New Separation Axioms Using the idea of "Gem-Set" in Topological Space

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Abstract

In this paper, we create a new set of topological space namely "Gem-Set" and immersed it with a new separation axioms in topological space and investigate the relationship between them

Keywords: "Gem-Set", separation axioms.

1. **Introduction and Preliminaries.**

The idea of "Gem-Set" is defined as: for a topological space (X,T) , and $A\subseteq X$, we defined A^{*x} with respect to space (X,T) as follows: $A^{*x} = \{y \in X : G \cap A \notin I_x$,for every $G \in T(y)\}$ where $T(y) = \{G \in T : y \in G\}$, I_x is an ideal on a topological space (X,T) at point x is defined by $I_x = \{U\subseteq X : x\in U^c\}$,where U is non-empty set of X.

Within this paper "Gem-Set" is studied with some its properties, a set of new separation axioms in topological spaces, namely "I^*-T_0-space", "I^*-T_1-space", "I^*-T_2-space", "I^{**}-T_0-space", "I^{**}-T_1-space","I^{**}-T_2-space" and the axioms R_i , i = 0,1,2,3 are proposed by using the idea of "Gem-Set", the relationship between them is studied. Also two mappings " I^*- map " and" I^{**}- map " are defined to carry properties of "Gem-Set" from a space to other space.

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A), respectively.

Definition 1.1.

A topological space(X,T) is called

- R_o –space[1,2,4]if and only if for each open set G and $x \in G$ implies $cl(\{x\}) \subseteq G$.
- R_1 –space[1,2,4]if and only if for each two distinct point x, y of X with $cl(\{x\})\neq cl(\{y\})$, then there exist disjoint open sets U,V such that $cl(\{x\}) \subseteq U$ and $cl(\{y\}) \subseteq V$
- R_2-space [2]if it is property regular space.
- R_3-space [3]if and only if (X,T) is a normal and R_1-space.

Remark 1.2[3]

Each separation axiom is defined as the conjunction of two weaker axiom :T_k- space =R_{k-1}-space and T_{k-1} space =R_{k-1}-space and T_0-space , k=1,2,3,4

Remark 1.3[3]

Every R_i –space is an R_{i-1}-space i = 0,1,2,3.

2. "Gem-Set" in Topological Space

Definition 2.1

For a topological space (X, T), $x\in X$, $Y\subseteq X$, we define an ideal ${}^{Y}I_x$ with respect to subspace (Y, T_Y) as follows: ${}^{Y}I_x = \{G \subseteq Y:x \in (X-G)\}$..

Remark 2.2

For a topological space (X, T) ,$Y\subseteq X$, for each $G \neq\emptyset$. Then ${}^{Y}I_x=\{G \subseteq Y:x \in (X-G)$,for each $x\in Y\}=\{G \subseteq Y:x \in (Y-G)$ for each $x\in Y\}$.

Proposition 2.3

For a topological space (X, T) , $Y\subseteq X$, for each $G \neq\emptyset$. Then ${}^{Y}I_x=\{G \subseteq Y: x \in (X-G) = \{G \cap Y$: for each$\emptyset\neq G\in I_x$ },

Definition 2.4

For a topological space (X, T) , $Y \subseteq X$, and $A \subseteq Y$, we defined ${}_Y A^{*x}$ with respect to subspace (Y, T_Y) as follows: for $A \subset Y$, ${}_Y A^{*x} = \{y \in Y : G \cap A \notin I_x$,for every $G \in T_Y(y)\}$ where $T_Y(y) = \{G \in T_Y : y \in G\}$.

Note 2.5

For a topological space (X,T) contains singleton point (say x),then $I_x=\varnothing$.

Proposition 2.6

Let (X, T) be a topological space, and let A, B be subsets of X, $x \in X$. Then

- $\varphi^{*x} = \varphi$
- $X^{*x}=X$, whenever $I_x=\varnothing$.
- $A \subset B$ implies $A^{*x} \subset B^{*x}$.
- For another ideal $I_y \supseteq I_x$ on X, $A^{*y} \subset A^{*x}$.
- If $x \in X$. Then $x \in A$ if and only if $x \in A^{*x}$.
- If $x \in A$,then $(A^{*x})^{*x}= A^{*x}$.
- If $x \in A, y \in B$ such that $x \neq y$, then $A^{*x} \cap B^{*y}=\varnothing$.
- If $x,y \in X$ such that $x \neq y$, then $y \in \{x\}^c$ implies $x \notin \{x\}^{*y}$ and $y \notin \{y\}^{*x}$
- $A^{*x} \cup B^{*x} =(A \cup B)^{*x}$
- $(A \cap B)^{*x} \subset A^{*x} \cap B^{*x}$.
- $A^{*x} \subset cl(A)$.

