Minimization of Transportation Cost Problems

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Abstract

This paper present seven initial methods of solving transportation problems. These methods were compared on a balanced transportation model of Dangote's Cement Factories and the result shows that the Vogel's method yields the best starting solution and gives an initial solution as the optimal with zero number of iteration. The North-West Corner Method although fast in obtaining the initial solution is not very efficient and accurate enough in finding initial solution as it pays no attention to the unit cost particularly as the solution is far from the optional. The objective value for the initial basic feasible solutions obtained from Russel's and Row-Minimum Method are in perfect agreement.

Keywords: Linear Programming, Transportation, Initial Solution, Minimization.

1. Introduction

A transportation problem is one of the sub-class of a linear programming problems where the objective is to transport various quantities of single homogenous product that are initially stored at various origin to different destinations in such a way that the total transportation cost is minimized (Taha 1995).

Whenever there is a physical movement of goods from the point of manufacturing to final consumer through a variety of channels of distribution, there is need to minimize the cost of transportation so as to increase the profit on sales. Transportation problem arises in all such cases and they aim at providing assistance to the top management in ascertaining how many units of a particular product should be transported from each supply origin to each demand destination so that the total prevailing demand for the company product is satisfied and at the same time transportation cost is minimized (Sharma 1997).

Transportation problem can be generalized into transshipment problem where shipment is possible from origin i to destination j. Transshipment can be formulated as a transportation problem with an increase number of origins and destinations (Kapoor 1997).

The general method of solution for a transportation problems is first to establish the problem matrix with each origin as a row and each destination as a column. Also an initial solution must be obtained that satisfied all constraints on the matrix model. A matrix with m origins and n destinations will have $m+n-1$ quantities of goods shipped from origin i to destination j. A test is performed on the matrix to see if there exist solutions that will provide a lower total cost to the company. If one exists, a new solution is found and this new solution is in turn tested for a possible better solution. Each solution which is tested and the new solution formed is called an

iteration. On a large scale, thousand of iterations may be required before an optimal solution can be obtained (Ocheche et al 2016).

2. Formulation of the Transportation Problem

Given m origin and n destinations, the transportation problem can be formulated as the following programming model

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} \tag{2.1}$$

$$\text{Subject to } \sum_{l=1}^{n} X_{ij} = a_i \text{ for } i = 1,2,\ldots,m \tag{2.2}$$

$$\sum_{i=1}^{m} X_{ij} = bj \text{ for } j = 1,2,\ldots,n \tag{2.3}$$

$$\text{(2.4)}$$

$$\text{and } X_{ij} \geq o \text{ for all } i \text{ and } j$$

Where X_{ij} is the amount of units shipped from origin i to destination j and C_{ij} is the cost of shipping one unit from origin i to destination j.

The amount of supply at origin is a_i and the amount at destination is b_j. The objective is to determine the unknown X_{ij} that will minimized the total transportation cost while satisfying all the supply and demand constraints (Ocheche et al 2016).

2.1 Balanced Transportation Problem

If the sum of the supplies of all the sources is equal to the sum of the demand of all the destinations, then the problem is termed as a Balanced Transportation problem. Here

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{2.5}$$

2.2 Unbalanced Transportation Problem

If the sum of the supplies of all the sources is not equal to the sum of the demands of all the destinations then the problem is termed as unbalanced transportation problems. Here

$$\text{(2.6)}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

An unbalanced transportation problem can be modified to a balanced one by introducing a dummy sink if

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \qquad \text{And a dummy source if} \qquad \sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

The inflow from the source to a dummy sink represents the surplus at the source. Similarly the flow from a dummy source to a sink presents the unfulfilled demand at the sink. The cost of transporting a unit from a dummy source to a dummy sink is assumed to be zero. The resulting problem is now balanced and be solved.

3. Solving Transportation Problem of Dangote's Cement Industries

Dangote group of company has a number of cement production plants. This research is focused on three of these plants; Dangote Cement Factory located at Gboko, Objana and Ibesse and the company's depots and wholesale outlets in cities of Lafia, Abuja, Jos and Minna.

