Stability of Double- Diffusive Hydromagnetic Walter's (Model B') Visco- Elastic Fluid Permeated with Suspended Particles and Variable Gravity in Porous Medium

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Abstract

The problem of double- diffusive hydro magnetic instability of Walter's (Model B') visco-elastic fluid permeated with suspended particles and variable gravity in porous medium is considered in the presence of rotation. It is found that principle of exchange of stabilities is valid under certain conditions. For stationary convection, the Walter's (Model B') visco-elastic fluid behaves like a Newtonian fluid. The effect of magnetic field has been shown graphically also.

Keywords: : Thermosolutal convection, Walter's (model B'), Variable gravity field, Rotation Magnetic field, Suspended particles, Porous medium.

1. Introduction

Due to the application of solid mechanics, transpiration, cooling, food preservative, cosmetic industry, blood flow and artificial dialysis, the problem of flow through porous medium has received a great deal of attention both in technological and bio- physical fields. The thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been discussed in detail by **Chandrasekhar [1].**

Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Benard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, “Columnar instability”. He added an aerosol to mark the flow pattern.

The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by **Wooding [3]** whereas **Scanlon and Segel [4]** have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer. **Veronese [5]** has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. **Sharma and Kumar [6]** have discussed the steady flow and heat transfer of a Walter’s (Model B’) fluid through a porous pipe of uniform circular cross-section with small suction. When a fluid flows through a porous medium, the gross effect is represented by the usual Darcy’s law. The effect of suspended particles on the stability of stratified fluids in porous medium might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. **Sharma and Kumar [7]** have discussed the thermal instability of fluid in porous medium in the presence of suspended particles and rotation and found that rotation has stabilizing effect and suspended particles have destabilizing effect on the system. Thermal instability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by **Pradhan and Samal[8]. Kent [9]** has discussed the effect of horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field, the system is known to be stable. **Sharma and Rana [10]** have discussed the thermosolutal instability of Rivlin- Ericeksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium and found that stable solute gradient has a stabilizing effect on the system while the magnetic field and rotation have stabilizing effect under certain conditions. **Bhatia and Steiner [11]** have discussed the problem of thermal instability of visco-elastic fluid in hydromagnetics and found that the magnetic field has a stabilizing influence on Maxwell fluid just as in the case of Newtonian fluid. **Sisodia and Gupta [9] and Srivastava and Singh [12]** have discussed the unsteady flow of a dusty elastic-viscous Rivlin- Ericksen fluid through channel of different cross-sections in the presence of the time dependent pressure gradient. The Rayleigh-Taylor instability of plasma in presence of a variable magnetic field and suspended particles in porous medium is discussed by **Sunil and Chand [13]. Rudraiah and Prabhamani[14]** analyzed the effect of thermal diffusion on the convective instability of two components fluid in a porous medium. **Pundir and Pundir[15]** have obtained unstable wave number ranges in the presence of a magnetic field which are known to be stable in its absence, showing thereby that magnetic acts as catalyst for instability in certain situations. In view of the fact that the study of visco-elastic fluids in porous medium may find applications in geophysics, bio-fluid dynamics and chemical technology, a number of research workers have contributed in this direction.

The effect of rotation on double-diffusive hydromagnetics instability of Walter's (Model B') visco-elastic fluid in porous medium seems to be best of our knowledge, uninvestigated so far. In the present paper, we have made an attempt to discuss the effect of magnetic field on double-diffusive hydromagnetics instability of Walter's (Model B') visco-elastic fluid permeated with suspended particles and variable gravity in porous medium.

2. Objectives and Research Design

The objective of the study is to discuss the effect of magnetic field on double-diffusive hydromagnetics instability of Walter's (Model B') visco-elastic fluid. This is an experimental research. The objective has been proved with numerical computation and solving various equations - The equations of motions, continuity and heat conduction of Walter's (model B') visco-elastic fluid.

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3. Constitutive Equations and The Equations of Motion

Consider a static state in which an incompressible visco-elastic Walter's(Model B') fluid is arranged in an isotropic and homogeneous porous medium confined between two infinite horizontal planes situated at $z = 0$ and $z = d$, which is acted upon by a uniform rotation * $(0, 0, \Omega)$, magnetic field H $(0,0, H)$ and variable gravity $g(0,0, -g)$, $g = \lambda g_0$ $(g_0 > 0)$ is the value of g at $z = 0$ and λ can be positive or negative as gravity increases or decreases upwards from its value g_0.The fluid layer heated and soluted from below leading to an adverse temperature gradient β = $\frac{T_0 - T_1}{d}$ and a uniform solutal gradient β' = $\frac{C_0 - C_1}{d}$ where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ and C_0 and C_1 are the constants solute concentrations of the lower and upper surface with $C_0 > C_1$.

