

# Mixed Convection of Unsteady Nanofluids Flow Past A Vertical Plane With Entropy Generation

Maurine Wafula<sup>1\*</sup> Mathew Kinyanjui<sup>2</sup>

1. School of Mathematical Sciences, Pan African University, Institute for basic Sciences, Technology and Innovation , P.O box 62000-00200, Nairobi, Kenya
2. School of Mathematical Sciences, Jomo Kenyatta University of Agriculture and Technology, P.O box 62000-00200, Nairobi, Kenya

## Abstract

A fully developed mixed convection nanofluid flow past accelerating vertical plane in the presence of a uniform transverse magnetic field has been studied. Three different types of water-based Nanofluids containing Titanium (iv) oxide, Copper and aluminum (iii) oxide are taken into consideration. The governing equations are solved numerically by shooting technique coupled with Runge-Kutta-Fehlberg integration scheme. Effects of the pertinent parameters on the nanofluid temperature and velocity are shown in figures followed by a quantitative discussion. The expression for entropy generation number and the Bejan number are also obtained based on the profiles. It is found that the magnetic field tends to decrease the nanofluid velocity.

**Keywords:** Nanofluids, Magnetohydrodynamics, Mixed convection, Heat transfer, Entropy generation.

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## Nomenclature

u Velocity components along the x direction, m/s  
v Velocity components along the y direction, m/s  
T Temperature of the nanofluid, K

$\rho_f$  Density of the base fluid, [Kgm<sup>-3</sup>]

$\rho_s$  Density of the nanoparticle, [Kgm<sup>-3</sup>]

$\rho_{nf}$  Density of nanofluid, [Kgm<sup>-3</sup>]

$\phi$  Volume fraction of the nanoparticle

$\mu_f$  Dynamic viscosity of the base fluid, kg/ms

$\sigma_f$  Electric conductivity of base fluid, S/m

$\sigma_s$  Electric conductivity of Nanoparticles, S/m

$\beta_f$  Base fluid thermal expansion coefficient, [K<sup>-1</sup>]

$\beta_s$  Nanoparticle thermal expansion coefficient, [K<sup>-1</sup>]

$\sigma_{nf}$  Electrical conductivity of nanofluid, S/m

$(\rho C_p)_s$  Heat capacitance of the nanoparticle, jKg<sup>-1</sup> K<sup>-1</sup>

$(\rho C_p)_f$  Heat capacitance of the base fluid, jKg<sup>-1</sup> K<sup>-1</sup>

$(\rho C_p)_{nf}$  Heat capacitance of nanofluid, jKg<sup>-1</sup> K<sup>-1</sup>

$\mu_{nf}$  Dynamic viscosity of the nanofluid, Kgm<sup>-1</sup> s<sup>-1</sup>

$K_{nf}$  Thermal conductivity of nanofluid, W/mK

## 1.Introduction

The study of convective heat transfer in Nanofluids has received considerable theoretical and practical interest due to their enhanced thermal conductivity as compared to the conventional fluids like oil, water, ethylene glycol among others. The field of nanotechnology opened new dimension for many technologies like cooling systems, power generation, biotechnology, medicine, domestic refrigerator and radiators among others. Thermal conductivity of base fluids is enhanced by adding solid particles such as metallic materials. By so doing, the resulting fluid is electrically conducting fluid. In presence of magnetic field, this kind of fluids has many applications in engineering. Nanofluids were first introduced by Choi [1]. He proposed to disperse small amounts of nanometer-sized ( $1 \times 10^{-9}$ ) solid particles in base fluids. Mixed convection flows is a combined forced and free convection flows. Such processes occur when the effects of buoyancy forces in forced convection or the effects of forced flow in free convection become significant. Effect of thermal radiation and viscous dissipation on a mixed convective flow past a vertical plate has been analyzed by [2]. Numerical Analysis of Mixed Convection of Nanofluids Inside a Vertical plate was investigated by [3]. [4] have examined the fully developed mixed convection flow in a vertical plane filled with nanofluids, their analysis showed that the analytical solution for the opposing flow is only valid for a certain region of the Rayleigh number in physical sense, besides the effects of the nanoparticle volume fraction on the temperature and the velocity distributions are exhibited. They confirmed that the nanoparticle volume fraction plays a key role for improving the heat and mass transfer characteristics of the fluids. [5] extended the work of [4] and considered the effect of magnetic field on the fully developed mixed convective flow in a vertical plane filled with Nano-fluids, they recorded that the fluid velocity and temperature are enhance due to the application of magnetic field. Fully developed heat transfer by mixed convection flow of nanofluid in a vertical plate has been investigated by [6]. Effect of wall conductivities on a fully developed mixed convection Magneto hydrodynamic nanofluid flow in vertical plates was investigated by [7],