Proof : Straight forward .

Proposition 2.7

Let a topological space (X, T) then for open set V, $V \cap A^{*x} = V \cap (V \cap A)^{*x} \subset (V \cap A)^{*x}$,for any $x \in X$.

Proof: Straight forward

Definition 2.8 Let (X,T) be a topological space and $A \subseteq X$ .We define ${}^{*x}pr(A)$, as following: ${}^{*x}pr(A)= A^{*x} \cup A$,for each $x \in X$.

Theorem 2.9 Let E and F be such sets of (X, T), $x \in X$. Then

- ${}^{*x}pr(\phi) = \phi$
- ${}^{*x}pr(X) = X$
- If $E \subseteq F$, then ${}^{*x}pr(E) \subseteq {}^{*x}pr(F)$.
- ${}^{*x}pr(E \cup F) = {}^{*x}pr(E) \cup {}^{*x}pr(F)$.
- ${}^{*x}pr(E \cap F) \subseteq {}^{*x}pr(E) \cap {}^{*x}pr(F)$.

Proof Straight forward.

Proposition 2.10

Let (X,T) be a topological space and $A \subset X$. If A is a closed set ,then ${}^{*x}pr(A)=A^{*x}=A=cl(A)$,for each $x \in A$.

Definition 2.11

A subset A of a topological space (X,T) is called prefected set if $A^{*x} \subseteq A$, for each $x \in X$**.**

Definition 2.12

A subset A of a topological space (X,T) is called coprefected set if A^c is a prefected set.

Lemma 2.13

Let (X,T) be a topological space ,then every a closed set is prefected set

Proof

Let A be a subset closed of X, then cl(A)=A .But $A^{*x} \subset cl(A)$, for each $x \in X$ [By proposition 2.6],so that $A^{*x} \subset cl(A)=A$,thus $A^{*x} \subseteq A$,for each $x \in X$ .Hence A is prefected set.

3. I^*-T_i , I^{**}-T_i, *i=0,1,2 and* R_i , *i =0,1,2and 3*

Definition 3.1.

A topological space(X,T) is called

- I^*-T_0-space if and only if if for each pair of distinct points x, y of X, there exist non-empty subsets A,B of X such that $y\notin A^{*x}$ or $x\notin B^{*y}$.
- I^*-T_1-space if and only if if for each pair of distinct points x, y of X, there exist non-empty subsets A,B of X such that $y\notin A^{*x}$ and $x\notin B^{*y}$.
- I^*-T_2-space if and only if for each pair of distinct points x,y of X, there exist subsets A,B of X such that $A^{*x}\cap B^{*y}=\emptyset$, with $y\notin A^{*x}$ and $x\notin B^{*y}$.
- I^{**}-T_0-space if and only if if for each pair of distinct points x, y of X, there exists non-empty subset A of X such that $y\notin A^{*x}$ or $x\notin A^{*y}$.
- I^{**}-T_1-space if and only if if for each pair of distinct points x, y of X, there exists non-empty subset A of X such that $y\notin A^{*x}$ and $x\notin A^{*y}$.
- I^{**}-T_2-space if and only if for each pair of distinct points x,y of X, there exists subset A of X such that $A^{*x}\cap A^{*y}=\emptyset$, with $y\notin A^{*x}$ and $x\notin A^{*y}$.

Definition 3.2

If (X,T) is a topological space and $Y\subseteq X$, we say that Y is an I^*-T_0-subspace(I^*-T_1-subspace) of X iff for each pair of distinct points y^1,y^2 of X, there exist non-empty subsets A,B of Y such that $y^2\notin A^{*y1}$ or (and) $y^1\notin B^{*y2}$.

Definition 3.3

Let (X,T) be a topological space ,for each $x\in X$, a non-empty subset A of X, is called a strongly set if and only if (A^{*x} is open set and $x\in A$).

Definition 3.4

A topological space (X, T) is said to be a strongly-T_I-space (briefly s-T_I-space)if and only if ,for each non-empty subset A of X is a strongly set.

Theorem 3.5

For a topological space (X, T) ,then the following properties hold:

1. Every T_0– space is a I^*-T_0–space.
2. Every T_1– space is a I^*-T_1–space.
3. Every T_2– space is a I^*-T_2–space.
4. Every T_0– space is a I^{**}-T_0–space.
5. Every T_1– space is a I^{**}-T_1–space.
6. Every T_2– space is a I^{**}-T_2–space.

