

# Study of Soft Set and Common Soft Point With Complete Metric Space

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**Abstract:** In this paper firstly we study systematic and critical study of the fundamentals of soft set theory, which include operations on soft sets and their properties. We define equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set. The purpose of this paper is to prove the existence of fixed point on a soft metric space.

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## 2. Introduction and Preliminaries:

The notion of soft sets introduced by Molodtsov [1] in (1999) as a general mathematical tool for dealing with uncertain objects. Molodtsov et al. bring out applications of soft set and soft theory in various areas, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration and probability [2]. Maji et al. [3] defined and studied several basic notions of soft set theory. C.A. Gman and Enginoglu [4] studied products of soft sets and uni-int decision functions. Certain De Morgan's laws in soft set theory with respect to different operations and extend the theoretical aspect of operations on soft sets studied by Sezgin and A. Atagun [5]. Shabir and Naz [6] initiated the study of soft topological spaces and showed that a soft topological space gives a parameterized family of topological spaces. They introduced the notions of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. In Shabir and Naz paper there were some incorrect results that Min [7] point out them and investigated the soft regular spaces and some properties of them. After them many useful literatures in soft set and soft topological space have been written by many authors such as products of soft sets and uni-int decision functions, soft first-countable spaces, soft second-countable spaces and soft separable spaces [8], properties of equivalence soft set relations [9] and soft mapping introduced in [10].

In this section we consider the basic definitions and properties of soft sets and soft topologies. we give some basic definitions and results on soft sets.

**Definition 2.1:** [1]. A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a function given by  $F : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the

universe  $U$ . For any parameter  $x \in A$ ,  $F(x)$  may be considered as the set of  $x$ -approximate elements of the soft set  $(F, A)$ .

**Definition 2.2:** [11]. Let  $(F,A)$  and  $(G,B)$  be two soft sets over  $U$ . The intersection of  $(F, A)$  and  $(G, B)$  is a soft set  $(H, C)$ , where  $C = A \cap B$  and for all  $e \in C$ ,  $H(e) = F(e) \cap G(e)$ . We write  $(F, A) \cap (G, B) = (H, C)$ .

**Definition 2.3:** [12]. Let  $(F,A)$  be a soft set over  $U$ . The relative complement of  $(F,A)$  is denoted by  $(F,A)^c$  and is defined by  $(F,A)^c = (F_c, A)$ , where  $F_c : A \rightarrow P(U)$  is a mapping given by  $F_c(a) = U - F(a)$  for all  $a \in A$ .

**Definition 2.4:** The complement of a soft set  $(A,D)$  is denoted by  $(A,D)^c$  and is defined by  $(A,D)^c = (A^c, D)$  where  $A^c : D \rightarrow S(X)$  mapping given by  $A^c(\alpha) = A(\alpha)^c, \forall \alpha \in D$ .

**Definition 2.5:** Let  $\mu$  be the set of real number and  $B(\mu)$  be the collection of all nonempty bounded subsets of  $\mu$  and  $E$  taken set of parameters. Then a mapping  $A: E \rightarrow B(\mu)$  is called a soft real set. It is denoted by  $(A,E)$ . If specifically  $(A,E)$  is a singleton soft set, then identifying  $(A,E)$  with the corresponding soft element, it will be called a soft real number and denoted  $\tilde{r}, \tilde{s}, \tilde{t}$  etc.  $\bar{0}, \bar{1}$  are the soft real number where  $\bar{0}(e) = 0, \bar{1}(e) = 1$  for all  $e \in E$ , respectively

**Definition 2.6:** Let  $U$  be a universe,  $E$  be a set of parameters and  $A \subseteq E$ .

- (1)  $(F,A)$  is called a relative null soft set with respect to  $A$ , denoted  $\Phi_A$  if  $F(e) = \phi$ .
- (2)  $(F,A)$  is called a relative whole soft set or  $A$  universal with respect to  $A$ , denoted  $u_A$ , if  $F(e) = U, \forall e \in A$ .
- (3) The relative whole soft set with respect to  $E$  denoted  $\tilde{U}_E$  is called the absolute soft set over  $U$ .

