Generalizations of Rakotch's Fixed Point Theorem For Soft Sets

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Abstract

In this paper we get some generalizations of Rakotch's fixed point theorem using the nation of ω -distance on soft metric space.

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2 INTRODUCTION & PRELIMINARIES

Banach fixed point theorem is one of the most crucial and fruitful tool to find solution of various problems in Mathematics. This result is generalized by many mathematicians in diverse directions. One of the most popular generalization is given by Rakotch [16] (Rakotch, E., 1962), Rakotch prove a fixed point theorem on taking monotone function.

On the other hand there are other mathematicians as well, who has generalized the notion of metric space by taking different conditions. In 1996, O. Kada [9] (Kada, O., Suzuki, T., Takahashi, W., 1996), T. Suzuki [17] (Suzuki, T., 1997) & W. Takahashi [19] (Takahashi, W., 1996), introduced the concept of ω −distance on a metric space, gave some examples, properties of ω −distance and they improved Caristi's fixed point (Caristi, J., 1976 [4]). Eventually, by the use of the concept of ω −distance they proved a fixed point theorem in a complete metric space.

In the year 1999, Molodtsov [13] initiated a novel concept of soft sets theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real life problems are progressing rapidly.

Definition 2.1: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set $X, i.e. F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 2.2: The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.This is denoted by $(F, A) \cap (G, B) = (H, C)$.

Definition 2.3: The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in A - B \\ G(\varepsilon), & if\ \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \cup (G, B) = (H, C)$.

Definition 2.4: The soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi$ (null set).

Definition 2.5: A soft set (F, A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 2.6: The difference (H, E) of two soft sets (H, E) and (H, E) over X denoted by $(H, E)\backslash(H, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.7: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \rightarrow P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 2.8: Let $\mathfrak{R}$ be the set of real numbers and $B(\mathfrak{R})$ be the collection of all nonempty bounded subsets of $\mathfrak{R}$ and E taken as a set of parameters. Then a mapping $F: E \rightarrow B(\mathfrak{R})$ is called a soft real set. It is denoted by (F, E). If specifically (F, E) is a singleton soft set, then identifying (F, E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

$\overline{0}, \overline{1}$ are the soft real numbers where $\overline{0}(e) = 0, \overline{1}(e) = 1$ for all $e \in E$, respectively.

Definition 2.9: For two soft real numbers

(i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.

(ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.

(iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.

(iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 2.10: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E\backslash\{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 2.11: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 2.12: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

(M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \tilde{\geq} \overline{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \overline{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

(M3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \cong \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M4) $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \tilde{\leq} \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 2.13 (Cauchy Sequence): A sequence $\{\tilde{x}_{\lambda,n}\}_n$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\varepsilon} \tilde{\geq} \overline{0}, \exists m \in N$ such that $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \tilde{\leq} \tilde{\varepsilon}, \forall\, i, j \geq m$, i.e. $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \to \overline{0}$, as $i, j \to \infty$.

Definition 2.14 (Soft Complete Metric Space): A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete, if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

Definition 2.15: Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space. A function $\tilde{\sigma} : (\tilde{X}, \tilde{d}, E) \times (\tilde{X}, \tilde{d}, E) \to [0, \infty]$ is called a ω -distance on $\tilde{X}$ if the following conditions are satisfied:

ω_1 $\tilde{\sigma}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \tilde{\leq} \tilde{\sigma}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{\sigma}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

ω_2 For any $\tilde{x}_{e_1} \in \tilde{X}$, $\tilde{\sigma}(\tilde{x}_{e_1}, .): \tilde{X} \to [0, \infty]$ is lower semi continuous.

ω_3 For any $\tilde{\varepsilon} \tilde{\geq} 0, \exists\, \tilde{\delta} = \tilde{\delta}(\tilde{\varepsilon}) > 0$ such that,

$\tilde{\sigma}(\tilde{z}_{e_3}, \tilde{x}_{e_1}) \leq \tilde{\delta}$ and $\tilde{\sigma}(\tilde{z}_{e_3}, \tilde{y}_{e_2}) \leq \tilde{\delta}$ imply $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \leq \tilde{\varepsilon}$.

The metric $\tilde{d}$ is a ω -distance on $\tilde{X}$.

Lemma 2.16: Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $\tilde{\sigma}$ **be a** ω -distance on $\tilde{X}$. Let $\{\tilde{a}^n_{\lambda_n}\}$ **and** $\{\tilde{b}^n_{\lambda_n}\}$ be sequences in $[0, +\infty)$ converging to zero and let $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$. Then the following condition hold

(i) **If** $\tilde{\sigma}(\tilde{x}^n_{\lambda_n}, \tilde{y}_{e_2}) \leq \tilde{a}^n_{\lambda_n}$ and $\tilde{\sigma}(\tilde{x}^n_{\lambda_n}, \tilde{z}_{e_3}) \leq \tilde{b}^n_{\lambda_n}$ for any $n \in \mathbb{N}$ then $\tilde{y}_{e_2} = \tilde{z}_{e_3}$.
In particular, if $\tilde{\sigma}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = 0$ and $\tilde{\sigma}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) = 0$ then $\tilde{y}_{e_2} = \tilde{z}_{e_3}$.