they reported that the case of a negative vertical temperature gradient. Entropy generation which is the measure of the destruction of available energy in a system plays an important role in the design and development of engineering processes such as pumps, heat exchangers, turbine and pipe networks. The energy utilization during the convection in any fluids flow as well as the improvement in thermal system is one of the fundamental problems of the engineering processes. An improvement of thermal system according to [8] provides better material processing, energy conservation and environmental effects. [9] Pioneered work on entropy generation. [10] Examined the entropy generation on an MHD flow and heat transfer over a flat plate with the convective boundary condition. [11] investigated heat transfer and entropy generation in fully developed mixed convection nanofluid flow past a vertical plates. From the studies cited above, much has been done on studies involving nanofluids but unsteady flow past a moving plane considering dissipative heat have not been investigated in one combined study and such is the motivation behind this work. The present study also analyses the entropy generation caused by hydromagnetic nanofluid flow.

## 2. Mathematical formulation

Consider an unsteady incompressible laminar two-dimensional MHD flow of a viscous electrically conducting water based nanofluids containing three types of nanoparticles, flowing past an accelerating vertical flat plane as shown in Figure 1. For the time  $t = 0$ , the fluid flow is steady. The unsteady state begin at  $t > 0$ . The velocity of the moving plane is  $U(x; t)$  along the infinite  $x$ -axis. The surface is convectively heated by hot fluid at temperature  $T_w(x)$ , while the temperature of the ambient cold fluid is  $T_\infty$ . A transverse magnetic field of strength  $B = B_0(1 - ct)^{1/2}$  is applied parallel to the  $y$ -axis, where  $B_0$  is constant magnetic field. The base fluid and the suspended nanoparticles are in thermal equilibrium. It is also assumed that induced magnetic field in the flow field is negligible in comparison with the applied magnetic field. Boussineq approximation holds and that, in addition to the Joulean heating, the volumetric heat generation by viscous friction is also significant. The pressure  $p$  is a function of  $x$  only and is a given constant..

The thermo-physical properties of the nanofluid are given in Table 1.

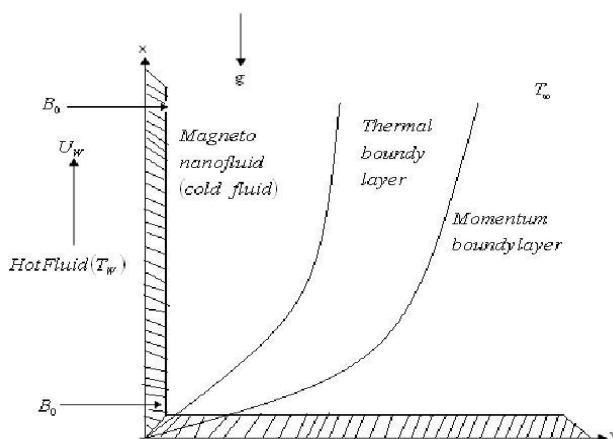


Figure 1. Geometry of the problem

The equations governing this type of flow are given as;

$$\frac{\partial u}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial y^2} \right) + g\beta_{nf}(T - T_f) - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}} \quad (2)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B_0^2 u^2}{(\rho C_p)_{nf}} \quad (3)$$

The last two terms in equation 3 indicate the effect of viscous dissipation and joule Heating respectively. The  $\mu_{nf}$ ,  $\rho_{nf}$ ,  $\sigma_{nf}$  and  $(\rho C_p)_{nf}$  as defined by [12], are given as;

$$\begin{aligned} \mu_{nf} &= \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \\ \sigma_{nf} &= \sigma_f \left[ 1 + \frac{3(\sigma-1)\phi}{(\sigma+2) - (\sigma-1)\phi} \right], \quad \sigma = \frac{\sigma_s}{\sigma_f} \end{aligned} \quad (4)$$

( $\phi = 0$  correspond to a base fluid or pure water) The expressions in equations 4 are restricted to spherical nanoparticles, where it does not account for other shapes of nanoparticles. The effective thermal conductivity of the nanofluid given by Oztop and Abu-Nada [13] is given by;

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (5)$$

Table 1: Thermophysical Properties of water and nanoparticles [13]

Physical Properties	Water/base fluid	Cu(Copper)	Al <sub>2</sub> O <sub>3</sub>	Ti <sub>2</sub> O <sub>4</sub>
$\rho(kgm^{-3})$	997.1	8933	3970	4250
$C_p(J / KgK)$	4179	385	765	686.2
k(WmK)	0.613	401	40	8.9538
$\phi$	0.0	0.05	0.15	0.2
$\sigma(S / m)$	$5.5 \times 10^{-6}$	$59.6 \times 10^6$	$35 \times 10^6$	$2.6 \times 10^6$

The initial and boundary conditions are [14]

$$t > 0: U_w(x, t) = \frac{ax}{(1-ct)}, v = 0, -k_f \frac{\partial T}{\partial y} = h_f [T_w(x, t) - T] \quad \text{at } y = 0, \tag{6}$$