Let p, ρ, T, C, α, α', v, v' and q(u, v, w) denote, respectively, pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, kinematic viscosity, kinematic visco-elasticity and velocity of fluid. K'= $6\pi\rho\, v\,\eta$, η being particle radius, is Stoke's drag coefficient, $\bar{x}$ = (x, y, z), E = ϵ + (1-ϵ)$\left(\frac{\rho s\ Cs}{\rho 0\ Cf}\right)$ is constant and E' is a constant analogous to E but corresponding to solute rather than heat.

The equations of motions, continuity and heat conduction of Walter's (model B') visco-elastic fluid are

$$\frac{1}{\epsilon}\left[\frac{\partial q}{\partial t} + \frac{1}{\epsilon}(q.\nabla)q\right] = -\frac{1}{\rho_0}\nabla\delta p + g\left(1+\frac{\delta\rho}{\rho_0}\right) - \frac{1}{\rho_0 k_1}\left(\mu - \mu'\frac{\delta}{\delta t}\right)q + \frac{K'N}{\epsilon\rho_0}(q_d - q) + \frac{\mu_e}{4\pi\rho_0}(\nabla \times H)\times H + \frac{2}{\epsilon}(q \times *) \quad (1)$$

$$\nabla.\, q = 0 \quad (2)$$

$$E\frac{\delta T}{\delta t} + (q.\nabla)T + \frac{mN C_{pt}}{\rho_0 C_f}\left[\varepsilon\frac{\partial}{\partial t} + q_d.\nabla\right]T = k_T \nabla^2 T \quad (3)$$

$$E\frac{\delta C}{\delta t} + (q.\nabla)C + \frac{mN C_{pt}}{\rho_0 C_f}\left[\varepsilon\frac{\partial}{\partial t} + q_d.\nabla\right]C = k_s \nabla^2 C \quad (4)$$

$$mN\left[\frac{\partial q_d}{\partial t} + \frac{1}{\epsilon}(q_d.\nabla)q_d\right] = K'N(q - q_d) \quad (5)$$

$$\in \frac{\partial H}{\partial t} + (\nabla . N q_d) = 0 \tag{6}$$

$$\in \frac{\partial N}{\partial t} = \nabla \times (q \times H) + \in \eta \nabla^2 H \tag{7}$$

$$\nabla . H = 0 \tag{8}$$

And effective density described by

$$P = \rho_0[1- \alpha(T - T_0) + \alpha' (C - C_0 0] \tag{9}$$

Where ρ_s, C_s, ρ_0, C_r denote density and heat capacity of the solid porous material and q_d ($\bar{x}$, t), N ($\bar{x}$, t) denote, respectively the filter velocity and number density of the suspended particles and $\bar{x}$ = (x, y, z), mN is the mass of particles per unit volume and C_{pt} is the heat capacity of the particles.

Here, we assume that the uniform particles are in spherical shape and relative velocities between the fluid and particles are small. The presence of particles adds an extra force term in the equation (1), which is proportional to the velocity difference between particles and fluid. The distances between particles are assumed quite large compared with their diameters. Therefore, the interparticle reactions are ignored. The buoyancy force on the particles is neglected.

4. **Basic State and Perturbation Equations**

The basic state of system is taken to be a quiescent layer with a uniform particle distribution N_0, i.e. , q(0,0,0), q_d (0,0,0) and N = N_0, N_0 is a constant. Thus, the initial state whose stability we wish to examine is characterized by

$$q\ (0, 0, 0),\ q_d = (0, 0, 0),\ T = T_0 - \beta z\ ,\ C = C_0 - \beta' z,\ \rho = \rho_0\,[\ 1 + \alpha\beta z - \alpha'\beta' z],\ N_0 = \text{constant} \tag{10}$$

Let $\delta\rho$, δp, q (u,v,w), q_d (1,r,s), h (hx, hy, H + hz), Θ, Γ denote, respectively the perturbations in density ρ, pressure p, velocity q(0,0,0), particle velocity q_d (0, 0, 0), magnetic field H (0,0,H), temperature T and solute concentration C. Linearizing the equation in perturbations and analyzing the perturbations in to normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, \xi, hz, \zeta] = [W(z), \Theta(z), \Gamma(z), X(z), K(z), Z(z)], \exp\,[ik_x x + ik_y y + nt] \tag{11}$$

Where k_x and k_y are the wave numbers in x and y direction respectively, and $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance.