Proof:(1)

Let $x,y\in X$ such that $x\neq y$ and let (X,T) is T_0–space. Then there exist an open set U such that, $x\in U$,$y\notin U$ or there exist an open set V such that, $y\in V$, $x\notin V$ and so , $U\cap\{y\}=\emptyset\in I_y$ or $V\cap\{x\}=\emptyset\in I_x$. Put A={x},B={y}.It is follows that $x\notin B^{*y}$ or $y\notin A^{*x}$. Hence let (X,T) be a I^*-T_0–space.

Proof:(2)

Let $x,y\in X$ such that $x\neq y$ and let (X,T) is T_1–space. Then there exist an open set U such that, $x\in U$,$y\notin U$, and there exist an open set V such that, $y\in V$, $x\notin V$ and so , $U\cap\{y\}=\emptyset\in I_y$ and $V\cap\{x\}=\emptyset\in I_x$. Put A={x},B={y}.It is follows that $x\notin B^{*y}$and $y\notin A^{*x}$. Hence let (X,T) be a I^*-T_1–space**.**

Proof:(3)

Let (X, T) be T_2– space. Then for each $x\neq y\in X$ there exist open sets U ,V such that $x\in U$, and $y\in V$ and $U\cap V=\emptyset$. But $U^{*x}\cap V^{*y}=\emptyset$ [Proposition 3.1.10]. Put A=U,B=V . It is follows that there exist subsets A,B of X such that $A^{*x}\cap B^{*y}=\emptyset$, with $y\notin A^{*x}$ and $x\notin B^{*y}$. Thus(X,T) is I^*-T_2–space.

Proof:(4) By the same proof of part(1) .

Proof:(5) Assume that (X,T) is an T_1–space and let $x,y\in X$ such that $x\neq y$. By assumption, Then there exist an open set U such that, $x\in U$,$y\notin U$, and there exist an open set V such that, $y\in V$, $x\notin V$. So that $U\cap\{y\}=\emptyset\in I_y$ and $V\cap\{y\}=\{y\}\in I_x$. Put A={y} and so $x\notin A^{*y}$ and $y\notin A^{*x}$. Hence (X,T) is I^{**}-T_1–space.

Proof:(6) By the same way of proof of part(3).

Remark 3.6

The converse of theorem need not be true as seen from the following examples.

Example 3.7

Let (X,T) be a topological space such that X={x,y,z}T={∅,X,{y,z},{x,z},{z}} and I_x={∅,{y},{z}{y,z}} ,I_y={∅,{x},{x,z},{z}}.Set A={x}. A^{*x}={x}, so that $y\notin A^{*x}$. Hence (X,T) is an I^*-T_0- space(I^{**}-T_0- space), but not T_0–space.

Example 3.8

Let (X,T) be a topological space such that X={x,y,z} T={∅,X,{y,z},{x,z},{z}}, and I_x={∅,{y},{z}{y,z}}, I_y={∅,{x}, {x,z} ,{z}}.Set A={x}.B={y},A^{*x}={x}, and B^{*y}={y} ,so that $y \notin A^{*x}$ and $x \notin B^{*y}$. Hence (X,T) is an I^*-T_1-space(I^{**}-T_1- space), but not T_1–space.

Example 3.9

Let (X,T) be a topological space such that X={x,y,z} T= {∅,X {y,z} ,{x,z} ,{z}}, and I_x={∅,{y},{z}{y,z}}, I_y={∅,{x}, {x,z} ,{z}} .Set A={x}.B={y},A^{*x}={x}, and A^{*y}={y} ,so that $A^{*x} \cap B^{*y} = \emptyset$. Hence (X,T) is an I^*-T_2- space(I^{**}-T_2- space), but not T_2–space.