$u \rightarrow 0, T \rightarrow T_\infty$  as  $y \rightarrow \infty$

Where  $T_w(x, t) = T_\infty + \frac{ax}{(1-ct)^2}$  is the temperature of the hot fluid and  $a$  and  $c$  are constants (where  $a >$

$0$  and  $c \geq 0$  with  $ct < 1$ ). The continuity equation (1) is automatically satisfied by introducing a stream function  $\psi(x, y)$  as,

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x} \tag{7}$$

The following similarity variables are introduced

$$\eta = y \left[ \frac{a}{v_f(1-ct)} \right]^{1/2}, \psi = \left[ \frac{av_f}{(1-ct)} \right]^{1/2} xf(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

Where  $\eta$  is the independent similarity variable,  $f(\eta)$  the dimensionless stream function and  $\theta(\eta)$  the dimensionless temperature. using equation(7) and (8),we have

$$u = \frac{ax}{1-ct} f'(\eta), \quad v = \left[ \frac{av_f}{(1-ct)} \right]^{1/2} f(\eta) \tag{9}$$

Substituting equation (9) in equations (2) and (3), we obtain the following ordinary differential equations

$$f''' - \phi_1 \left[ (f'^2 - ff'') + \lambda \left( f' + \frac{1}{2} \eta f'' \right) \right] - \phi_2 Haf' + Ri\theta = 0 \tag{10}$$

$$\frac{1}{Pr} \left( \frac{k_{nf}}{k_f} \right) \theta'' + \phi_3 \frac{f\theta'}{2} + \frac{Ec}{(1-\phi)^{2.5}} (f'')^2 + HaEc\phi_4 \lambda (f'-1)^2 = 0 \tag{11}$$

Where

$$\begin{aligned} \phi_1 &= (1-\phi)^{2.5} \left[ (1-\phi) + \frac{\rho_s}{\rho_f} \right] \\ \phi_2 &= (1-\phi)^{2.5} \left[ 1 + \frac{3(\phi-1)\phi}{(\phi+2) - (\phi-1)\phi} \right] \\ \phi_3 &= \left[ 1 - \phi + \phi(\rho C_p)_s / (\rho C_p)_f \right] \\ \phi_4 &= (1-\phi + \phi\sigma_s / \sigma_f)(f'-1)^2 = 0 \end{aligned} \tag{12}$$

and  $Ha = \frac{\sigma_f B_0^2 x(1-ct)}{\alpha \rho_f}$  is the magnetic parameter representing the ratio of electromagnetic (Lorentz

force) to the viscous force,  $\lambda = \frac{c}{a}$  is the unsteadiness parameter,  $Pr = \frac{\nu_f}{\alpha_f}$  is the Prandtl number and

$Ec = \frac{U_\infty^2}{(C_p)_f (T_w - T_\infty)}$  is the Eckert number.  $Ri = \frac{Gr_x}{Re_x}$  is the Richardson number where;

$Re_x = \frac{U_\infty x}{\nu_f}$  is the Reynolds number and  $Gr = \frac{\beta_f g (T_w - T_\infty) (1 - ct) x}{U_\infty^2}$ . The prime denotes the

differentiation with respect to  $\eta$ . The corresponding boundary conditions are;

$$f(0) = 0, f'(0) = 1, \theta'(0) = Bi[1 - \theta(0)], f' \rightarrow 0, \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty. \quad (13)$$

Where  $Bi = \frac{h_f}{k_f} \left( \frac{(1 - ct)x \nu_f}{a} \right)^{\frac{1}{2}}$  is the Biot number. Now, the Biot number and the magnetic parameter are functions of  $x$ . To have similarity equations, all parameters must be a constant and not a function of  $x$ . we therefore assume

$$h_f = b(x(1 - ct))^{-\frac{1}{2}}, \quad \sigma_f = d(x(1 - ct))^{-1} \quad (14)$$

where  $b$  and  $d$  are constants.

### 3. Numerical solution

The numerical solution for the governing equations (10) and (11) with the boundary conditions (13) is obtained by shooting technique. The corresponding higher order nonlinear differential equations becomes

The corresponding higher order nonlinear differential equations becomes;

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= \phi_1 \left[ (y_2^2 - y_1 y_3) + \lambda (y_2 + \frac{1}{2} \eta y_3) \right] + \phi_2 Ha y_2 + Ri y_4 \\ y_4' &= y_5 \\ y_5' &= -Pr \left( \frac{k_f}{k_{nf}} \right) \left[ \phi_3 \frac{y_1 y_5}{2} + \frac{Ec}{(1 - \phi)^{2.5}} (y_3)^2 + Ha Ec \phi \lambda (y_2 - 1)^2 \right] \end{aligned} \quad (15)$$

with boundary conditions

$$y_1(0) = 0, \quad y_2(0) = 1, \quad y_3(0) = \gamma, \quad y_4(0) = \delta, \quad y_5(0) = -Bi(1 - \delta) \quad (16)$$

Where  $\delta$  and  $\gamma$  are unknowns which are to be determined such that the boundary conditions

$y_2(\infty) = 0$  and  $y_5(\infty) = 0$  are satisfied.  $\delta$  and  $\gamma$  are guessed using shooting by iterations

until the boundary conditions are satisfied. The resulting differential equations can be solved using Runge-Kutta-Fehlberg fourth order scheme.