We eliminate the physical quantities using the non- dimensional parameters a = kd, $\sigma = \frac{nd^2}{\nu}$ and $D^* = Nd$ and dropping (*) for convenience, we obtain the following system of equations

$$(D^2 - a^2 - E_1 p_1 \sigma)\Theta = -\frac{\beta d^2}{k_T}\left(\frac{B+\sigma\tau_1}{1+\sigma\tau_1}\right)W \tag{12}$$

$$(D^2 - a^2 - E'_1 q\sigma)\Gamma = -\frac{\beta' d^2}{k_S}\left(\frac{B'+\sigma\tau_1}{1+\sigma\tau_1}\right)W \tag{13}$$

$$\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1-\sigma F}{P_t}\right](D^2 - a^2)W = -\frac{ga^2 d^2\alpha}{\nu}\Theta + \frac{ga^2 d^2\alpha'}{\nu}\Gamma$$

$$+\frac{\mu_e H d}{4\pi\rho_0 \nu}(D^2 - a^2)DK \quad -\frac{2\Omega d^2}{\epsilon\nu}DZ \tag{14}$$

$$\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1-\sigma F}{P_t}\right]Z = \frac{\mu_e H d}{4\pi\rho_0 \nu}DX + \frac{2\Omega d}{\epsilon\nu}DW \tag{15}$$

$$(D^2 - a^2 - \sigma p_2)K = -\frac{HdDW}{\epsilon\eta}) \tag{16}$$

$$(D^2 - a^2 - \sigma p_2)X = -\frac{HdDZ}{\epsilon\eta} \tag{17}$$

Apply the operator $(D^2 - a^2 - \sigma p_2)$ on equation (15) to eliminate X between equations (15) and (17), we obtain

$$\left[\left\{\frac{\sigma}{\in}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1-\sigma F}{P_t}\right\}(D^2-a^2-\sigma\rho_2)+\frac{Q}{\in}D^2\right]Z$$

$$=\frac{2\Omega d}{\in v}(D^2-a^2-\sigma p_2)DW \qquad (18)$$

Eliminating Θ, Γ, Z and K between the equation (13), (14), (16) and (18), we obtain the final stability governing equation

$$\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1-\sigma F}{p_t}\right](D^2-a^2-E_1p_1\sigma)(D^2-a^2-E_1q\sigma)(D^2-a^2-\sigma p_2)(D^2-a^2)W$$

$$-\lambda Ra^2\left(\frac{B+\sigma\tau_1}{1+\sigma\tau_1}\right)(D^2-a^2-E_1q\sigma)(D^2-a^2-\sigma p_2)W$$

$$+\lambda Sa^2\left(\frac{B+\sigma\tau_1}{1+\sigma\tau_1}\right)(D^2-a^2-E_1q\sigma)(D^2-a^2-\sigma p_2)W$$

$$+\frac{Q}{\epsilon}(D^2-a^2-E_1p_1\sigma)(D^2-a^2-E_1q\sigma)(D^2-a^2)D^2W$$

$$+\frac{T_A(D^2-a^2-E_1p_1\sigma)(D^2-a^2-E_1q\sigma)(D^2-a^2-E_1p_2\sigma)D^2W}{\epsilon^2\left[\left\{\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1-\sigma F}{P_t}\right\}(D^2-a^2-\sigma p_2)+\frac{Q}{\epsilon}D^2\right]} \qquad (19)$$

R = $\frac{g_0\alpha\beta d^4}{vk_T}$ is the thermal Rayleigh number, S = $\frac{g_0\alpha'\beta' d^4}{vk_s}$ is the analogous solute Rayleigh number,

$T_A=\left(\frac{2\Omega d^2}{v}\right)^2$ is the Taylor numbers, Q = $\frac{\mu_e H^2 d^2}{4\pi\rho_0 v\eta}$ is Chandrasekhar number and $\tau=\frac{m}{K'}$, $\tau_1=\frac{\tau\upsilon}{d^2}$,

$M=\frac{mN}{\rho_0}$, $p_t=\frac{k_1}{d^2}$, $E_1=E+b\epsilon$, $E_1=E'+b'\epsilon$, $B=b+1$, $B'=b'+1$, $F=\frac{\upsilon'}{d^2}$,