**Definition 2.7:** [6]. Let  $X$  be an initial universe set and  $E$  be the fixed nonempty set of parameter with respect to  $X$ . Let  $\tau$  be the collection of soft sets over  $X$ ; then  $\tau$  is called a soft topology on  $X$  if  $\tau$  satisfies the following axioms:

- (1)  $\emptyset$  and  $X$  belong to  $\tau$ .
- (2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- (3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The pair  $((X, E), \tau)$  is called soft topological space. The members of  $\tau$  are said to be soft open in  $X$ . A soft set  $(F, E)$  over  $X$  is said to be soft closed in  $X$  if its relative complement  $(F,E)^c$  belongs to  $\tau$ .

**Definition 2.8:** [13]. Let  $(F,A)$  and  $(G,B)$  be two soft sets over  $U$ , then the Cartesian product of  $(F,A)$  and  $(G,B)$  is defined as  $(F,A) \times (G,B) = (H, A \times B)$ , where  $H : A \times B \rightarrow P(U \times U)$  and  $H(a, b) = F(a) \times G(b)$  for all  $(a, b) \in A \times B$ .

**Definition 2.9:** [13]. Let  $(F,A)$  be a soft set over  $U$ , and  $R$  be a relation from  $(F,A)$  to itself, then

- (1)  $R$  is reflexive if  $H(a, a) \in R$ , for all  $a \in A$ .
- (2)  $R$  is symmetric if  $H(a, b) \in R$ , then  $H(b, a) \in R$ , for all  $(a, b) \in A \times A$ .
- (3)  $R$  is transitive if  $H(a, b) \in R$  and  $H(b, c) \in R$ . Then  $H(a, c) \in R$ , for all  $a, b, c \in A$ .

**Definition 2.10:** [14]. Let  $((X, \tau), E)$  be a soft topological space. A subcollection  $B$  of  $\tau$  is called a basis for  $\tau$  if every member of  $\tau$  can be expressed as a union of members of  $B$ . A subcollection  $S$  of  $\tau$  is said to be a sub basis for  $\tau$  if the family of all finite intersections of members of  $S$  forms a basis for  $\tau$ .

The following Theorem has been stated in [14], which it has a mistype, and authors in [15], corrected it as follows

**Definition 2.11:** [16]. Let  $((X, \tau), E)$  be a soft topological space over  $X$  and let  $(F,A)$  be a soft set over  $X$ .

- (1) The soft closure of soft set  $(F, A)$ , denoted by  $(F,A)$  is the intersection of all soft closed

super sets of  $(F,A)$ . Clearly  $(F, A)$  is the smallest soft closed set over  $X$  which contains  $(F, A)$ .

- (2) The soft boundary of soft set  $(F, A)$  is denoted by  $(F,A)$  and is defined as  $(F,A) = (F,A) \cap ((F,A)c)$ .

- (3) The soft interior of soft set  $(F,A)$  is denoted by  $(F,A)^\circ$ . and is defined as the union of all soft open sets contained in  $(F,A)$ . Thus,  $(F,A)^\circ$  is the largest soft open set contained in  $(F,A)$ .

**Definition 2.12:-** for two soft real numbers

- I.  $\tilde{r} \leq \tilde{s}$  if  $\tilde{r}(e) \leq \tilde{s}(e)$ , for all  $e \in E$ .
- II.  $\tilde{r} \geq \tilde{s}$  if  $\tilde{r}(e) \geq \tilde{s}(e)$ , for all  $e \in E$ .
- III.  $\tilde{r} < \tilde{s}$  if  $\tilde{r}(e) < \tilde{s}(e)$ , for all  $e \in E$ .
- IV.  $\tilde{r} > \tilde{s}$  if  $\tilde{r}(e) > \tilde{s}(e)$ , for all  $e \in E$ .

**Definition 2.13:** A sequence  $\{ \tilde{x}_{\lambda_n} \}_n$  of soft point in  $(\bar{X}, \bar{d}, E)$  is considered as a Cauchy Sequence in  $\bar{X}$  if corresponding to every  $\tilde{\epsilon} \gtrsim \bar{0}$ ,  $\exists m \in \mathbb{N}$  such that  $d(\tilde{x}_{\lambda_i}, \tilde{x}_{\lambda_j}) \lesssim \tilde{\epsilon}$ ,  $\forall i, j \geq m$ , i.e.  $d(\tilde{x}_{\lambda_i}, \tilde{x}_{\lambda_j}) \rightarrow \bar{0}$ , as  $i, j \rightarrow \infty$ .