#### 4. Results and Discussion

The effect of various thermo physical parameters on the nanofluid velocity, temperature, heat transfer rate as well as shear stress at the plate are presented in graphs and tables. The Prandtl number for the base fluid is kept constant as  $Pr=6.2$ . Computations are carried out for solid volume fraction  $\phi$  in a range of  $0 \leq \phi \leq 0.2$  for regular fluid,  $\phi=0$  and  $Ha = 0$  corresponds to absence of magnetic field.

The copper nanoparticles are used in all figures except those which focus on the influence of the type of applied nanoparticles.

##### 4.1 Effect of Parameters on the Velocity Profiles

In order to get a clear understanding of the problem ,effects of different values of magnetic parameter  $Ha$ , Grashof number  $Gr$  and Eckert number  $Ec$  on the fluid velocity and temperature are discussed. Figure 2 displays the variation in the nanofluid velocity for three types of water-based nanofluids  $Al_2O_3$ -water,  $TiO_2$ - water and  $Cu$ -water. It is noted that the velocity is maximum at the moving plate surface but decreases gradually to zero at the free stream far away from the plate. It is also observed that  $Cu$ -water nanofluid has the thinnest momentum boundary thickness and tends to flow closer to the convectively heated plate surface. From Figure 3, it is observed that the fluid velocity decreases for increasing value of Hartmann number  $Ha$ . Figure 4 displays the effect of unsteadiness parameter  $\lambda$  on the fluid velocity. The fluid velocity decreases as the values of  $\lambda$  increases. For unsteady flow  $\lambda > 0$  and for steady flow  $\lambda = 0$ . From Figure 5, it is observed that the fluid velocity decreases with increasing values of Grashof number  $Gr$ .

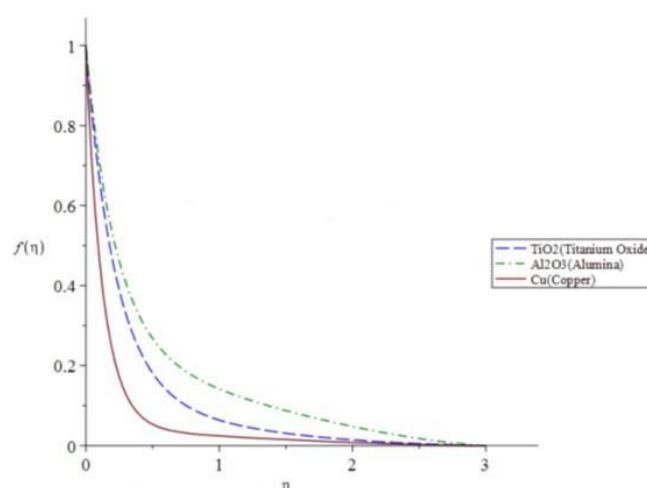


Figure 2: Velocity profile for different nanofluids when  $\phi = 0.1$  and  $Ha = \lambda = 1$

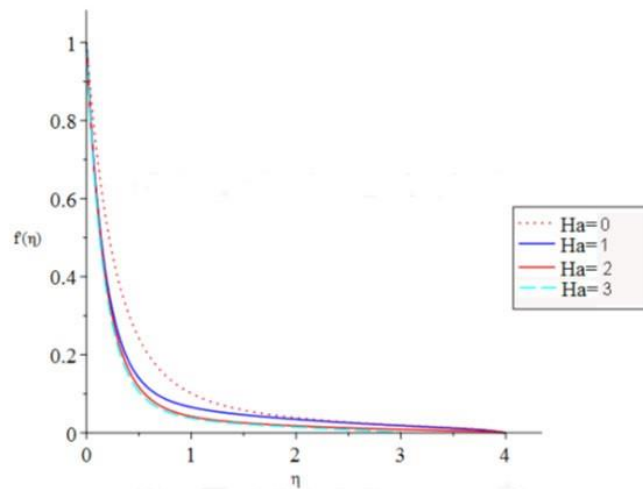


Figure 3: Velocity profile for different  $Ha$  when  $\phi = 0.1$  and  $\lambda = 1$

