RELATION INVOLVING $\Psi$ FUNCTION AND I FUNCTION OF TWO VARIABLE

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ABSTRACT

In the paper, we established relation involving $\Psi$ function and I- Function of two variable $I[z_1, z_2]$. Some more results known and new are also obtained as a particular case of the relation.

Key Word: $\Psi$ function, kampe de feriet function, I- function of two variable.

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Recently number of relation involving $\Psi$ function and I – Function in this paper, we use following method. Some relations involving the $\Psi$ function and I- Function of two variable have been established. Further, more results involving kampe de feriet function and general hyper geometric series. We obtained as particular care of these relations.

Definition and Notation:

The multivariable extension of the kampe de feriet function in defined and represented as 

$$
F_{\nu,\nu';...;\nu_{n}}^{p,q;...;q_{n}}[z_{1},...,z_{n}] = F_{\nu,\nu';...;\nu_{n}}^{p,q;...;q_{n}} \left[ (a_{1})^{(l)}; (b_{1})^{(l)}; \ldots; (a_{s})^{(l)}; (b_{s})^{(l)} \right]
$$

$$
= \sum_{a_{1},...,a_{n}=0}^{\infty} g(s_{1},...,s_{n}) \frac{z_{1}^{N_{1}}}{s_{1}!} \ldots \frac{z_{n}^{N_{n}}}{s_{n}!} \quad \ldots \quad (1.1)
$$

$$
g(s_{1},...,s_{n}) = \frac{\prod_{i=1}^{p} (a_{i})_{s_{i}} \prod_{j=1}^{q} (b_{j})_{s_{j}} \ldots \prod_{l=1}^{l_{n}} (d_{l})_{s_{l}}}{\prod_{i=1}^{p} (c_{i})_{s_{i}} \prod_{j=1}^{q} (e_{j})_{s_{j}} \ldots \prod_{l=1}^{l_{n}} (f_{l})_{s_{l}}},
$$

and for convergence $1 + l + m_{l} - p - q_{k} \geq 0, k = 1, ..., n$.

The equality hold when in addition, either $p > 1$ and $\max |z_{1}|^{p-1} + \ldots + |z_{n}|^{p-1} < l$ or $p \leq l$ and $\max \{ |z_{1}|, \ldots, |z_{n}| \} < 1$.
The I- Function of two variable is defined by Sharma and Mishra [1991] represented by following manner:

\[ I[z_1, z_2] = I[z_1, z_2] = I = \sum_{j=1}^{\infty} \left( \sum_{n=1}^{\infty} \left( \sum_{k=0}^{\infty} \frac{R_{n,k}}{(u+n)_{k_1} k_2} \right) \right) \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} (-1)^{k+1} \phi(z_1, z_2) \theta_1(z_1) \theta_4(z_1) \frac{s^{k_1+1}}{k_1! k_2} \left( \left( \delta_k f_k k_1 \right)^{-1} \right). \] ....(1.7)

Where \( z_i = z(h_i k_i) = \frac{d_{n+1}}{\delta n} \) .... (1.8)

and \( z_2 = \frac{f_{n+1}}{F_{n+1}} \) .... (1.9)

If \( \{n \delta, k_1, k_2\} \) in an arbitrary bounded sequence of complex number, we have

\[ \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \left[ \psi(m+k_1+k_2) - \psi(u+k_1+k_2) \right] \frac{s^{k_1+1}}{k_1! k_2} \] ....(1.10)

Again from [1] with above (1.10) and known Results

\[ \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \left[ \psi(c+n) - \psi(c) \right] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \left[ \psi(c-a) + \psi(c-b) - \psi(c) - \psi(c-a-b) \right] \] ....(1.11)
The following extension of the above given [1] to the case of two variable are obtained easily.

Let \( \{ R_{k_i} \}_{k_i=0}^\infty \) be an arbitrary sequence of complex number also let \( u_\alpha (\mu, \rho; z) = \sum_{k_i=0}^\infty \sum_{k_j=0}^\infty \frac{\Gamma(\mu - \rho) \cdot z^{k_i + k_j}}{k_i! \cdot k_j!} \).

Where \( \rho = \rho(k_i, k_j) \) is a given with \( \Re(\rho(k_i, k_j)) \geq 0 \).

