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RELATION INVOLVING ψ FUNCTION AND I FUNCTION OF TWO VARIABLE

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ABSTRACT

In the paper, we established relation involving ψ function and I- Function of two variable $I\left[z_1, z_2\right]$. Some more result known and new are also obtained as a particular cases of these relation.

Key Word: ψ function. kampe de feriet function, I- function of two variable.

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Recently number of relation involving ψ function and I – Function in this paper, we use following method. Some relations involving the ψ function and I- Function of two variable have been established. Further, more results involving kampe de feriet function and general hyper geometric series. We obtained as particular care of these relations.

Definition and Notation:

The multivariable extension of the kampe de fereit function in defined and represented as [ ].

$$F^{p:q_1;.......;q_n}_{i:m_1;...........m_n}\left[z_1,..........,z_n\right] = F^{p:q_1;;q_n}_{i:m_1;;m_n}\left[\begin{matrix}(a_p):(b^{(1)}_{q_1});........;\left(b^{(n)}_{q_n}\right): \\ (\alpha_l):\left(\beta^{(l)}_{m_i}\right);........;\left(\beta^{(n)}_{m_n}\right)\end{matrix} z_1........z_n\right].$$

$$= \sum_{s_1.....s_n=0}^{\infty} g\left(s_1,.........,s_n\right)\frac{z_1^{s_1}}{s_1!}......\frac{z_n^{s_n}}{s_n!} \qquad .... \quad (1.1).$$

$$g\left(s_1,..........,s_n\right) = \frac{\prod_{j=1}^{p}\left(a_j\right)_{s_{1+.......s_n}}\prod_{i=1}^{q_1}\left(b_i^1\right)_{s_1}............\prod_{s_1}^{q_n}\left(b_i^{(n)}\right)_{s_n}}{\prod_{i=1}^{l}\left(\alpha_i\right)_{s_{1+.......+s_n}}\prod_{i=1}^{m_1}\left(\beta_i^1\right)s_1..............\prod_{j=1}^{m_n}\left(\beta_j^{(n)}\right)_{s_n}}.$$

and for convergence $1+ l + m_k - p - q_k \geq 0$, $k = 1, ...n$.

The equality hold when in addition, either $p>1$ and $|z_1|^{1/(p-1)} + ... + |z_n|^{1/(p-1)} < 1$ or $p \leq l$ and $\max\{|z_1|,.........|z_n|\} < 1$

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The I- Function of two variable is defined by Sharma and Mishra [1991] represented by following manner:

$$I\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = I[z_1, z_2] = I^{0,\ n\ :m_1\ \ n_1;m_2,n_2}_{p_i, q_{i:r};\ p_i', q', r';\ p_i'', q_i'', r''}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} \left[(a_j;\alpha_j;A_j)_{1,n_1}\right],\left[(a_{ji};\alpha_{ji};A_{ji})_{n+1,p_i}\right];\left[(e_i,E_j)_{1,n_2}\right]\left[(e_{ji}'',E_{ji}'')_{n_1+1,p_2''}\right];\left[(f_j,F_j)_{1,m_1}\right] \\ :\left[(c_j:y_j)_{1,n_1}\right]\left[(c_{ji};y'_{ji})_{n_1+1,p_i}\right];\left[(b_{ji};\beta_{ji};B_{ji})_{1,q_1}\right]\left[(d_j,\delta_j)_{1,m_1}\right];\left[(d_{ji},\delta_{ji})_{m_1+1,q_i}\right]\left[(f_{ji}'',F_{ji}'')_{m_2+1,q_i''}\right] \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(\xi,\eta)\theta_2(\xi)\theta_3(\eta) z_1^{\xi_1} z_2^{\eta_1} d\xi d\eta \text{ . } where\ \ \omega=\sqrt{-1}\,.$$

….(1.3)

$$\phi(\xi,\eta) = \frac{\prod_{j=1}^{n}\left[(1-a_j+\alpha_j\xi+A_j\xi)\right]}{\sum_{i=1}^{n}\left[\prod_{j=n+1}^{p_i}\left[(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\right]\prod_{j=1}^{q_i}\left[(1-f_{ji}''+F_{ji}^{n})\right]\right]} = \qquad \text{….(1.4)}$$