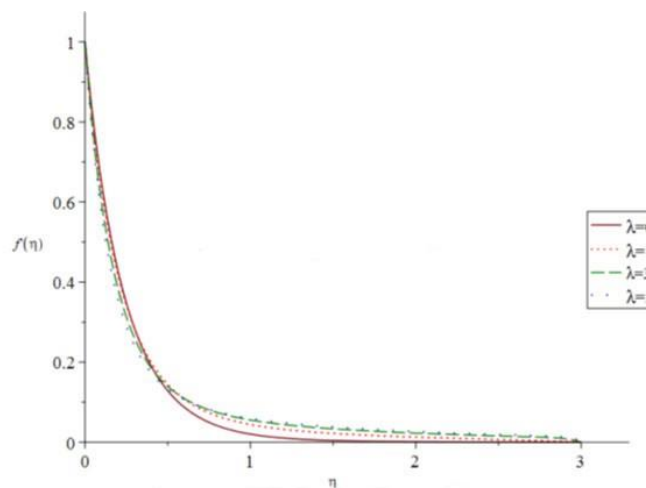


Figure 4: Velocity profile for different  $\lambda$  when  $Gr = \phi = 0.1, Ha=1$  and  $\lambda = 1$

#### 4.2 Effects of parameters on temperature profiles

From Figure 6, it is observed that the temperature is high in Cu-water nanofluid as compared to  $Al_2O_3$  and  $TiO_2$ . This can be explained from the fact that copper has a high thermal conductivity as compared to  $Al_2O_3$  and  $TiO_2$ . Figure 7 displays the effect of Biot number  $Bi$  on fluid temperature. The fluid temperature rises as the Biot number increases. It is observed from Figure 8 that the fluid temperature increases for increasing values of Eckert number  $Ec$ . From Figure 9, it is observed that the fluid temperature rises as the magnetic field becomes stronger.



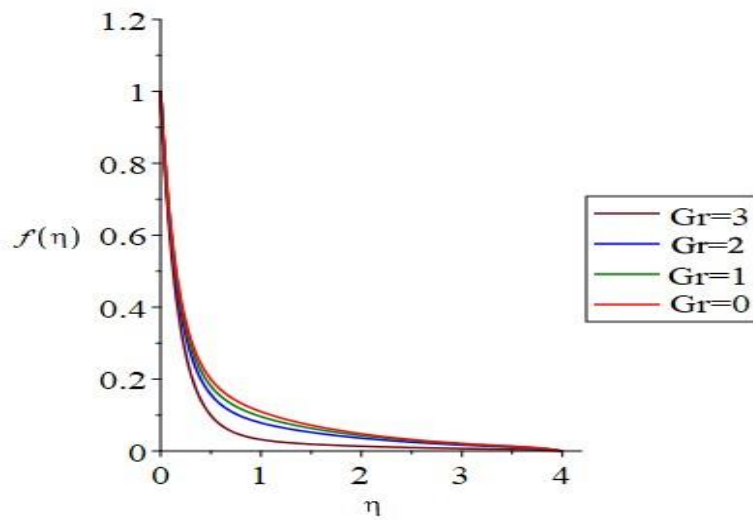


Figure 5: Velocity profile for different Gr when  $\phi = 0.1, Ha=1$  and  $\lambda = 1$

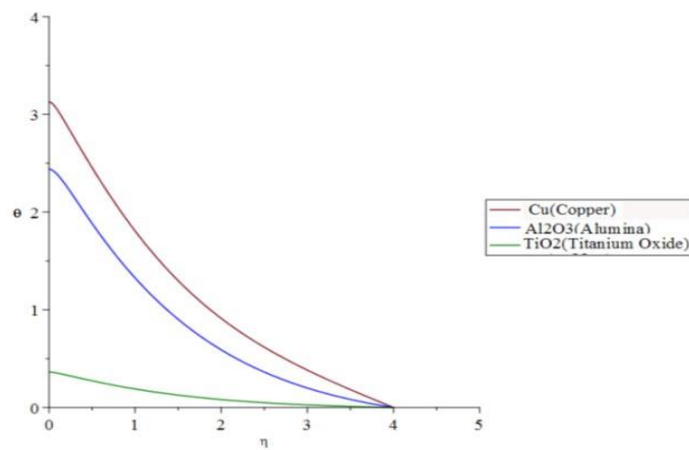


Figure 6: Temperature profile for different nanofluids when  $Bi = \phi=0.1$  and  $Ha = \lambda = 1$

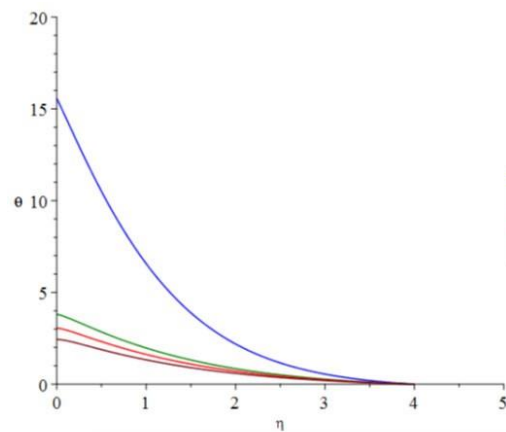


Figure 7: Temperature profiles for different Bi when  $Ha=\lambda=1$  and  $\phi = 0.1$