Dangote Transport Company ships truckloads of cement from three factories to four warehouses. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in the transportation model in Table 1.

The model seeks the minimum-cost shipping schedule between the factories and the warehouses. This is equivalent to determining the quantity X_{ij} shifted from factory i to warehouse j (i=1,2,3: j=1,2,3,4).

Table1. Supply and demand with unit transportation cost

	Destinations / Sources	Warehouse				Supply S_i
		Lafia	Abuja	Jos	Minna	
Factories	Ibesse	30	70	60	40	24
	Gboko	20	40	30	20	38
	Obajana	40	30	80	50	76
Demand	d_j	18	43	52	25	138

4. Methods for Finding Initial Solution Used in the Research

Seven initial methods were used to solve transportation problems from Dangote Cement Factories and a brief explanation on each of the methods is as follows:

4.1 North-West Corner Method (NWC)

This method starts at the northwest corner cell or upper left corner cell of a transportation table. It allocates as much as possible to the selected cell and adjust the associated amount of supply and demand by subtracting the allocated amount. This results in crossing out a row or column.

If exactly one row or column is left uncrossed out, we stop, otherwise, we move to the cell which is to the right if a column has just been crossed out or below if a row has been crossed out (Taghrid et al 2009). We denote this method as M 1 for the purpose of this research.

4.2 Least Cost Method

The Least Cost Method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the supply and demand are adjusted accordingly. We next look for the uncrossed out cell with smallest unit cost and repeat the process until exactly one row or column is left uncrossed (Taghrid et al 2009) We denote this method as M2 for the research purpose.

4.3 Vogel's Approximation Method (Vam)

Vogel's Approximation Method, denoted as M3 in this research, is an improved version of the least cost method that generally has been obtained to produce better starting solutions. It has been claimed to be the best initial method (John et al 2001). The complete steps in implementing the methods are as follows:

Step 1:   For each row (column) determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step II: Identify the row or column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the identified row or column, adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, one of the two is crossed out and the remaining row (column) is assigned zero supply (demand).

Step III: (a) If exactly one row or column with zero supply or demand (column) remains uncrossed out stop.

(b) If one row (column) with positive supply remains uncrossed out, determine the basic variables in the row (column) by the least cost method. Stop.

(c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least-cost method. Stop.

(d) Otherwise go to step (I) (Kapoor 1997)

4.4 Row-Column Minimum Method

Row-Column Minimum method is a combination of both row and column minimum methods. It is developed in this research with the intention of taking the advantages of the two methods. It starts with allocation in the first row and first column simultaneously choosing the lowest cost cells one after the other of the first row or column so that the capacities of the first supply and first demand are both satisfied. Next the satisfied row and column are both crossed out, and the amount of supply and demand are adjusted accordingly. The method next considers allocation in the second row and second column, choosing the lowest cost cells and repeating the process until exactly both the capacities of the columns and the rows are exhausted. This method is denoted as M4.

4.5 Column Minimum Method

Column Minimum Method denoted as M5 in the research, starts with first column and chooses the lowest cost cell of first column so that either the demand of the fist distribution centre is satisfied or the capacity of the ith supply is satisfied or both. Three eases arise as follows:

i.   If the demand of the first distribution centre is satisfied cross out the first column and move right to the second column.

ii.   If the capacity of ith supply is satisfied, cross out the ith row and reconsider the first column with the remaining demand.

iii. If the demands of the first distribution centre as well as the capacity of the ith supply are completely satisfied, cross out the column as well as the ith row and move right to the second column.

 Continue the process for the resulting reduced transportation table until the last column (John et al 2001).

4.6 Russell's Approximation Method

Russell's approximation method, denoted as M6, is a more recently proposed method that seems very promising. Computation takes a longer time because each cell is involved or considered. The complete steps followed in implementing the method are as follows:

Step I: For each source i remaining under consideration determine its u_i which is the largest cost (c_{ij}) still remaining in that row.

