reduces the effect of induced magnetic field and viscosity and as such influencing the flow velocity positively. It can be seen from figure 3 that velocity profile decreases slightly with increases values of the magnetic field parameter. This happen because the Lorentz force which opposes the motion of the blood flow in the artery and as a result slow down the flow velocity. Also, it can be seen from figures 2,5 and 6 that the velocity profiles increase with increases values of the pressure gradient, shear thinning and Reynold number.

Similarly,figures 7, 8, 9, 10 and 11 shows the variation of the temperature profiles along the radial direction for different values of the shear thinning, Eckert number, third grade parameter, slip velocity and magnetic field parameter. It is reviewed from figures 9 and 10 that temperature profile decrease with increases values of third grade parameter and slip velocity. Also, temperature profiles increase with shear thinning, Eckert number and and magnetic fireld parameter as shown in figures 7, 8 and 11 respectively.

Figures 12, 13, and 14 shows the variation of the volume flow rate,shear stress and resistance to blood flow along the axial direction for different values of the slip velocity. The figures reviewed that increase in slip velocity significantly increase the volume flow rate and shear stress but reduce the flow resistance. Also, figures 15, 16 and 17 depicts the variation of volume flow rate, shear stress and resistance to flow along the axial direction for different values of the magnetic field parameter. From the figures, it was shown that magnetic field parameter increases with shear stress and resistance to flow but reduce the flow rate.

5 Conclusion

It is very important to understand theoretically the dynamics of blood flow and heat transfer through a stenosed artery in human physiological system. In view of this, mathematical analysis for the magnetic field parameter and slip velocity influence on blood flow and heat transfer through a stenosed artery using a third grade fluid model, brings out many interesting fluid mechanical phenomena, arising due to the combined effect of those parameters. It is seen from the findings that slip velocity significantly increases the flow velocity, flow rate and shear stress but reduces the flow resistance and heat transfer rate. Magnetic field parameter slightly reduces the flow velocity, flow rate and wall shear stress but increases the heat transfer rate and flow resistance.

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