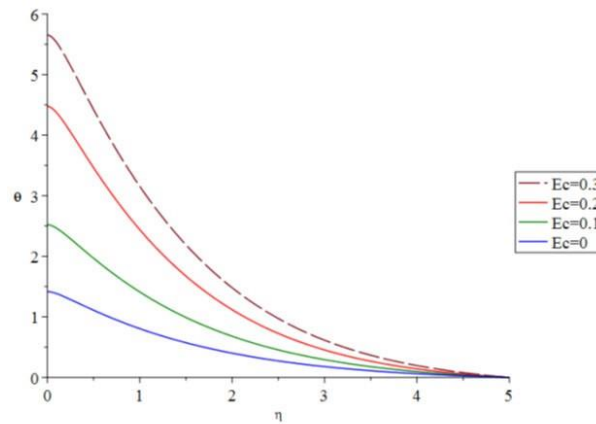


Figure 8: Temperature profiles for different  $Ec$  when  $Ha=\lambda = 1$   $Bi=\phi = 0.1$

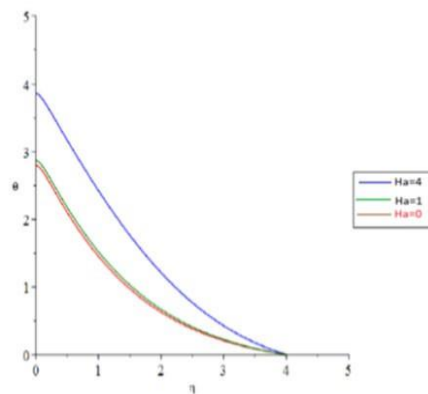


Figure 9: Temperature profiles for different  $Ha$  when  $\lambda = 1$   $Bi=\phi = 0.1$

### 4.3 Entropy Generation

Entropy generation is caused by the non-equilibrium state of the fluid resulting from the heat changes between the two media. This entropy generation is due to the irreversible nature of heat transfer and fluid friction within the fluid and the solid boundaries. From the known temperature and velocity fields, volumetric entropy generation can be calculated. According to [15], the local volumetric rate of entropy generation for an electrically conducting nanofluid in the presence of magnetic field is given by

$$E_G = \frac{k_{nf}}{T_\infty^2} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu_{nf}}{T_\infty} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B_0^2}{T_\infty} u^2 \quad (17)$$

The first term in equation (17) is the irreversibility due to the heat transfer, the second term is entropy generation due to viscous dissipation and the third term is local entropy generation due to the effect of magnetic field. The non-dimensional entropy generation number is defined as

$$N_s = \frac{T_w^2 a^2 E_G}{k_f (T_w - T_\infty)^2} \quad (18)$$

On the use of (8), the entropy generation number in the non-dimensional form can be obtained as follows

$$N_s = \text{Re} \left[ \frac{k_{nf}}{k_f} \theta'^2 + \frac{1}{(1-\phi)^{2.5}} \frac{Br}{\Omega} (f''^2 + \phi_2 Ha f'^2) \right] \quad (19)$$

Where  $\text{Re} = \frac{U_w a}{\nu_f x}$  is the Reynolds number,  $Br = \frac{\mu_f U_w^2}{k_f (T_w - T_\infty)}$  the Brinkman number which

represents the ratio of direct heat conduction from the surface to the viscous heat generated by shear in the boundary layer and  $\Omega = \frac{T_w - T_\infty}{T_w}$  the non-dimensional temperature difference. The entropy

generation number  $N_s$  can be written as a summation of the entropy generation due to heat transfer denoted by  $N_1$  and the entropy generation due to fluid friction with magnetic field denoted by  $N_2$  given as

$$N_1 = \text{Re} \frac{k_{nf}}{k_f} \theta'^2 \quad (20)$$

$$N_2 = \frac{\text{Re}}{(1-\phi)^{2.5}} \frac{Br}{\Omega} (f''^2 + \phi_2 Ha f'^2)$$

In order to obtain an idea of whether entropy generation due to heat transfer dominates over entropy generation due to the fluid friction and magnetic field or vice versa, the Bejan number  $Be$  is defined to be the ratio of entropy generation due to heat transfer to the entropy generation number Paoletti et al [16]

$$Be = \frac{\text{entropy generation due to heat transfer}}{\text{entropy generation number}} = \frac{N_1}{N_s} \quad (21)$$