Then \( v_\alpha (\alpha, \beta, \mu, \eta; \rho; z) = \sum_{k_i=0}^\infty \sum_{k_j=0}^\infty R_{k_i, k_j} \cdot \frac{\Gamma(u + \rho) \Gamma(u + \alpha + \beta + \eta + \rho)}{\Gamma(u + \eta + \rho) \Gamma(u + \alpha + \beta + \eta + \rho)} \cdot \sum_{n=1}^\infty \left( \frac{1}{n} \right) \cdot \frac{\alpha + \eta \cdot \alpha + \rho}{k_i! \cdot k_j!} \cdot z^{k_i + k_j} \). … (1.12)

Again let \( v_\alpha (\alpha, \beta, \mu, \eta; \rho; z) = \sum_{k_i=0}^\infty \sum_{k_j=0}^\infty R_{k_i, k_j} \cdot \frac{\Gamma(u + \rho) \Gamma(u + \alpha + \beta + \eta + \rho)}{\Gamma(u + \eta + \rho) \Gamma(u + \alpha + \beta + \eta + \rho)} \cdot \sum_{n=1}^\infty \left( \frac{1}{n} \right) \cdot \frac{\alpha + \eta \cdot \alpha + \rho}{k_i! \cdot k_j!} \cdot z^{k_i + k_j} \). … (1.13)

2. **Main Results:**

We establish the following results:

\[
\sum_{n=1}^\infty \left( \frac{1}{n} \right) \cdot \frac{\alpha + \eta \cdot \alpha + \rho}{k_i! \cdot k_j!} \cdot z^{k_i + k_j} = \sum_{h_1=1}^p \sum_{h_2=1}^p \sum_{h_3=1}^p (-1)^{h_1 + h_2} \phi(\xi_1, \xi_2) \theta_3(\xi_1) \theta_4(\xi_2) \left( \frac{s_n \cdot F_{h_1}}{h_1} \right) \cdot \psi(u + a \xi_1 + c_1 \xi_2) - \psi(\alpha + u + c_1 \xi_1 + c_2 \xi_2) \cdot \frac{z^{\xi_1 + \xi_2}}{k_i! \cdot k_j!}.
\]

… (2.1)

**Proofs of Main Result**

To Establish (2.1), We assume

\[ p(k_1, k_2) = c_1 \xi_1 + c_2 \xi_2 \]

and \( \xi_1, \xi_2 \) are given by (1.7 and 1.8)

Next we assume

\[ R_{k_1, k_2} = (-1)^{k_1 + k_2} \cdot \phi(\xi_1, \xi_2) \theta_3(\xi_1) \theta_4(\xi_2) \left( \frac{s_n \cdot F_{h_1}}{h_1} \right)^{-1}. \]

Where \( \phi(\xi_1, \xi_2), \theta_3(\xi_1) \) and \( \theta_4(\xi_2) \) are given by (1.5) and (1.9), respectively. Now we multiply both sides of (1.9) by \( z^{\xi_1 + \xi_2 - k_1 - k_2} \) and then sum up the resulting equation from \( h_1 = 1 \) to \( p_2 \) and \( h_2 = 1 \) to \( p_3 \) and then interpret with the help of (1.6) to arrive at the result given in (2.1)
To establish (2.2), we again assume that \( p = p(k_1, k_2) = c_1 \xi_1^p + c_2 \xi_2^p \), \( (c_1, c_2) \) are constants) and \( \xi_1, \xi_2 \) are given by (1.7) and (1.8). In new of the series representation of \( H \left[ \begin{array}{c} x \\ y \end{array} \right] \), given in (1.6), we further let

\[
R_{k_1, k_2} = (-1)^{k_1+k_2} \phi(\xi_1, \xi_2) \theta_3(\xi_1) \theta_4(\xi_2) \left( \delta_{h_1}, F_{h_2} \right)^{-1}.
\]

Where \( \phi(\xi_1, \xi_2) \) and \( \theta_i(\xi_1) \) are given by (1.5) and (1.9), in (1.21) and then multiply both sides of (1.21) by \( \xi_1^{k_1} + \xi_2^{k_1-k_2} \) and sum up both sides of the resulting equation from \( h_1 = 1 \) to \( p_2 \) and \( h_2 = 1 \) to \( p_3 \) and then interpret with the help of (1.6) to arrive at result (2.2)

Similarly, the result in (2.3) is established with the help of (1.22) and (1.23)

Reference