(ii) **If** $\tilde{\sigma}(\tilde{x}^n_{\lambda_n}, \tilde{y}^n_{\lambda_n}) \leq \tilde{a}^n_{\lambda_n}$ and $\tilde{\sigma}(\tilde{x}^n_{\lambda_n}, \tilde{z}_{e_3}) \leq \tilde{b}^n_{\lambda_n}$ for any $n \in \mathbb{N}$ then $\{\tilde{y}^n_{\lambda_n}\}$ converges to $\tilde{z}_{e_3}$.

(iii) **If** $\tilde{\sigma}(\tilde{x}^n_{\lambda_n}, \tilde{x}^m_{\lambda_m}) \leq \tilde{a}^n_{\lambda_n}$ for any $n, m \in \mathbb{N}$ with $m > n$, then $\{\tilde{x}^n_{\lambda_n}\}$ is a Cauchy sequence.

(iv) **If** $\tilde{\sigma}(\tilde{y}_{e_2}, \tilde{x}^n_{\lambda_n}) \leq \tilde{a}^n_{\lambda_n}$ for any $n \in \mathbb{N}$, then $\{\tilde{x}^n_{\lambda_n}\}$ is a Cauchy sequence.

Definition 2.17: Let $(\tilde{X},\tilde{d},E)$ be a soft metric space**.** A finite sequence $\{\tilde{x}^0_{\lambda_0},\tilde{x}^1_{\lambda_1},\dots,\tilde{x}^n_{\lambda_n}\}$ of points of $\tilde{X}$ is called an ϵ –chain joining $\tilde{x}^0_{\lambda_0}$ and $\tilde{x}^n_{\lambda_n}$ if

$$\tilde{d}\left(\tilde{x}^{i-1}_{\lambda_{i-1}},\tilde{x}^i_{\lambda_i}\right)<\epsilon \text{ for each } \epsilon>0, i=1,2,3,\dots,n.$$

Definition 2.18: A soft metric space $(\tilde{X},\tilde{d},E)$ is said to be an ϵ –chainable if for pair $(\tilde{x}_{e_1},\tilde{y}_{e_2})$ of its points the exists an ϵ –chain joining $\tilde{x}_{e_1}$ and $\tilde{y}_{e_2}$.

Every connected metric space is ϵ –chainable but the converse is not always true. However, for compact spaces both are equivalent. The following result was proved in (Takahashi, W., 1996),

Lemma 2.19: Let $\epsilon\in(0,+\infty)$ and let $(\tilde{X},\tilde{d},E)$ be an ϵ −chainable metric space. Then the function $\tilde{\sigma}:\tilde{X}\times\tilde{X}\to[0,+\infty)$ defined by

$\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu) = \inf[\sum_{i=1}^n \tilde{d}(\tilde{x}^{i-1}_{\lambda_{i-1}},\tilde{x}^i_{\lambda_i}) / \{\tilde{x}^0_{\lambda_0},\tilde{x}^1_{\lambda_1},\dots,\tilde{x}^n_{\lambda_n}\}$ *is an* ϵ –chain joining $\tilde{x}_\lambda$ and $\tilde{y}_\mu$] is a ω −distance on $\tilde{X}$.

Definition 2.21: Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $\tilde{\sigma}$ be a ω −distance on $\tilde{X}$. We denote by $\mathcal{F}$ the family of functions $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)$ satisfying the following condition:

I. $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)=\tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu)\right)$, i.e., $\tilde{\alpha}$ is dependent on the ω −distance $\tilde{\sigma}$ on $\tilde{X}$.
II. $o\le\tilde{\alpha}(\tilde{\sigma})<1$ for every $\tilde{\sigma}>0$.
III. $\tilde{\alpha}(\tilde{\sigma})$ is monotonically decreasing function of $\tilde{\sigma}$.

Now we introduce the following definition.