**Definition 2.14:** A soft metric space  $(\tilde{X}, \tilde{d}, E)$  is called complete, if every Cauchy Sequence in  $\tilde{X}$  converges to some point of  $\tilde{X}$ .

### 3. MAIN RESULTS:

Let  $(f, \varphi)$  be a Continuous self map, defined on a soft complete metric space  $(\tilde{X}, \tilde{d})$  Satisfies the following condition;

$$\begin{aligned} \tilde{d}((f, \varphi)\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega) &\leq \alpha \frac{\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{y}_\omega) + \tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega) \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)}{\tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega)} \\ &+ \beta \frac{\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) \tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{y}_\omega) \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)}{\tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega)} \\ &+ \gamma [(\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{y}_\omega))] + \delta[(\tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega) + \tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega))] \\ &+ \eta \tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega) \end{aligned}$$

For all  $\tilde{x}_\omega, \tilde{y}_\omega \in SP(\tilde{X})$ ,  $\tilde{x}_\omega \neq \tilde{y}_\omega$ , and for some  $\alpha, \beta, \gamma, \delta, \eta \in [1, \infty)$  with  $(\alpha + 2\beta + 2\delta + \eta < 1)$  Then  $(f, \varphi)$  has soft point in  $\tilde{X}$ .

**Theorem 3.1:** Let  $(f, \varphi)$  be a Continuous self map, defined on a soft complete metric space  $(\tilde{X}, \tilde{d})$  Satisfies the following condition;

$$\begin{aligned} \tilde{d}((f, \varphi)\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega) &\leq \alpha \left[ \frac{\tilde{d}^3(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) + \tilde{d}^3(\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega)}{1 + \tilde{d}^2(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega^n)} \right] \\ &+ \beta [\tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{y}_\omega) \cdot \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)] + \gamma \tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega) + \eta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) + \\ &\tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{y}_\omega)] + \delta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{y}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)] \end{aligned} \quad (3.1.1)$$

For all  $\tilde{x}_\omega, \tilde{y}_\omega \in SP(\tilde{X})$ ,  $\tilde{x}_\omega \neq \tilde{y}_\omega$ , and for some  $\alpha, \beta, \gamma, \delta, \eta \in [1, \infty)$  with  $(\alpha + 2\beta + 2\delta + \eta < 1)$  Then  $(f, \varphi)$  has soft point in  $\tilde{X}$ .

**Proof:** Let  $\tilde{x}_\omega^0$  be any arbitrary soft point in  $\tilde{X}$ , and we define a sequence  $\{\tilde{x}_\omega^n\}$  be means of iterates of  $(f, \varphi)$  be setting  $(f, \varphi)_{\tilde{x}_\omega^n}^n = \tilde{x}_\omega^n$ , Where  $n$  is a positive integer, if  $\tilde{x}_\omega^n = \tilde{x}_\omega^{n+1}$  for some  $n$ , Then  $\tilde{x}_\omega^n$  is a soft point of  $(f, \varphi)$  taking  $\tilde{x}_\omega^n \neq \tilde{x}_\omega^{n+1}$  for all  $n$ .

$$\begin{aligned} \tilde{d}(\tilde{x}_\omega^{n+1}, \tilde{x}_\omega^n) &= \tilde{d}[(f, \varphi)\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^{n-1}] \\ \tilde{d}(\tilde{x}_\omega^{n+1}, \tilde{x}_\omega^n) &\leq \alpha \left[ \frac{\tilde{d}^3(\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^n) + \tilde{d}^3(\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^{n-1})}{1 + \tilde{d}^2(\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^n)} \right] \\ &+ \beta [\tilde{d}(\tilde{x}_\omega^{n-1}, (f, \varphi)\tilde{x}_\omega^{n-1}) \cdot \tilde{d}(\tilde{x}_\omega^{n-1}, (f, \varphi)\tilde{x}_\omega^n)] + \gamma \tilde{d}(\tilde{x}_\omega^n, \tilde{x}_\omega^{n-1}) + \\ &\eta [\tilde{d}(\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^n) + \tilde{d}(\tilde{x}_\omega^{n-1}, (f, \varphi)\tilde{x}_\omega^{n-1})] \\ &+ \delta [\tilde{d}(\tilde{x}_\omega^n, (f, \varphi)\tilde{x}_\omega^{n-1}) + \tilde{d}(\tilde{x}_\omega^{n-1}, (f, \varphi)\tilde{x}_\omega^n)] \end{aligned} \quad (3.1.2)$$