Step II: For each destination column j remaining under consideration, determine its v_j which is the largest unit cost (c_{ij}) still remaining in that column.

Step III: For each variable x_{ij} not previously selected in these rows and columns, calculate $\Delta_{ij} = C_{ij} - U_i - V_j$. Allocation is then made to the cell having the largest negative value of Δ_{ij}. At The allocation will result in crossing out a row or a column, according as to whether the supply in the row or the demand in the column is satisfied. The process is repeated until all demands and supplies are satisfied.

4.7 Row Minimum Method

denoted as M7 in the research, starts with the first row and chooses the lowest cost cell in the row so that either the capacity of the first supply is exhausted or the demand of the jth distribution centre is satisfied or both. Three cases arise as follows.

i.   If the capacity of the first supply is completely satisfied, cross out the first row and proceed to the second row.

ii. If the demand at j^{th} distribution centre is satisfied, cross out the jth column and reconsider the first row with the remaining capacity.

iii. If the capacity of the first supply as well as demand at j^{th} distribution centre are completely satisfied, cross out the row as well as the column and move down to the second row.

Continue this process for the resulting reduced transportation table units until the last row (John et al 2001) The above seven initial methods were applied to the Dangote Company in table 1 as follows:

Table 3. The Starting Solution using North West Corner Method (M_1)

Factories	Destinations / Sources	Warehouse				Factory Supply Si
		Lafia	Abuja	Jos	Minna	
	Ibesse	30	70	60	40	24
		18 →	6			
	Gboko	20	40	30	20	38
			37 →	1		
	Obajana	40	30	80	50	76
				51 →	25	
		18	43	52	25	138
Demand	(Regiments)					

The initial basic feasible solution is obtained as follows:

$$\text{Min Z=} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$Z= 30X_{11} + 70X_{12} + 40X_{22} + 30X_{23} + 80X_{33} + 50X_{34}$

$= ₦7,800$

Table 3. The Starting Solution using least cost Mothod (M_2)

Factories	Destinations / Sources	Warehouse				Factory Supply Si
		Lafia	Abuja	Jos	Minna	
	Ibesse	30	70	60	40	24
				19	5	
	Gboko	20	40	30	20	38
		18			20	
	Obajana	40	30	80	50	76
			43	33		
Demand	d_j	18	43	52	25	138

From the basic feasible solution, the total transportation

Cost $Z = \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} C_{ij} X_{ij}$ is obtained as follows

$Z = 60X_{13} + 40X_{14} + 20X_{21} + 20X_{24} + 30X_{32} + 80X_{33}$

$= ₦6,030$

Table 4. The starting Solution using Vogel's Method (M_3)

	Destinations⟍Sources	Warehouse				Factory Supply S_i	Row Penalty
		Lafia	Abuja	Jos	Minna		
Factories	Ibesse	30 10	70	60 14	40	24	10
	Gboko	20	40	30 38	20	38	10
	Obajana	40 8	30 43	80	50 25	76	10
Demand	d_j	18	43	52	25	138	
	Column Penalty	10	40	30	20		

From the initial basic feasible solution, is obtained as follows:

Cost $Z = \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} C_{ij} X_{ij}$

$Z = 30X_{11} + 60X_{13} + 30X_{23} + 40X_{31} + 30X_{32} + 50X_{34}$

$Z = ₦5,140$

Table 5. The Starting Solution Using Row-Column Minimum Method (M_4)

Sources \ Destinations	Warehouse				Factory Supply S_i
	Lafia	Abuja	Jos	Minna	
Ibesse	30 __18__	70	60	40 __6__	24
Gboko	20	40 __19__	30	20 __19__	38
Obajana	40	30 __24__	80 __52__	50	76
Demand d_j	18	43	52	25	