Definition 2.21: Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $\tilde{\sigma}$ be a ω −distance on $\tilde{X}$. A mapping

$(f,\varphi):\tilde{X}\to\tilde{X}$ is called a ω −Rakotch contraction if there exists a function $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\in\mathcal{F}$ such that

$$\tilde{\sigma}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right)\le\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu) \quad \text{for all } \tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X}.$$

3 MAIN RESULTS

Theorem 3.1: Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and $\tilde{\sigma}$ be an ω-distance on $\tilde{X}$. Suppose the soft mapping $(f,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be an ω-Rakotch contraction there exists a mapping $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\in\mathcal{F}$ such that:

$$\tilde{\sigma}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{2}\end{array}\right\} \quad \ldots \textbf{(3.1.1)}$$

For each $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}$,. Then (f,φ) has a unique fixed point in $\tilde{X}$, *i. e.* there exist $\tilde{x}_\lambda^* \in \tilde{X}$ such that $(f,\varphi)\tilde{x}_\lambda^* = \tilde{x}_\lambda^*$ and moreover hold $\tilde{\sigma}(\tilde{x}_\lambda^*,\tilde{x}_\lambda^*) = 0$.

Proof: Let $\tilde{x}_\lambda^0$ be any soft point in $SP(X)$.

Set $\tilde{x}_{\lambda_1}^1 = (f,\varphi)(\tilde{x}_\lambda^0) = \left(f(\tilde{x}_\lambda^0)\right)_{\varphi(\lambda)}$

$$\tilde{x}_{\lambda_2}^2 = (f,\varphi)(\tilde{x}_{\lambda_1}^1) = \left(f^2(\tilde{x}_\lambda^0)\right)_{\varphi^2(\lambda)}$$

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$$\tilde{x}_{\lambda_{n+1}}^{n+1} = (f,\varphi)(\tilde{x}_{\lambda_n}^n) = \left(f^{n+1}(\tilde{x}_\lambda^0)\right)_{\varphi^{n+1}(\lambda)}, - -$$

From (3.1.1) we have

$$\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}) = \tilde{\sigma}\left((f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1}),(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right)$$

$$\leq \tilde{\alpha}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n),\tilde{\sigma}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1})\right),\tilde{\sigma}\left(\tilde{x}_{\lambda_n}^n,(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right),\\ \frac{\tilde{\sigma}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1})\right)+\tilde{\sigma}\left(\tilde{x}_{\lambda_n}^n,(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right)}{2}\end{array}\right\}$$

$$\leq \tilde{\alpha}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n),\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n),\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}),\\ \frac{\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)+\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})}{2}\end{array}\right\}$$

$$\leq \tilde{\alpha}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\max\left\{\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n),\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}),\frac{\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)+\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})}{2}\right\}$$

Since $\tilde{\alpha}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n) \leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\right)$

Then we have

$$\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\right)\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n),\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}),\\ \frac{\tilde{\sigma}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)+\tilde{\sigma}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})}{2}\end{array}\right\} \quad \ldots(3.1.2)$$

Now the following two cases are arise:

Case I: If $\max\{\tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n}), \tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})\} = \tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})$ then from (3.1.2) we have

$$\tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) \leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n})\right)\tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})$$

This is contraction.

Case II: If $\max\{\tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n}), \tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})\} = \tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n})$ then from (3.1.2) we have

$$\tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) \leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n})\right)\tilde{\sigma}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n}) \leq \cdots$$

$$\leq \cdots \ldots \ldots \ldots \ldots \ldots \leq \prod_{k=0}^{n-1} \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^{k}_{\lambda_k}, \tilde{x}^{k+1}_{\lambda_{k+1}})\right)\tilde{\sigma}(\tilde{x}^{0}_{\lambda_0}, (f,\varphi)\tilde{x}^{0}_{\lambda_0})$$

and

$$\tilde{\sigma}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n}) = \tilde{\sigma}\left((f,\varphi)(\tilde{x}^{n}_{\lambda_n}), (f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}})\right)$$

$$\leq \tilde{\alpha}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n-1}_{\lambda_{n-1}}) \max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n-1}_{\lambda_{n-1}}), \tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n}, (f,\varphi)(\tilde{x}^{n}_{\lambda_n})\right), \tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}})\right), \\ \frac{\tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n}, (f,\varphi)(\tilde{x}^{n}_{\lambda_n})\right) + \tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}})\right)}{2}\end{matrix}\right\}$$

$$\leq \cdots \ldots \ldots \ldots \ldots \ldots \leq \prod_{k=0}^{n-1} \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^{k+1}_{\lambda_{k+1}}, \tilde{x}^{k}_{\lambda_k})\right)\tilde{\sigma}\left((f,\varphi)(\tilde{x}^{0}_{\lambda_0}), \tilde{x}^{0}_{\lambda_0}\right)$$

It follows that

$$\begin{cases}\tilde{\sigma}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) < \tilde{\sigma}\left(\tilde{x}^{0}_{\lambda_0}, (f,\varphi)(\tilde{x}^{0}_{\lambda_0})\right) \\ and \\ \tilde{\sigma}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n}) < \tilde{\sigma}\left((f,\varphi)(\tilde{x}^{0}_{\lambda_0}), \tilde{x}^{0}_{\lambda_0}\right)\end{cases} \quad \text{for all } n = 1,2,3,\ldots$$