Now we know that  $N_s = N_1 + N_2$ , Substituting for  $N_s$  in Equation (21) and dividing through

by  $N_1$ , Bejan number is defined as

$$Be = \frac{1}{1 + \Phi} \quad (22)$$

Where  $\Phi = \frac{N_2}{N_1}$  is the irreversibility ratio. Heat transfer dominates for  $0 \leq \Phi < 1$  and fluid friction with magnetic effect dominates when  $\Phi > 1$ . The contribution of both heat transfer and fluid friction to entropy generation are equal when  $\Phi = 1$ . The Bejan number  $Be$  takes the values between 0 and 1 Cimpean et al [17]. The value of  $Be=1$  is the limit at which the heat transfer irreversibility dominates,  $Be=0$  is the opposite limit at which the irreversibility is dominated by the combined effects of fluid friction and magnetic field and  $Be=0.5$  is the case in which the heat transfer and the fluid friction with magnetic field entropy production rates are equal.

#### 4.4 Effect of parameters on entropy generation

It is observed from figure 10 that the entropy generation number increases near the moving plate with an increase in magnetic parameter  $Ha$ . Figure 11 indicates that the the entropy generation number  $N_S$  increases with an increase in the volume fraction parameter  $\phi$ . Increasing the volume fractions of the solid nanoparticles leads to an increase in the viscous force of the nanofluids. Figure 12 illustrates the effects of the Biot number  $Bi$  on the entropy generation. Near the surface of the plate, the effects of  $Bi$  on  $N_S$  are prominent. From Figures 10-12, we can conclude that the entropy generation for the nanofluids is more than that for the base fluid ( $\phi = 0$ ). This is because the metallic nanoparticles have high thermal conductivity.

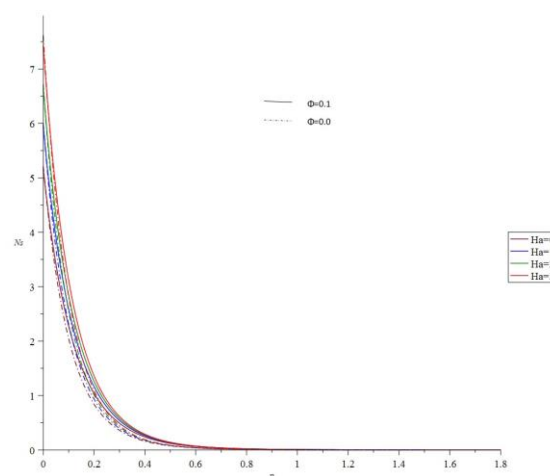


Figure 10:  $N_S$  for different  $Ha$  when  $Bi=0.1, Re=1, Br\Omega^{-1} = 1$  and  $\lambda = 1$  for Cu- water

nanofluid

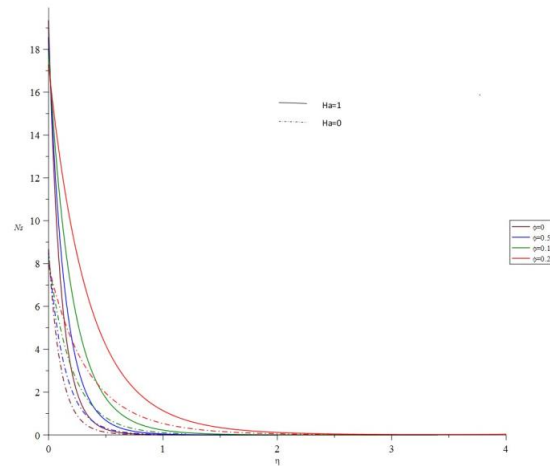


Figure 11:  $N_s$  different  $\phi$  when  $Re=1$ ,  $\lambda = 1$  and  $Br\Omega^{-1} = 1$

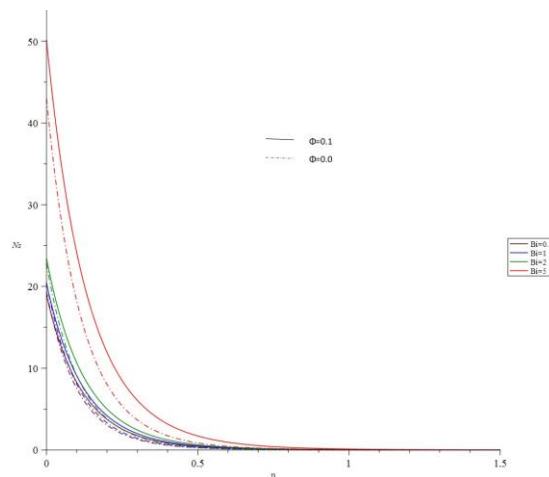


Figure 12:  $N_s$  different  $Bi$  when  $Re=1$ ,  $\lambda = 1$  and  $Br\Omega^{-1} = 1$

#### 4.5 Effects of parameters on Bejan number

To study whether heat transfer entropy generation dominates over the fluid friction and magnetic field entropy generation or vice versa, the Bejan number is plotted for the physical parameters. Figure 13 indicates that as magnetic parameter increases, the Bejan number decreases. The entropy generation due to fluid friction and magnetic field is fully dominated by heat transfer entropy generation near the plate. In figure 14, an increase in Biot number  $Bi$  results in an increase in Bejan. Figure 15 reveals that the Bejan number  $Be$  increases with increasing volume fraction parameter  $\phi$ .