$p_1=\frac{\upsilon}{k_T}$, $q=\frac{\upsilon}{k_s}$ and $p_2=\frac{\upsilon}{\eta}$

Let both the boundaries are free and perfect conductors of heat and the adjoining medium is assumed to be electrically non- conducting. In this case, the appropriate boundary conditions are

$$W = D^2 W = \Gamma = \Theta = DZ = 0 \text{ at } z = 0 \text{ and } z = 1 \tag{20}$$

The components of h are continuous and tangential components are zero outside the fluids, so on the boundaries, we have

$$DK = 0, \tag{21}$$

It is clear that all the even order derivatives of W vanish on the boundaries after using the boundary conditions (20) and (21). Therefore, the proper solution of equation (19) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{22}$$

Where W_0 is constant. Using (22), equation (19) yields

$$R_1 = \left[\frac{i\sigma^2}{\epsilon}\left(1 + \frac{M}{1+i\sigma\pi^2\tau_1}\right) + \frac{1-i\sigma_1\pi^2 F}{p}\right]\left[\frac{1+X}{\lambda X}\right]\left[\frac{1+i\sigma\pi^2\tau_1}{B+i\sigma\pi^2\tau_1}\right](1+X+i\sigma_1 E_1 p_1)$$

$$+ S_1\left[\frac{B'+i\sigma\pi^2\tau_1}{B+i\sigma\pi^2\tau_1}\right]\left(\frac{1+X+i\sigma_1 E_1 p_1}{1+X+i\sigma_1 E_1 q}\right) + \frac{Q_1(1+X)}{\lambda X\epsilon}\left[\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right]\left(\frac{1+X+i\sigma_1 E_1 p_1}{1+X+i\sigma_1 E_1 p_2}\right)$$

$$+ \frac{T_{A1}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)(1+X+i\sigma_1 E_1 p_1)(1+X+i\sigma_1 p_2)}{\lambda X\epsilon^2\left[\left\{\frac{i\sigma_1}{\epsilon}\left(1+\frac{M}{1+i\sigma\pi^2\tau_1}\right)+\frac{1+i\sigma_1\pi^2 F}{p}\right\}(1+X+i\sigma_1 p_2)+\frac{Q_1}{\epsilon}\right]} \tag{23}$$

Where $R_1 = \frac{R}{\pi^4}$, $S_1 = \frac{S}{\pi^4}$, $Q_1 = \frac{Q}{\pi^2}$, $T_{A1} = \frac{T_A}{\pi^4}$, $\frac{\sigma}{\pi^2} = i\sigma_1$, $x = \frac{a^2}{\pi^2}$ and $p = \pi^2 p_t$

5.Findings and Conclusion

(a) Stationary and Convection:

At stationary convection, the marginal state will be characterized by $\sigma = 0$,

$$R_1 = \frac{1}{B}\left[\frac{1+X}{\lambda X}\left\{\frac{1+X}{p} + \frac{Q_1}{\epsilon} + \frac{T_{A1}\,p(1+X)}{\epsilon^2\left(1+X+\frac{pQ_1}{\epsilon}\right)}\right\} + S_1 B'\right] \tag{24}$$

The above relation express the modified Rayleigh number R_1 as a function of the parameters S_1, Q_1, T_{A1}, p and dimensionless wave number x. We observe that visco-elastic parameter F vanishes with σ , therefore, visco-elastic Walter's (model B') fluid behave like an ordinary Newtonian fluid. To examine the effect of magnetic field, rotation, suspended particles, stable solute gradient and medium permeability, we examine $\frac{dR_1}{dQ_1}, \frac{dR_1}{dT_{A1}}, \frac{dR_1}{dB}, \frac{dR_1}{dS_1}$ and $\frac{dR_1}{dp}$ analytically.

From equation (24), we obtain

$$\frac{dR_1}{dQ_1} = \frac{(1+X)}{\lambda XB}\left[\frac{1}{\epsilon} - \frac{p^2 T_{A1}(1+X)}{\epsilon^3\left(1+X+\frac{pQ_1}{\epsilon}\right)^2}\right] \tag{25}$$

Which shows that the magnetic field has stabilizing and destabilizing effect on the system and gravity increase upward ($\lambda > 0$) when