Now we prove that $\tilde{\sigma}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{n}_{\lambda_n}) \leq C$ for some $C > 0$ and $n = 1,2,3,\ldots$

In fact, $\tilde{\sigma}(\tilde{x}^{1}_{\lambda_1}, \tilde{x}^{n+1}_{\lambda_{n+1}}) = \tilde{\sigma}\left((f,\varphi)(\tilde{x}^{0}_{\lambda_0}), (f,\varphi)(\tilde{x}^{n}_{\lambda_n})\right)$

$$\leq \tilde{\alpha}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{n}_{\lambda_n}) \max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{n}_{\lambda_n}), \tilde{\sigma}\left(\tilde{x}^{0}_{\lambda_0}, (f,\varphi)(\tilde{x}^{0}_{\lambda_0})\right), \tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n}, (f,\varphi)(\tilde{x}^{n}_{\lambda_n})\right), \\ \frac{\tilde{\sigma}\left(\tilde{x}^{0}_{\lambda_0}, (f,\varphi)(\tilde{x}^{0}_{\lambda_0})\right) + \tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n}, (f,\varphi)(\tilde{x}^{n}_{\lambda_n})\right)}{2}\end{matrix}\right\}$$

$$\leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{n}_{\lambda_n})\right)\tilde{\sigma}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{n}_{\lambda_n})$$

and by the triangle inequality

$$\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n}) \leq \tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^1_{\lambda_1}) + \tilde{\sigma}(\tilde{x}^1_{\lambda_1},\tilde{x}^{n+1}_{\lambda_{n+1}}) + \tilde{\sigma}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^n_{\lambda_n})$$

$$\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n}) \leq \tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^1_{\lambda_1}) + \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n})\right)\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n}) + \tilde{\sigma}\left((f,\varphi)(\tilde{x}^0_{\lambda_0}),\tilde{x}^0_{\lambda_0}\right)$$

$$\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n}) < \frac{\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^0_{\lambda_0})\right)+\tilde{\sigma}\left((f,\varphi)(\tilde{x}^0_{\lambda_0}),\tilde{x}^0_{\lambda_0}\right)}{1-\tilde{\alpha}\left(\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^n_{\lambda_n})\right)\right)}$$

Now if $\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^n_{\lambda_n})\right) \geq \tilde{z}_0$ for a given $\tilde{z}_0 > 0$, then by the monotonicity of $\tilde{\alpha}(\tilde{\sigma})$ it follows that

$$\tilde{\alpha}\left(\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^n_{\lambda_n})\right)\right) \leq \tilde{\alpha}(\tilde{z}_0)$$

and therefore

$$\tilde{\sigma}(\tilde{x}^0_{\lambda_0},\tilde{x}^n_{\lambda_n}) < \frac{\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^0_{\lambda_0})\right)+\tilde{\sigma}\left((f,\varphi)(\tilde{x}^0_{\lambda_0}),\tilde{x}^0_{\lambda_0}\right)}{1-\tilde{\alpha}(\tilde{z}_0)} = C$$

On the other hand if $\tilde{\sigma}(\tilde{x}^k_{\lambda_k},\tilde{x}^{k+1}_{\lambda_{k+1}}) \geq \epsilon_0, k = 0,1,2,\dots(n-1)$ for any $\epsilon_0 > 0$ then by monotonicity of $\tilde{\alpha}$ it follow that

$$\tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^k_{\lambda_k},\tilde{x}^{k+1}_{\lambda_{k+1}})\right) \leq \tilde{\alpha}(\epsilon_0)$$

and hence $\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}) \leq [\tilde{\alpha}(\epsilon_0)]^n\tilde{\sigma}\left((f,\varphi)(\tilde{x}^0_{\lambda_0}),\tilde{x}^0_{\lambda_0}\right)$

But $0 < \tilde{\alpha}(\epsilon_0) < 1$ by **lemma 2.16** we have

$$\lim_{n\to\infty}\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}) = 0$$

We shall show that $\{\tilde{x}^n_{\lambda_n}\}$ is a Cauchy sequence in $(\tilde{X},\tilde{d},E)$. for $m > 0$

$$\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+m}_{\lambda_{n+m}}) \leq \prod_{k=0}^{n-1}\tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}^k_{\lambda_k},\tilde{x}^{k+m}_{\lambda_{k+m}})\right)\tilde{\sigma}(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^0_{\lambda_0}))$$