$$\begin{aligned} &\leq \alpha \left[ \frac{\{ \tilde{d}(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n}^n) + \tilde{d}(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n-1}^{n-1}) \} \{ \tilde{d}^2(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n}^n) + \tilde{d}^2(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n-1}^{n-1}) + \tilde{d}(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n}^n) \cdot \tilde{d}(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n-1}^{n-1}) \}}{1 + \tilde{d}^2(\tilde{x}_{\omega n}^n, (f, \varphi)\tilde{x}_{\omega n}^n)} \right] \\ &\quad + \beta [\tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n}^n) \cdot \tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n+1}^{n+1})] + \gamma \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1}) + \eta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \\ &\quad \tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n}^n)] + \delta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n}^n) + \tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n+1}^{n+1})] \\ &\leq \alpha \left[ \frac{\{ \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n}^n) \} \{ \tilde{d}^2(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \tilde{d}^2(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n}^n) + \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n}^n) \}}{\tilde{d}^2(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1})} \right] \\ &\quad + \beta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1})] + \gamma \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1}) + \eta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n}^n)] + \\ &\quad + \delta [\tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n+1}^{n+1})] \\ &\leq \alpha [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1})] + \beta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1})] + \gamma \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1}) \\ &\quad + \eta [\tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n}^n)] + \delta [\tilde{d}(\tilde{x}_{\omega n-1}^{n-1}, \tilde{x}_{\omega n}^n) + \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1})] \end{aligned}$$

$$\tilde{d}(\tilde{x}_{\omega n+1}^{n+1}, \tilde{x}_{\omega n}^n) \leq (\alpha + \beta + \delta + \eta) \tilde{d}(\tilde{x}_{\omega n+1}^{n+1}, \tilde{x}_{\omega n}^n) + (\gamma + \delta + \eta) \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1}) \quad (3.1.3)$$

$$\{1 - (\alpha + \beta + \delta + \eta)\} \tilde{d}(\tilde{x}_{\omega n+1}^{n+1}, \tilde{x}_{\omega n}^n) \leq (\gamma + \delta + \eta) \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1}) \quad (3.1.4)$$

$$\tilde{d}(\tilde{x}_{\omega n+1}^{n+1}, \tilde{x}_{\omega n}^n) \leq \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1})$$

⋮

⋮

$$\left[ \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} \right]^{\text{ntimes}} \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n-1}^{n-1})$$

By the triangular inequality, We have  $m > n$

$$\begin{aligned} \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega m}^m) &\leq \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega n+1}^{n+1}) + \tilde{d}(\tilde{x}_{\omega n+1}^{n+1}, \tilde{x}_{\omega n+2}^{n+2}) + \dots + \tilde{d}(\tilde{x}_{\omega m-1}^{m-1}, \tilde{x}_{\omega m}^m) \\ &\leq (\tilde{K}^n + \tilde{K}^{n+1} + \tilde{K}^{n+2} + \dots + \tilde{K}^{n-1}) \tilde{d}(\tilde{x}_{\omega}^0, \tilde{x}_{\omega 1}^1) \end{aligned}$$

$$\text{Where } \tilde{K} = \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} < 1 \quad (3.1.5)$$

$$1 - (\alpha + \beta + \delta + \eta) \leq (\gamma + \delta + \eta)$$

$$(\alpha + \beta + 2\delta + 2\eta + \gamma) < 1$$

$$\therefore \tilde{d}(\tilde{x}_{\omega n}^n, \tilde{x}_{\omega m}^m) \leq \frac{\tilde{K}^n}{1 - \tilde{K}} \tilde{d}(\tilde{x}_{\omega}^0, \tilde{x}_{\omega 1}^1) \quad (3.1.6)$$

So  $\{\tilde{x}_{\omega n}^n\}$  is soft Cauchy sequence in  $\tilde{X}$ . So by completeness of  $\tilde{X}$ , there is a point

$\tilde{u}_\omega \in \tilde{X}$ . Such that  $\tilde{x}_{\omega n}^n \rightarrow \tilde{u}_\omega$  as  $n \rightarrow \infty$ .

Further the continuity of  $(f, \varphi)$  in  $\tilde{X}$ .