Along with other convergence condition as mention by Sharma and Mishra (1991). z_1, z_2 are not equal to zero and an empty product in interpreted as unity.

$$\theta_2(\xi) = \frac{\prod_{j=1}^{n_1}\Gamma(1-c_j-\gamma_j\xi)\prod_{j=1}^{m}\Gamma(d_{ji}'+\delta_i\xi)}{\sum_{i''=1}^{n_2}\left[\prod_{j=n_1+1}^{p_i'}\Gamma(c_{ji}-\gamma_{ji})\ \underset{j=m_{1+1}}{\overset{\infty_2}{\pi}}\ \prod_{j=m_1+1}^{q_i'}\Gamma(1-d_{ji}'+\delta_{ji}\xi)\right]}\,.$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{n_2}\Gamma(1-e_j-E_j\eta)\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)}{\sum_{i'''=1}^{r'''}\left[\prod_{j=n_2+1}^{p_i''}\Gamma(e_{ji}''-E_{ji}''\eta)\prod_{j=m_2+1}^{q_i''}\Gamma(1-f_{ji}''+F_{ji}''\eta)\right]}\,.$$

We required the following series representation of the I- function of two variable [4,pp-84-85] with the help of H- Function two variable on the simplify we get the definition of H- Function and the represented as summation series.

$$I[z_1, z_2] = \sum_{h=1}^{p_2}\sum_{h_2=1}^{p_3}\left\{\sum_{k=1}^{\infty}\sum_{k_2=0}^{\infty}(-1)^{k_1+k_2}\phi(\xi_1,\xi_2)\theta_3(\xi)\theta_4(\xi_1)x^{\xi_1}y^{\xi_2}\left((\delta_{h_1}, f_{h_2})k_1k_2\right)^{-1}\right\}. \qquad \text{….(1.7)}$$

Where $\xi_1 = \xi(h_1,k_1) = \frac{d_{n_1+k_1}}{\delta_{n_1}}$ …. (1.8)

and $\xi_2 = \xi(h_2,k_2) = \frac{f_{n_1+k_2}}{F_{n_2}}$ …. (1.9)

If $\left\{{}^{R}k_1,k_2\right\}_{k_1,k_{2=0}}^{\infty}$ in an arbitrary bounded sequence of complex number, we have

$$= \sum_{n=1}^{\infty}\frac{(v)_n}{n(v)_n}\left\{\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{R_{k_1k_2}}{(u+n)_{k_1+k_2}}\frac{z^{k_1+k_2}}{\lfloor k_1\lfloor k_2}\right\}$$

$$= \sum_{k_1,k_2=0}^{\infty}\left[\frac{R_{k_1k_2}}{(u)_{k_1+k_2}}\left\{\psi(m+k_1+k_2)-\psi(u+k_1+k_2-v)\right\}\frac{z^{k_1+k_2}}{\lfloor k_1\lfloor k_2}\right] \qquad \text{….(1.10)}$$

Again from [1] with above (1.10) and known Results

$$\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\left[\psi(c+n)-\psi(c)\right] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[\psi(c-a)+\psi(c-b)-\psi(c)-\psi(c-a-b)\right] \qquad \text{….(1.11)}$$

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The following extension of the above given [1] to the case of two variable are obtained easily.

Let $\left\{R_{k_1}, R_{k_2}\right\}_{k_1,k_2=0}^{\infty}$ be an arbitrary sequence of complex number also let $u_n\{\mu,\rho;z\} = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\Gamma(\mu-\rho).z^{k_1+k_2}}{\Gamma(\mu+\eta+\rho)\lfloor k_1 .. \lfloor k_2}$.