If $\tilde{\sigma}(\tilde{x}^k_{\lambda_k},\tilde{x}^{k+m}_{\lambda_{k+m}}) \geq \epsilon_0$ for any given $\epsilon_0 > 0$ and $k = 0,1,2,\dots(n-1)$ then

$$\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+m}_{\lambda_{n+m}}) \leq [\tilde{\alpha}(\epsilon_0)]^n\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^0_{\lambda_0})\right) \to 0 \text{ as } n \to \infty$$

and by **lemma 2.16** we have that $\{\tilde{x}^n_{\lambda_n}\}$ is a Cauchy sequence. Since $(\tilde{X},\tilde{d},E)$ is complete, $\{\tilde{x}^n_{\lambda_n}\}$ converges to some $\tilde{x}^*_{\lambda}$. Since $\tilde{x}^m_{\lambda_m} \to \tilde{x}^*_{\lambda}$ and $\tilde{\sigma}(\tilde{x}^n_{\lambda_n},.)$ is lower semi continuous,

$$\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_{\lambda}) \leq \lim_{n\to\infty}\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^m_{\lambda_m}) \leq \tilde{\alpha}^n(\epsilon_0)\tilde{\sigma}\left(\tilde{x}^0_{\lambda_0},(f,\varphi)(\tilde{x}^0_{\lambda_0})\right)$$

So $\lim_{n\to\infty}\tilde{\sigma}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_{\lambda}) = 0.$

On the other hand,

$$\tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)=\tilde{\sigma}\left((f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}}),(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)$$

$$\leq \tilde{\alpha}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{*}_{\lambda}\right)\max\left\{\begin{array}{c}\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{*}_{\lambda}\right),\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},(f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)\right),\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right),\\ \frac{\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},(f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)\right)+\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)}{2}\end{array}\right\}$$

$$\leq \tilde{\alpha}\left(\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{*}_{\lambda}\right)\right)\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{*}_{\lambda}\right)$$

$$<\tilde{\sigma}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{*}_{\lambda}\right)$$

So $\lim_{n\to\infty}\tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)=0.$

and by **lemma 2.16** we have $(f,\varphi)(\tilde{x}^{*}_{\lambda})=\tilde{x}^{*}_{\lambda}$.

Now $\tilde{\sigma}(\tilde{x}^{*}_{\lambda},\tilde{x}^{*}_{\lambda})=\tilde{\sigma}\left((f,\varphi)(\tilde{x}^{*}_{\lambda}),(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)$

$$\leq \tilde{\alpha}(\tilde{x}^{*}_{\lambda},\tilde{x}^{*}_{\lambda})\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}^{*}_{\lambda},\tilde{x}^{*}_{\lambda}),\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right),\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right),\\ \frac{\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)+\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)}{2}\end{array}\right\}$$

$$<\ \tilde{\sigma}(\tilde{x}^{*}_{\lambda},\tilde{x}^{*}_{\lambda})$$

So $\quad\tilde{\sigma}\left(\tilde{x}^{n}_{\lambda_n},\tilde{x}^{n}_{\lambda_n}\right)=0.$

If $\tilde{y}_{\mu}=(f,\varphi)\left(\tilde{y}_{\mu}\right)$ then

$$\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},\tilde{y}_{\mu}\right)=\tilde{\sigma}\left((f,\varphi)(\tilde{x}^{*}_{\lambda}),(f,\varphi)\left(\tilde{y}_{\mu}\right)\right)$$

$$\leq \tilde{\alpha}\left(\tilde{x}^{*}_{\lambda},\tilde{y}_{\mu}\right)\max\left\{\begin{array}{c}\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},\tilde{y}_{\mu}\right),\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right),\tilde{\sigma}\left(\tilde{y}_{\mu},(f,\varphi)\left(\tilde{y}_{\mu}\right)\right),\\ \frac{\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},(f,\varphi)(\tilde{x}^{*}_{\lambda})\right)+\tilde{\sigma}\left(\tilde{y}_{\mu},(f,\varphi)\left(\tilde{y}_{\mu}\right)\right)}{2}\end{array}\right\}$$

$$<\tilde{\alpha}\left(\tilde{x}^{*}_{\lambda},\tilde{y}_{\mu}\right)$$

and $\tilde{\sigma}\left(\tilde{x}^{*}_{\lambda},\tilde{y}_{\mu}\right)=0$. So by **lemma 2.16** we have $\tilde{x}^{*}_{\lambda}=\tilde{y}_{\mu}$.

Theorem 3.2: Let $\left(\tilde{X},\tilde{d},E\right)$ be a complete soft metric space and $\tilde{\sigma}$ be an ω-distance on $\tilde{X}$. Suppose the soft mapping $(f,\varphi):\left(\tilde{X},\tilde{d},E\right)\to\left(\tilde{X},\tilde{d},E\right)$ such that for some integer $\mathrm{m}\in\mathbb{N}$, $(f,\varphi)^{\mathrm{m}}$ be an ω-Rakotch contraction. Then (f,φ) has a unique fixed point, i.e. there exists $\tilde{x}^{*}_{\lambda}\in\tilde{X}$ such that $(f,\varphi)\tilde{x}^{*}_{\lambda}=\tilde{x}^{*}_{\lambda}$ and moreover holds $\tilde{\sigma}(\tilde{x}^{*}_{\lambda},\tilde{x}^{*}_{\lambda})=0$.