Implies  $(f, \varphi)(\tilde{u}_\omega) = (f, \varphi)$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \tilde{x}_{\omega n} \\ &== \lim_{n \rightarrow \infty} (f, \varphi)(\tilde{x}_{\omega n}) \\ &= \lim_{n \rightarrow \infty} \tilde{x}_{\omega n+1} \\ &= \tilde{u}_\omega \end{aligned}$$

$\therefore \tilde{u}_\omega$  is soft point of  $(f, \varphi)$  in  $\tilde{X}$ .

**Theorem 3.2:** Let  $(f, \varphi)$  be the self map defined on a soft complete metric space

$(\tilde{X}, \tilde{d})$ . Such that (3.1.1) holds, if for some positive integer  $p$ ,  $(f, \varphi)^p$  is continuous, Then  $(f, \varphi)$  has a soft point not unique. Satisfies the following condition;

$$\begin{aligned} &\tilde{d}((f, \varphi)\tilde{x}_\omega, (f, \varphi)^m \tilde{y}_\omega) \\ &\leq \alpha \left[ \frac{\tilde{d}^3(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) + \tilde{d}^3(\tilde{x}_\omega, (f, \varphi)^m \tilde{y}_\omega)}{1 + \tilde{d}^2(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega)} \right] + \beta [\tilde{d}(\tilde{y}_\omega, (f, \varphi)^m \tilde{y}_\omega) \cdot \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)] \\ &\quad + \gamma \tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega) + \eta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)\tilde{x}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)^m \tilde{y}_\omega)] + \\ &\quad \delta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)^m \tilde{y}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)\tilde{x}_\omega)] \end{aligned} \quad (3.2.1)$$

For all  $\tilde{x}_\omega, \tilde{y}_\omega \in SP(\tilde{X})$ ,  $\tilde{x}_\omega \neq \tilde{y}_\omega$ , and for some  $\alpha, \beta, \gamma, \delta, \eta \in [1, \infty)$  with  $(\alpha + 2\beta + 2\delta + \eta < 1)$  Then has  $(f, \varphi)$  soft point in  $\tilde{X}$ .

**Proof:** We define a sequence  $\{\tilde{x}_{\omega k}\}$  as in Theorem (3.1.1) clearly it converges to some point  $\tilde{u}_\omega$  is  $(\tilde{X}, \tilde{d})$ . Therefore its subsequence  $\{\tilde{x}_{\omega k}\}$ ,  $(\omega_k = k_p)$  also converges to  $\tilde{u}_\omega$ .

$$\begin{aligned} (f, \varphi)_{\tilde{u}_\omega}^p &= (f, \varphi)^p \lim_{k \rightarrow \infty} \tilde{x}_{\omega k} \\ &= \lim_{k \rightarrow \infty} \{ (f, \varphi)^p(\tilde{x}_{\omega k}) \} \\ &= \lim_{k \rightarrow \infty} \tilde{x}_{\omega k+1} \\ &= \tilde{u}_\omega \end{aligned}$$

Therefore  $\tilde{u}_\omega$  is a soft point of  $(f, \varphi)^p$ . Now we show that  $(f, \varphi)^p \tilde{u}_\omega = \tilde{u}_\omega$ . Let  $m$  be the smallest positive integer, Such that  $(f, \varphi)^m \tilde{u}_\omega = \tilde{u}_\omega$ , but  $(f, \varphi)^q \tilde{u}_\omega \neq \tilde{u}_\omega$  for  $q=1, 2, 3, \dots, m-1$ .

If  $m > 1$  then by (3.1.1)