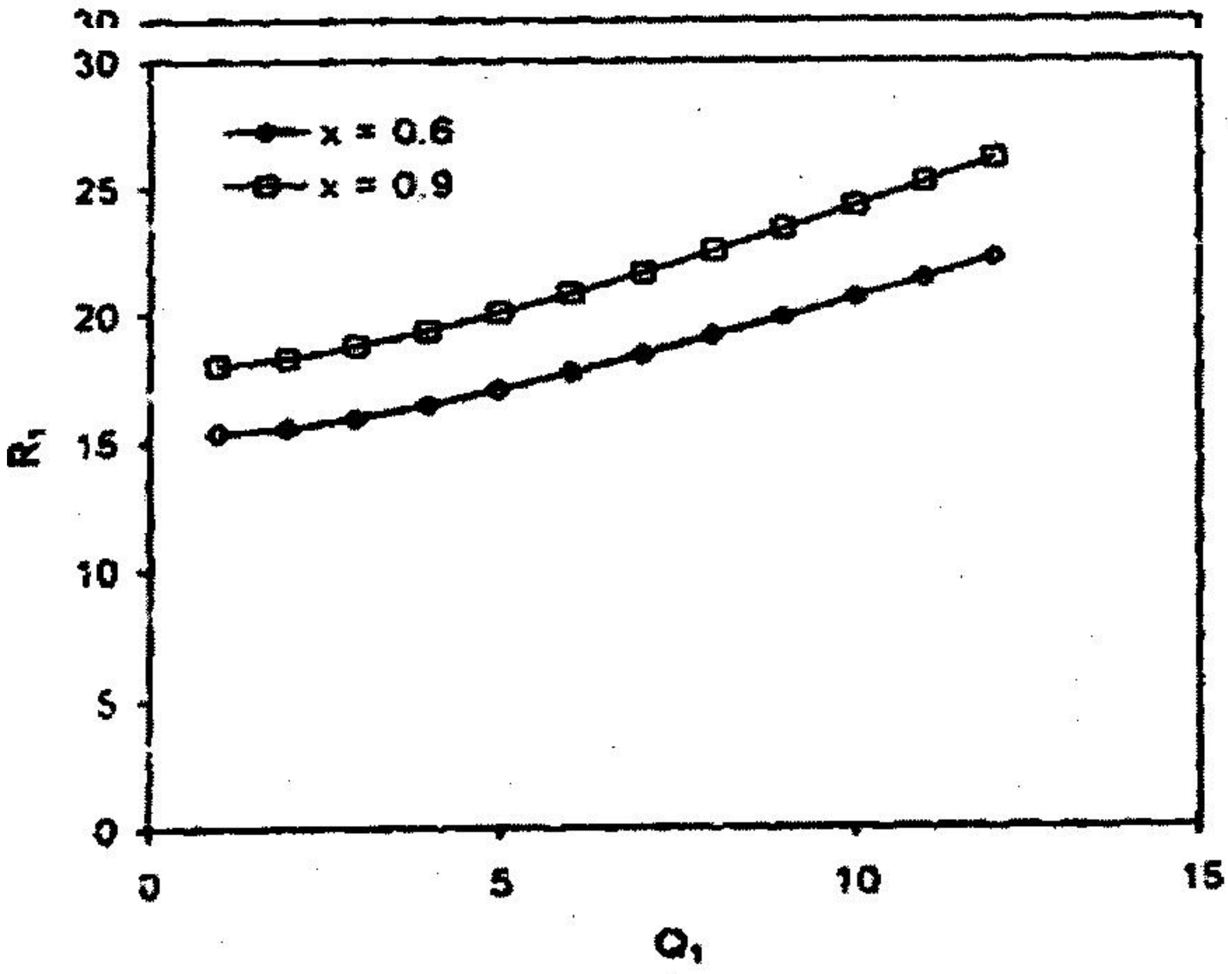
$$T_{A1} < \frac{\epsilon^2\left(1+X+\frac{pQ_1}{\epsilon}\right)^2}{p^2(1+X)} \quad \text{or} \quad T_{A1} > \frac{\epsilon^2\left(1+X+\frac{pQ_1}{\epsilon}\right)^2}{p^2(1+X)}$$

Also, from equation (25), we observe that, in the absence of rotation, magnetic field has a stabilizing effect on the system.

6. Numerical Computation

The dispersion relation (24) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

In Fig 1, R_1 is plotted against Q_1 for $\lambda = 0.2$, $p = 0.2$, $S_1 = 10$, $\epsilon = 0.5$, $B' = 2$, $B = 3$, $T_{A1} = 15$ for fixed wave numbers $x = 0.6$ and $x = 0.9$. This shows that magnetic field has a stabilizing effect on the system.



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