Proof: Since for some m $\in \mathbb{N}$, $(f,\varphi)^{m}$ is a ω-Rakotch contraction, then there exists a function $\tilde{\alpha}(\tilde{x}_\lambda, \tilde{y}_\mu) \in \mathcal{F}$ such that:

$$\tilde{\sigma}\left((f,\varphi)^m(\tilde{x}_\lambda),(f,\varphi)^m(\tilde{y}_\mu)\right)$$

$$\leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda))+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right)}{2}\end{matrix}\right\} \quad \cdots$$

(3.2.1)

For each $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$. Hence by **Theorem 3.1** there exist a unique $\tilde{x}_\lambda^* \in \tilde{X}$ such that $\tilde{x}_\lambda^* = (f,\varphi)^m(\tilde{x}_\lambda^*)$ for m $\in \mathbb{N}$ and $(f,\varphi)(\tilde{x}_\lambda^*) = (f,\varphi)\left((f,\varphi)^m(\tilde{x}_\lambda^*)\right) = (f,\varphi)^m\left((f,\varphi)(\tilde{x}_\lambda^*)\right)$ it follows that $\tilde{x}_\lambda^* = (f,\varphi)(\tilde{x}_\lambda^*)$.

Theorem 3.3: Let $\tilde{X}$ be a non empty set, $\tilde{d}$ and $\tilde{\rho}$ two metrics on $\tilde{X}$, $\tilde{\sigma}$ and $\tilde{\tau}$ their respective ω-distance on $\tilde{X}$ and $(f,\varphi) : (\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$ be a mapping. Suppose that:

a. $\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu) \leq \tilde{\tau}(\tilde{x}_\lambda,\tilde{y}_\mu)$ for all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$,
b. $(\tilde{X},\tilde{d},E)$ is a complete soft metric space,
c. $(f,\varphi) : (\tilde{X},\tilde{\rho}) \to (\tilde{X},\tilde{\rho})$ is a ω-Rakotch contraction, i.e., there exists $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu) \in \mathcal{F}$ such that

$$\tilde{\tau}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq$$

$$\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{matrix}\tilde{\tau}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\tau}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)),\tilde{\tau}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right),\\ \frac{\tilde{\tau}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{\tau}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{2}\end{matrix}\right\}$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$.

Then there exist exists $\tilde{x}_\lambda^* \in \tilde{X}$ such that $(f,\varphi)\tilde{x}_\lambda^* = \tilde{x}_\lambda^*$ and moreover holds $\tilde{\sigma}(\tilde{x}_\lambda^*,\tilde{x}_\lambda^*) = 0$.

Proof: Let $\tilde{x}_{\lambda_0}^0$ be any soft point in $SP(X)$ and $\tilde{x}_{\lambda_n}^n = (f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1}) = \left(f^n(\tilde{x}_\lambda^0)\right)_{\varphi^n(\lambda)}, n \in \mathbb{N}$.

From (c), that $\{\tilde{x}_{\lambda_n}^n\}$ is a Cauchy sequence in $(\tilde{X},\tilde{\rho})$. By (a) and **lemma 2.16**, $\{\tilde{x}_{\lambda_n}^n\}$ is a Cauchy sequence in $(\tilde{X},\tilde{d},E)$ and by (b) it converges. The rest of the proof is similar to **Theorem 3.1.**

Theorem 3.4: Let $\epsilon \in (0,\infty)$ be and let $(\tilde{X},\tilde{d},E)$ be a complete soft ϵ -chainable metric space. If $(f,\varphi) : (\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$ be a mapping satisfying the condition, $0 < \tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) < \epsilon$

Implies

$$\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{array}{c}\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)),\tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right),\\ \frac{\tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{2}\end{array}\right\} \quad \ldots(3.4.1)$$

For all $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}$, and $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu) \in \mathcal{F}$. Then (f,φ) has a unique fixed point $\tilde{x}_\lambda^* \in \tilde{X}$ such that $\tilde{x}_\lambda^* = (f,\varphi)(\tilde{x}_\lambda^*)$.