$$\begin{aligned} \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) &= \tilde{d}[(f, \varphi) \tilde{u}_\omega, (f, \varphi)^m \tilde{u}_\omega] = \tilde{d}[(f, \varphi) \tilde{u}_\omega, (f, \varphi)(f, \varphi)^{m-1} \tilde{u}_\omega] \\ \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) &\leq \alpha \left[ \frac{\tilde{d}^3(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}^3(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega)}{1 + \tilde{d}^2(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)} \right] + \\ &\quad + \beta [\tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi)(f, \varphi)^{m-1} \tilde{u}_\omega) \cdot \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] \\ &\quad + \gamma \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) + \eta [\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi)(f, \varphi)^{m-1} \tilde{u}_\omega)] \\ &\quad + \delta [\tilde{d}(\tilde{u}_\omega, (f, \varphi)(f, \varphi)^{m-1} \tilde{u}_\omega) + \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] \tag{3.2.2} \\ &\leq \alpha \left[ \frac{\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega)}{\tilde{d}^2(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)} \{ \tilde{d}^2(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}^2(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) + \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) \} \right] \\ &\quad + \beta [\tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega) \cdot \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] + \gamma \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) \\ &\quad + \eta [\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega)] + \delta [\tilde{d}(\tilde{u}_\omega, \tilde{u}_\omega) + \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] \\ &\leq \alpha \left[ \frac{\{ \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}(\tilde{u}_\omega, \tilde{u}_\omega) \} \{ \tilde{d}^2(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}^2(\tilde{u}_\omega, \tilde{u}_\omega) + \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) \tilde{d}(\tilde{u}_\omega, \tilde{u}_\omega) \}}{\tilde{d}^2(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)} \right] \\ &\quad + \beta [\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] + \gamma \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) \\ &\quad + \eta [\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega)] + \delta [\tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] \\ &\leq \alpha \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \beta \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \gamma \tilde{d}(\tilde{u}_\omega, (f, \varphi)^{m-1} \tilde{u}_\omega) + \eta [\tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega) + \\ &\quad \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega)] + \delta [\tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega) + \tilde{d}(\tilde{u}_\omega, (f, \varphi) \tilde{u}_\omega)] \\ \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) &\leq (\alpha + \beta + \delta + \eta) \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) + (\gamma + \delta + \eta) \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega) \\ \{1 - (\alpha + \beta + \delta + \eta)\} \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) &\leq (\gamma + \delta + \eta) \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega) \tag{3.2.3} \end{aligned}$$

$$\tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) \leq K \tilde{d}((f, \varphi)^{m-1} \tilde{u}_\omega, \tilde{u}_\omega)$$

$$\text{Where } K = \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)}$$

$$\{1 - (\alpha + \beta + \delta + \eta)\} \leq (\gamma + \delta + \eta)$$

$$(\alpha + \beta + 2\delta + 2\eta + \gamma) < 1$$

$$\text{Thus we write } \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) \leq K^m \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega)$$

Since  $K^m < 1$

$$\therefore \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega) \leq \tilde{d}((f, \varphi) \tilde{u}_\omega, \tilde{u}_\omega)$$

Which is contradiction. Hence  $((f, \varphi) \tilde{u}_\omega = \tilde{u}_\omega)$ . i.e.  $\tilde{u}_\omega$  is a Soft point of  $(f, \varphi)$ . But  $(f, \varphi)$  has not unique soft point.

**Theorem 3.3:** Let  $(f, \varphi)_1$  and  $(f, \varphi)_2$  be two self maps, defined on a soft complete metric space  $(\tilde{X}, \tilde{d})$ . Satisfies the following condition;

$$\begin{aligned} \tilde{d}\{(f, \varphi)_1 \tilde{x}_\omega, (f, \varphi)_2 \tilde{y}_\omega\} \leq & \alpha \left[ \frac{\tilde{d}^3(\tilde{x}_\omega, (f, \varphi)_1 \tilde{x}_\omega) + \tilde{d}^3(\tilde{x}_\omega, (f, \varphi)_2 \tilde{y}_\omega)}{1 + \tilde{d}^2(\tilde{x}_\omega, (f, \varphi)_1 \tilde{x}_\omega)} \right] \\ & + \beta [\tilde{d}(\tilde{y}_\omega, (f, \varphi)_2 \tilde{y}_\omega) \cdot \tilde{d}(\tilde{y}_\omega, (f, \varphi)_1 \tilde{x}_\omega)] + \gamma \tilde{d}(\tilde{x}_\omega, \tilde{y}_\omega) + \eta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)_1 \tilde{x}_\omega) + \\ & \tilde{d}(\tilde{y}_\omega, (f, \varphi)_2 \tilde{y}_\omega)] + \delta [\tilde{d}(\tilde{x}_\omega, (f, \varphi)_2 \tilde{y}_\omega) + \tilde{d}(\tilde{y}_\omega, (f, \varphi)_1 \tilde{x}_\omega)] \end{aligned} \quad (3.3.1)$$

For all  $\tilde{x}_\omega, \tilde{y}_\omega \in SP(\tilde{X})$ ,  $\tilde{x}_\omega \neq \tilde{y}_\omega$ , and for some  $\alpha, \beta, \gamma, \delta, \eta \in [1, 0)$  with  $(\alpha + 2\beta + 2\delta + \gamma + \eta < 1)$  Then  $(f, \varphi)_1$  and  $(f, \varphi)_2$  has soft point in  $\tilde{X}$  and by the (3.2.1)  $(f, \varphi)_1$  and  $(f, \varphi)_2$

Are continuous on  $SP(\tilde{X})$ .