The Initial Basic Feasible Solution is obtained as follows:

$$\text{Cost } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$$Z = 30X_{11} + 40X_{14} + 40X_{22} + 20X_{24} + 30X_{32} + 80X_{33}$$

$$Z = ₦6,800$$

Table 6. The Starting Solution Using Column-Minimum Method (M_5)

Sources \ Destinations	Warehouse				Factory Supply S_i
	Lafia	Abuja	Jos	Minna	
Ibesse	30	70	60 __24__	40	24
Gboko	20 __18__	40	30 __20__	20	38
Obajana	40	30 __43__	80 __8__	50 __25__	76
Demand d_j	18	43	52	25	

The Initial Basic Feasible Solution is obtained as follows:

$$\text{Cost } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ii} X_{ii}$$

$Z = 60X_{13} + 20X_{21} + 30X_{13} + 30X_{32} + 80X_{33} + 50X_{34}$

$Z = ₦5,580$

Table 7. The Starting Solution Using Russell's Method (M_6)

Factories	Destinations / Sources	Warehouse				Factory Supply Si
		Lafia	Abuja	Jos	Minna	
	Ibesse	30 18	70	60	40 6	24
	Gboko	20	40	30 19	20 19	38
	Obajana	40	30 43	80 33	50	76
Demand	d_j	18	43	52	25	

The Initial Basic Feasible Solution is obtained as follows:

$$\text{Cost } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$\triangle_{ij} = C_{ij} - U_i - V_j$

$Z = 30X_{11} + 40X_{14} + 30X_{23} + 20X_{24} + 30X_{32} + 80X_{33}$

$Z = ₦5,660$

Table 8. The starting Solution Using Row-minimum Method (M_7)

	Destinations / Sources	Warehouse				Factory Supply S_i
		Lafia	Abuja	Jos	Minna	
Factories	Ibesse	30	70	60	40	24
		18			6	
	Gboko	20	40	30	20	38
				19	19	
	Obajana	40	30	80	50	76
			43	33		
Demand	d_j	300	43	52	25	

The Initial Basic Feasible Solution is obtained as follows:

$$\text{Cost } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$$Z = 30X_{11} + 40X_{14} + 30X_{23} + 20X_{24} + 30X_{32} + 80X_{33}$$

$Z = ₦5,660$

5.   Comparison between the Initial Methods

Table 9. Comparism of result between the initial Methods

Methods	Initial Feasible Solution	Optional	Number of Iterations
North West Corner Method (M1)	7800	5140	3
Least Cost Method (M2)	6030	5140	4
Vogel Method (M3)	5140	5140	0
Row-Column Minimum Method (M4)	6800	5140	4
Column Minimum Method (M5)	5580	5140	2
Russel Method (M6)	5660	5140	3
Row-Minimum Method (M7)	5660	5140	3

6. Discussion of the Result

The seven initial methods differ in the quality of the starting basic solution they produce in the sense that a better starting solutions yields a smaller objective value. The main virtue of the North-West Corner Method is that it is quick and easy, however it pays no attention to the unit cost usually the result is far from the optimal.   Vogel's Method yields the best starting solution and gives an initial solution as the optimal. Row-Column Minimum Method, even though sensitive to unit cost, is not very efficient and accurate enough in finding the initial feasible solution. It takes four numbers of iterations to give an optional solution. Column- Minimum Method takes two number of iterations to give an optimal solution but the initial solution is not as good as Vogel's Method.

Russel's and Row-Minimum methods yield the same initial solution but Russel's method differ from Row-Minimum method in the sense that it is slow in implementing it's step because computation takes a longer time. The objective function of the Least Cost Method is more than the Vogel's method and it takes four numbers of iterations to give an optimal solution.

7. Conclusion

The research has shown that the Vogel's (M3) method gives the best initial methods for a transportation problem; it provides a lower total cost for the company. Therefore production and marketing managers of Dangote's cement factories may adopt the method so as to maximized profit and also the designers of software of transportation problems may use the method in their packages.

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