Proof: Since $(\tilde{X},\tilde{d},E)$ is soft ϵ -chainable for every $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}$ we define the function

$\tilde{\sigma}: \tilde{X} \times \tilde{X} \to [0,\infty)$ as follows:

$\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu) = \inf[\sum_{i=1}^{n}\tilde{d}(\tilde{x}_{\lambda_{i-1}}^{i-1},\tilde{x}_{\lambda_i}^{i}) / \{\tilde{x}_{\lambda_0}^{0},\tilde{x}_{\lambda_1}^{1},\ldots,\tilde{x}_{\lambda_n}^{n}\}$ *is an* ϵ –chain joining $\tilde{x}_\lambda$ and $\tilde{y}_\mu$]

From **lemma 2.16** $\tilde{\sigma}$ is a ω –distance on $\tilde{X}$ satisfying $\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) \leq \tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu)$. Given $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}$ and any ϵ –chain $\{\tilde{x}_{\lambda_0}^{0},\tilde{x}_{\lambda_1}^{1},\ldots,\tilde{x}_{\lambda_n}^{n}\}$ with $\tilde{x}_{\lambda_0}^{0} = \tilde{x}_\lambda$ and $\tilde{x}_{\lambda_n}^{n} = \tilde{y}_\mu$ we have for $i = 1,2,\ldots,n$.

$$\tilde{d}\left((f,\varphi)(\tilde{x}_{\lambda_{i-1}}^{i-1}),(f,\varphi)(\tilde{x}_{\lambda_i}^{i})\right) \leq \tilde{\alpha}(\tilde{x}_{\lambda_{i-1}}^{i-1},\tilde{x}_{\lambda_i}^{i})\max\left\{\begin{array}{c}\tilde{d}(\tilde{x}_{\lambda_{i-1}}^{i-1},\tilde{x}_{\lambda_i}^{i}),\tilde{d}\left(\tilde{x}_{\lambda_{i-1}}^{i-1},(f,\varphi)(\tilde{x}_{\lambda_{i-1}}^{i-1})\right),\\ \tilde{d}\left(\tilde{x}_{\lambda_i}^{i},(f,\varphi)(\tilde{x}_{\lambda_i}^{i})\right),\\ \frac{\tilde{d}\left(\tilde{x}_{\lambda_{i-1}}^{i-1},(f,\varphi)(\tilde{x}_{\lambda_{i-1}}^{i-1})\right)+\tilde{d}\left(\tilde{x}_{\lambda_i}^{i},(f,\varphi)(\tilde{x}_{\lambda_i}^{i})\right)}{2}\end{array}\right\}$$

$$< \tilde{\alpha}(\epsilon)\epsilon$$

$$< \epsilon$$

Hence $(f,\varphi)(\tilde{x}_{\lambda_0}^{0}),\ldots,(f,\varphi)(\tilde{x}_{\lambda_n}^{n})$ is an ϵ –chain joining $(f,\varphi)(\tilde{x}_\lambda)$ and $(f,\varphi)(\tilde{y}_\mu)$ and

$$\tilde{\sigma}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq \sum_{i=1}^{n}\tilde{d}\left((f,\varphi)(\tilde{x}_{\lambda_{i-1}}^{i-1}),(f,\varphi)(\tilde{x}_{\lambda_i}^{i})\right)$$

$$\leq \sum_{i=1}^{n}\tilde{\alpha}\left(\tilde{d}(\tilde{x}_{\lambda_{i-1}}^{i-1},\tilde{x}_{\lambda_i}^{i})\tilde{d}(\tilde{x}_{\lambda_{i-1}}^{i-1},\tilde{x}_{\lambda_i}^{i})\right)$$

Since $\{\tilde{x}_{\lambda_0}^{0},\tilde{x}_{\lambda_1}^{1},\ldots,\tilde{x}_{\lambda_n}^{n}\}$ is an arbitrary ϵ –chain we have

$$\tilde{\sigma}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{array}{c}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{2}\end{array}\right\}$$

Hence by **Theorem 3.1**, (f,φ) has a unique fixed point $\tilde{x}_\lambda^* \in \tilde{X}, \tilde{x}_\lambda^* = (f,\varphi)(\tilde{x}_\lambda^*)$.

Theorem 3.5: Let $\epsilon \in (0,\infty)$ be and let $(\tilde{X},\tilde{d},E)$ be a complete soft ϵ -chainable metric space. If $(f,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a mapping satisfying the condition, $\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)<\epsilon$ Implies

$$\tilde{\sigma}\left((f,\varphi)^m(\tilde{x}_\lambda),(f,\varphi)^m(\tilde{y}_\mu)\right)$$

$$\leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda))+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right)}{2}\end{matrix}\right\} \quad \ldots(3.5.1)$$

For every $\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X}$, for $m\in\mathbb{N}$ and $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\in\mathcal{F}$, then (f,φ) has a unique fixed point in $\tilde{X}$.