**Proof :** Chose  $\tilde{x}_\omega^0$  be any soft point is  $SP(\tilde{X})$ .

$$\tilde{x}_{\omega_1} = (f, \varphi)(\tilde{x}_\omega^0) = (f(\tilde{x}_\omega^0))_{\omega\varphi}$$

$$\tilde{x}_{\omega_2}^2 = (f, \varphi)(\tilde{x}_{\omega_2}^1) = (f(\tilde{x}_\omega^0))_{\omega\varphi^2}$$

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$$\tilde{x}_{\omega_{n+1}}^{n+1} = (f, \varphi)(\tilde{x}_{\omega_n}^n) = (f(\tilde{x}_\omega^0))_{\omega\varphi^{n+1}}$$

$$\text{We have } \tilde{d}(\tilde{x}_{\omega_{n+1}}^{n+1}, \tilde{x}_{\omega_n}^n) = \tilde{d}((f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1})$$

$$\begin{aligned} \tilde{d}(\tilde{x}_{\omega_{2n+1}}^{2n+1}, \tilde{x}_{\omega_{2n}}^{2n}) \leq & \alpha \left[ \frac{\tilde{d}^3(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}) + \tilde{d}^3(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1})}{1 + \tilde{d}^2(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n})} \right] \\ & + \beta [\tilde{d}(\tilde{x}_{\omega_{2n-1}}^{2n-1}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1}) \cdot \tilde{d}(\tilde{x}_{\omega_{2n-1}}^{2n-1}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n})] + \gamma \tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, \tilde{x}_{\omega_{2n-1}}^{2n-1}) + \\ & \eta [\tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}) + \tilde{d}(\tilde{x}_{\omega_{2n-1}}^{2n-1}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1})] + \delta [\tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1}) + \\ & \tilde{d}(\tilde{x}_{\omega_{2n-1}}^{2n-1}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n})] \end{aligned} \quad (3.3.2)$$

$$\leq \alpha \left[ \frac{\{\tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}) + \tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1})\} \{\tilde{d}^2(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}) + \tilde{d}^2(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1}) + \tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n}) \cdot \tilde{d}(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_2 \tilde{x}_{\omega_{2n-1}}^{2n-1})\}}{\tilde{d}^2(\tilde{x}_{\omega_{2n}}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega_{2n}}^{2n})} \right]$$



$$\begin{aligned}
 & + \beta [\tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n}) \cdot \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n+1}^{2n+1})] + \gamma \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n-1}^{2n-1}) + \eta [\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \\
 & \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n})] + \delta [\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n}^{2n}) + \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n+1}^{2n+1})] \\
 \leq & \\
 \alpha & \left[ \frac{\{\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n}^{2n})\} \{\tilde{d}^2(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \tilde{d}^2(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n}^{2n}) + \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n}^{2n})\}}{\tilde{d}^2(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1})} \right] \\
 & + \beta [\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1})] + \gamma \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n-1}^{2n-1}) + \eta [\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n})] \\
 & + \delta [\tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n+1}^{2n+1})] \\
 \leq & \alpha \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \beta \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \gamma \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n-1}^{2n-1}) \\
 & + \eta [\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) + \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n})] + \delta [\tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n}) + \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1})] \\
 & \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega 2n+1}^{2n+1}) \leq (\alpha + \beta + \delta + \eta) \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega 2n+1}^{2n+1}) + \\
 & (\gamma + \delta + \eta) \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n}) \\
 (3.3.3)
 \end{aligned}$$

$$\{1 - (\alpha + \beta + \delta + \eta)\} \tilde{d}(\tilde{x}_{\omega 2n}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega 2n+1}^{2n+1}) \leq (\gamma + \delta + \eta) \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n}) \quad (3.3.4)$$

$$\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, (f, \varphi)_1 \tilde{x}_{\omega 2n+1}^{2n+1}) \leq \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n})$$