The entropy generation due to heat transfer is dominated as  $\phi$  evolves. number.

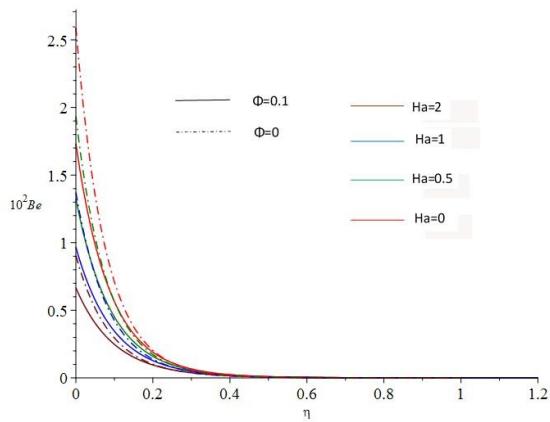


Figure 13: Bejan for different Ha when  $\phi = 0.1$ ,  $Bi=0.1$  and  $Br\Omega^{-1} = \lambda = 1$ .

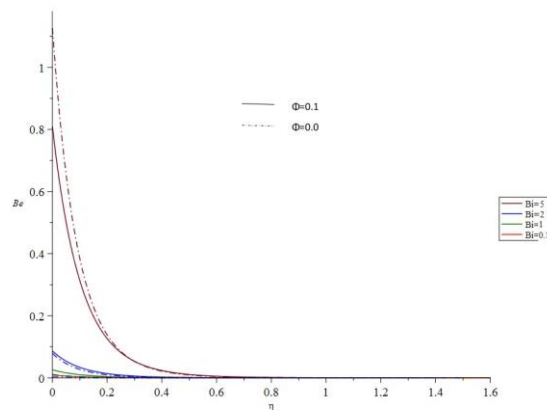


Figure 14: Bejan for different Biot Bi when  $\phi = 0.1$  and  $Br\Omega^{-1} = \lambda = 1$ .

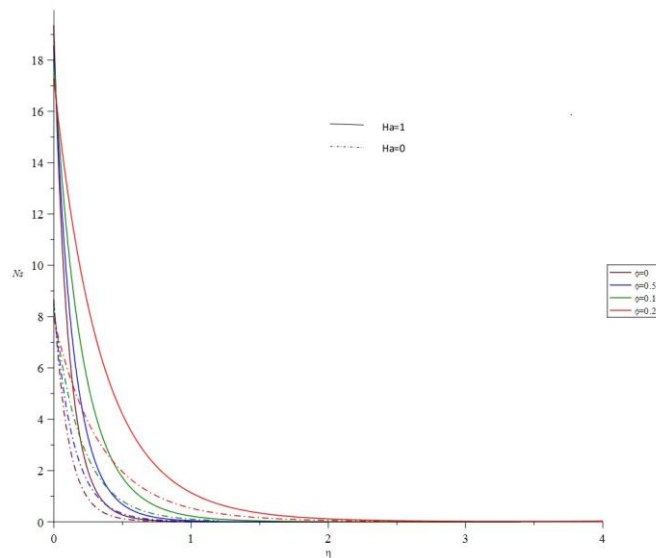


Figure 15: Bejan number for different  $\phi$  when  $\lambda = 1$  and  $Bi=0.1$

## 5 . Conclusion

Analysis of hydromagnetic nanofluid flow past a vertical plane has been done in this study. The influences of the different types of nanoparticles on the flow of a viscous incompressible electrically conducting nanofluid with convective boundary condition in the presence of a transverse magnetic field with viscous dissipation was examined. Conclusions of the results obtained by varying various parameters. The variations of these parameters affected the velocity and temperature in the boundary layer. These variations in turn affected the entropy generation. It was observed that the velocity of nanofluid decreases as the strength of magnetic field increases. Also, the velocity and temperature of nanofluid reduce due to increasing unsteadiness parameter. In the presence of uniform magnetic field, the fluid velocity enhances whereas the temperature of the fluid falls as the volume fraction parameter increases. Alumina-water shows a thicker velocity boundary than Cu-water nanofluid. The entropy generation depends on the thermal conductivity of the nanoparticles in the base fluid. The presence of metallic nanoparticles creates the entropy more in the nanofluid flow compared to the regular fluid. The entropy generation depends on the thermal conductivity of the nanoparticles in the base fluid. Nanofluids are highly susceptible to the effects of magnetic field compared to conventional base fluid due to the complex interaction of the electrical conductivity of nanoparticles with that of base fluid.

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