Where $\rho=\rho(k_1,k_2)$ is a given with $\mathrm{Re}(\rho(k_1,k_2))\geq 0$.

$$\text{then } \sum\frac{(\alpha)n}{n}u_n(\mu,\rho;z) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left\{R_{k_1-k_2}\left[\psi(u+\rho)-\psi(-\alpha+u+\rho)\right]\frac{z^{k_1+k_2}}{\lfloor k_1\cdot\lfloor k_2}\right\}. \quad \ldots. \quad (1.12)$$

Again let

$$v_n(\alpha,\beta,\mu,\eta,\rho;z) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}R_{k_1,k_2}\frac{\Gamma(u+\rho)\Gamma(u+\alpha+\beta+\eta+\rho)}{\Gamma(u+\eta+\rho)\Gamma(u+\alpha+\beta+\rho+\eta+\rho)}\times 3F_2\left[\begin{matrix}\alpha+\eta,\alpha+\rho; & -n\\ \mu+\alpha+\beta+\eta+\rho;\alpha\end{matrix};1\right]\frac{z^{k_1+k_2}}{\lfloor k_1\lfloor k_2}. \quad \ldots.(1.13)$$

2. Main Results:

We establish the following results:

$$\sum_{n=1}^{\infty}\frac{(\alpha)n}{n}I^{o,n\,:\,m_1,n_1;\,m_2,n_2}_{p_i,q_i:r;\,p_i,q_i;\,p_i,q_i,r}\left[\begin{matrix}z_1\\ z_2\end{matrix}\left|\begin{matrix}(q_i,u_j,c_1,s_2),(a_j,\alpha_i,A_j)\\ (b_i,\beta_j,B_j)_{1,s_1}\ (1-u-n;c_1,c_2)\end{matrix}\right.\right].$$

$$=\sum_{h_1=1}^{b_2}\sum_{h_3=1}^{b_3}\left\{\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(-1)^{k_1+k_2}\phi(\xi_1\xi_2)\theta_3(\xi_1)\theta_4(\xi_2)\left(s_{n_1},F_{h_2}\right)^{-1}\times\left[\psi(u+a\xi_1+c_2\xi_2)-\psi(-\alpha+u+c_1\xi_1+c_2\xi_2)\right]\frac{z^{\xi_1+\xi_2}}{\lfloor k_1\lfloor k_2}\right\}$$

.

….(2.1)

Proofs of Main Result

To Establish (2.1), We assume

$$p(k_1,k_2)=c_1\xi_1+c_2\xi_2 \qquad (c_1, c_2 \text{ are constants})$$

and ξ_1,ξ_2 are given by (1.7 and (1.8)

Next we assume

$$R_{k_1,k_2}=(-1)^{k_1+k_2}\phi(\xi_1,\xi_2)\theta_3(\xi_1)\theta_4(\xi_2)\left(\delta_{h_1}F_{h_2}\right)^{-1}.$$

Where $\phi(\xi_1,\xi_2),\theta_3(\xi_1)$ and $\theta_4(\xi_2)$ are given by (1.5) and (1.9), respectively. Now we multiply both sides of (1.9) by $z^{\xi_1+\xi_2-k_1-k_2}$ and then sum up the resulting equation from $h_1=1$ to p_2 and $h_2=1$ to p_3 and then interpret with the help of (1.6) to arrive at the result given in (2.1)

To establish (2.2), we again assume that $p = p(k_1, k_2) = c_1\xi_1 + c_2\xi_2$, ($c_1, c_2$ are constants) and ξ_1, ξ_2 are given by (1.7) and (1.8). In new of the series representation of $H\begin{bmatrix} x \\ y \end{bmatrix}$, given in (1.6), we further let

$$R_{k_1,k_2} = (-1)^{k_1+k_2}\,\phi(\xi_1,\xi_2)\,\theta_3(\xi_1)\,\theta_4(\xi_2)\left(\delta_{h_1}, F_{h_2}\right)^{-1}.$$

Where $\phi(\xi_1,\xi_2)$ and $\theta_3(\xi_1)$ are given by (1.5) and (1.9), in (1.21) and then multiply both sides of (1.21) by $z^{\xi_1+\xi_2-k_1-k_2}$ and sum up both sides of the resulting equation from $h_1 = 1$ to p_2 and $h_2 = 1$ to p_3 and then interpret with the help of (1.6) to arrive at result (2.2)

Similarly, the result in (2.3) is established with the help of (1.22) and (1.23)

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