⋮  
⋮

$$\left[ \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} \right]^{n \text{ times}} \tilde{d}(\tilde{x}_{\omega 2n-1}^{2n-1}, \tilde{x}_{\omega 2n}^{2n})$$

$$\tilde{d}(\tilde{x}_{\omega 2n}^{2n}, \tilde{x}_{\omega 2n+1}^{2n+1}) \leq K^{2n} \tilde{d}(\tilde{x}_{\omega}^0, \tilde{x}_{\omega 1}^1)$$

Where  $K = \frac{(\gamma + \delta + \eta)}{1 - (\alpha + \beta + \delta + \eta)} < 1$

$$1 - (\alpha + \beta + \delta + \eta) \leq (\gamma + \delta + \eta)$$

$$(\alpha + \beta + 2\delta + 2\eta + \gamma) < 1$$

Similarly we can show that

$$\tilde{d}(\tilde{x}_{\omega 2n+1}^{2n+1}, \tilde{x}_{\omega 2n+2}^{2n+2}) \leq K^{2n+1} \tilde{d}(\tilde{x}_{\omega}^0, \tilde{x}_{\omega 1}^1)$$

Now it can be easily seen that  $\{\tilde{x}_{\omega}^n\}$  is a Cauchy sequence. Let  $\tilde{x}_{\omega}^n \rightarrow \tilde{u}_{\omega}$ , Then the subsequence  $\{\tilde{x}_{\omega p}^n\}$  also converges to  $\tilde{u}_{\omega}$  for  $\omega_p = 2p$ .

$$\begin{aligned} \text{Now } ((f, \varphi)_1, (f, \varphi)_2 \tilde{u}_{\omega}) &= (f, \varphi)_1, (f, \varphi)_2(\lim_{p \rightarrow \infty} \tilde{x}_{\omega p}^n) \\ &= (\lim_{p \rightarrow \infty} \tilde{x}_{\omega p+1}^n) \\ &= \tilde{u}_{\omega} \end{aligned}$$

We now show that  $(f, \varphi)_2 \tilde{u}_{\omega} \neq \tilde{u}_{\omega}$

If  $(f, \varphi)_2 \tilde{u}_{\omega} \neq \tilde{u}_{\omega}$ , then

$$\tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) = \tilde{d}((f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})$$

$$\tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) \leq \alpha \left[ \frac{\{\tilde{d}^3((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}^3((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})\}}{1 + \tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega})} \right]$$

$$+ \beta[\tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) \tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega})] + \gamma \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})]$$

$$+ \eta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})] + \delta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) +$$

$$\tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega})] \tag{3.3.5}$$

$$\leq \alpha \left[ \frac{\{\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})\} \{\tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega}) \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})\}}{\tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega})} \right]$$

$$+ \beta[\tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) \tilde{d}(\tilde{u}_{\omega}, \tilde{u}_{\omega})] + \gamma [\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})] +$$

$$\eta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) + \tilde{d}(\tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})] + \delta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}(\tilde{u}_{\omega}, \tilde{u}_{\omega})]$$

$$\leq \alpha \left[ \frac{\{\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) + \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})\} \{\tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) + \tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega}) + \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, (f, \varphi)_2 \tilde{u}_{\omega})\}}{\tilde{d}^2((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})} \right]$$

$$+ \gamma [\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})] + 2 \eta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})]$$

$$\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) \leq \alpha \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) + \gamma [\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})] + 2 \eta[\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega})]$$

$$\tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) \leq (\alpha + \gamma + 2 \eta) \tilde{d}((f, \varphi)_2 \tilde{u}_{\omega}, \tilde{u}_{\omega}) \tag{3.3.6}$$

Which is contradiction .

$$\therefore (\alpha + \beta + 2\delta + 2\eta + \gamma) < 1 \quad \text{so} \quad (\alpha + \gamma + 2 \eta) < 1$$

Hence we have  $(f, \varphi)_2 \tilde{u}_{\omega} = \tilde{u}_{\omega}$

Now  $(f, \varphi)_1 (f, \varphi)_2 \tilde{u}_{\omega} = (f, \varphi)_2 \tilde{u}_{\omega} = \tilde{u}_{\omega}$

Thus  $\tilde{u}_{\omega}$  is the common soft point of  $(f, \varphi)_1$  and  $(f, \varphi)_2$ , But uniqueness is not possible.

**Conclusion:** In this paper we proof some theorem of soft point in complete metric space. We have discussed in detail the fundamentals of soft set theory such as soft subset, soft operations and their properties etc.

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