Proof: As in **Theorem 3.4** we define $\tilde{\sigma}$ as follows:

$$\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu)=$$
$$\inf\left[\sum_{i=1}^{n}\tilde{d}((\tilde{x}_\lambda)_{i-1},(\tilde{x}_\lambda)_i)/\{\tilde{x}_{\lambda_0},\tilde{x}_{\lambda_1},\ldots,\tilde{x}_{\lambda_n}\}\ is\ an\ \epsilon-\text{chain joining } \tilde{x}_\lambda \text{ and } \tilde{y}_\mu\right]$$

By **lemma 2.16** $\tilde{\sigma}$ is a $\omega-$ distance on $\tilde{X}$ satisfying $\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)\leq\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu)$. as in **Theorem 3.3** we have that $(f,\varphi)^m$ satisfies the condition

$$\tilde{\sigma}\left((f,\varphi)^m(\tilde{x}_\lambda),(f,\varphi)^m(\tilde{y}_\mu)\right)$$

$$\leq \tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)^m(\tilde{x}_\lambda))+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)^m(\tilde{y}_\mu)\right)}{2}\end{matrix}\right\} \quad \ldots \textbf{(3.5.1)}$$

For all $\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X}, m\in\mathbb{N}$ and therefore by **Theorem 3.4** we conclude that $(f,\varphi)^m$ has a unique $\tilde{x}_\lambda^*\in\tilde{X}$ such that $\tilde{x}_\lambda^*=(f,\varphi)^m\,\tilde{x}_\lambda^*$. It follows that (f,φ) has a unique fixed point $\tilde{x}_\lambda^*$ and moreover $\tilde{\sigma}(\tilde{x}_\lambda^*,\tilde{x}_\lambda^*)=0$.

Finally, using the ideas of M. Telci-K. Tas [18] we obtain a generalization of Rakotch's theorem on non-complete metric spaces.

Theorem 3.6: Let $(\tilde{X},\tilde{d},E)$ be a non-complete soft metric space and $\tilde{\sigma}$ be an ω-distance on $\tilde{X}$. Suppose the soft mapping $(f,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a ω-Rakotch contraction and suppose that there exists a point $\tilde{x}_\lambda^*\in\tilde{X}$ such that

$$\theta(\tilde{x}_\lambda^*)=\inf\{\theta(\tilde{x}_\lambda)/\tilde{x}_\lambda\in\tilde{X}\}$$

where $\theta(\tilde{x}_\lambda)=\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)\tilde{x}_\lambda)$ for all $\tilde{x}_\lambda\in\tilde{X}$. Then $\tilde{x}_\lambda^*$ is a fixed point of (f,φ).

Proof: Suppose that $\tilde{x}_\lambda^*\neq(f,\varphi)\tilde{x}_\lambda^*$, Since otherwise $\tilde{x}_\lambda^*$ would be a fixed point of (f,φ). Now (f,φ) is a ω-Rakotch contraction, then there exists a function $\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\in\mathcal{F}$ such that:

$$\tilde{\sigma}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq$$

$$\tilde{\alpha}(\tilde{x}_\lambda,\tilde{y}_\mu)\max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)),\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right),\\ \frac{\tilde{\sigma}(\tilde{x}_\lambda,(f,\varphi)\tilde{x}_\lambda)+\tilde{\sigma}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{2}\end{matrix}\right\}$$

For all $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}$. and so

$$\theta((f,\varphi)\tilde{x}_\lambda^*) = \tilde{\sigma}((f,\varphi)\tilde{x}_\lambda^*,(f,\varphi)^2\tilde{x}_\lambda^*)$$

$$\leq$$

$$\tilde{\alpha}(\tilde{x}_\lambda^*,(f,\varphi)\tilde{x}_\lambda^*)\max\left\{\begin{matrix}\tilde{\sigma}(\tilde{x}_\lambda^*,(f,\varphi)\tilde{x}_\lambda^*),\tilde{\sigma}(\tilde{x}_\lambda^*,(f,\varphi)(\tilde{x}_\lambda^*)),\tilde{\sigma}((f,\varphi)\tilde{x}_\lambda^*,(f,\varphi)^2(\tilde{x}_\lambda^*)),\\ \frac{\tilde{\sigma}\left(\tilde{x}_\lambda^*,(f,\varphi)(\tilde{x}_\lambda^*)\right)+\tilde{\sigma}\left((f,\varphi)\tilde{x}_\lambda^*,(f,\varphi)^2(\tilde{x}_\lambda^*)\right)}{2}\end{matrix}\right\}$$

$$\leq \tilde{\alpha}\left(\tilde{\sigma}(\tilde{x}_\lambda^*,(f,\varphi)\tilde{x}_\lambda^*)\right)\tilde{\sigma}(\tilde{x}_\lambda^*,(f,\varphi)\tilde{x}_\lambda^*)$$

$$< \tilde{\sigma}(\tilde{x}_\lambda^*,(f,\varphi)\tilde{x}_\lambda^*) = \theta(\tilde{x}_\lambda^*)$$

This is a contradiction. So result is proved.

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