Remark 3.10

The converse of theorem 3.5 , need not be true. But it is true generally, if (X,T) is a s-T_I-space

Theorem 3.11

If (X,T) is an I^*-T_o -space and $Y \subseteq X$, then Y is I^*-T_o –subspace

Proof

Let (X,T) is an I^*-T_o -space and Y is a subspace of X. Let y^1 and y^2 be two distinct points of Y. Since $Y \subseteq X$ and y^1, y^2 are distinct points of X. Again, since X is an I^*-T_o -space, there exist non-empty subset A,B of X such that $y^2 \notin B^{*y1}$ or $y^1 \notin A^{*y2}$. Suppose, $y^1 \notin A^{*y2}$, so that there exists an T-open set U such that, $y^1 \in U$, $U \cap A \in I_{y2}$. Put $U' = U \cap Y$ is T_Y-open and $A' = A \cap Y$, so that U' containing y^1 and $U' \cap A' \subseteq U \cap A \in I_{y2}$. It is follows that $y^1 \notin A'^{*y2}$.So by definition, we have that Y is I^*-T_o–subspace

Theorem 3.12

If (X,T) is an I^*-T_1-space and $Y \subseteq X$, then Y is I^*-T_1 –subspace

Proof

Let (X,T) is an I^*-T_1 -space and Y is a subspace of X. Let y^1 and y^2 be two distinct points of Y. Since $Y \subseteq X$ and y^1, y^2 are distinct points of X. Again, since X is an I^*-T_1 -space, there exist a subset A,B of X such that $y^2 \notin B^{*y1}$ and $y^1 \notin A^{*y2}$, so that there exist an T-open set U such that, $y^1 \in U$, $U \cap A \in {}^X I_{y2,}$ and there exist an T-open set V such that, $y^2 \in V$, $V \cap B \in I_{y1}$. Put $U' = U \cap Y$ and $V' = V \cap Y$ are T_Y-open, $A' = A \cap Y$, $B' = B \cap Y$,so that U'containing y^1,V'containing y^2,thus $U' \cap A' \subseteq U \cap A \in I_{y2}$and $V' \cap B' \subseteq V \cap B \in I_{y1}$.It is follows that $y^2 \notin B'^{*y1}$ and $y^1 \notin A'^{*y2}$.So by definition, we have that Y is I^*-T_1–subspace

Theorem 3.13

A topological space (X, T) is an R_0-space if and only if for each $x \in X$ and U open set such that $x \in U$, then $cl(\{x\}^{*x}) \subseteq U$.

Proof

Let $x \in X$ and *U* open set such that $x \in U$. By assumption ,then $cl(\{x\}) \subseteq U$. But $\{x\}^{*x} \subseteq cl(\{x\})$ [By Proposition 2.8].Therefore $cl(\{x\}^{*x}) \subseteq cl(cl(\{x\}))$ implies $cl(\{x\}^{*x}) \subseteq cl(\{x\})$.Thus $cl(\{x\}^{*x}) \subseteq U$.

Conversely, to prove (X, T) is R_0-space, let $U \in T$ and $x \in U$.Since, $\{x\} \subseteq \{x\}^{*x}$. Then $cl(\{x\}) \subseteq cl(\{x\}^{*x}) \subseteq U$. Thus $cl(\{x\}) \subseteq U$. Therefore (X,T) is R_0 – space.

Theorem 3.14

A topological space(X,T) is R_1 – space if and only if , for each $x,y \in X$ and $A \in X$, such that $x \neq y$ and $cl(\{x\}) \neq cl(\{y\})$,then ,there exist disjoint open sets U,V such that $cl(\{x\}^{*x}) \subseteq U$ and $cl(\{y\}^{*x}) \subseteq V$.

Proof

Let $x,y \in X$ and $A \in X$, with $x \neq y$, and $cl(\{x\}) \neq cl(\{y\})$. By assumption ,then there exist disjoint open sets U,V such that $cl(\{x\}) \subseteq U$ and $cl(\{y\}) \subseteq V$. But $\{x\}^{*x} \subseteq cl(\{x\})$ and $\{y\}^{*x} \subseteq cl(\{y\})$ [By Proposition 2.8].Therefore $cl(\{x\}^{*x}) \subseteq cl(cl(\{x\}))$ and $cl(\{y\}^{*x}) \subseteq cl(cl(\{y\}))$ .This implies $cl(\{x\}^{*x}) \subseteq cl(\{x\})$ and $cl(\{y\}^{*x}) \subseteq cl(\{y\})$.Thus $cl(\{x\}^{*x}) \subseteq U$ and $cl(\{y\}^{*x}) \subseteq V$.

Conversely ,let $x,y \in X$ such that $x \neq y$ and $cl(\{x\}) \neq cl(\{y\})$.By assumption , then there exist disjoint open sets U,V such that $cl(\{x\}^{*x}) \subseteq U$ and $cl(\{y\}^{*x}) \subseteq V$. Now ,since, $\{x\} \subseteq \{x\}^{*x}$ and $\{y\} \subseteq \{y\}^{*x}$. Then ,$cl(\{x\}) \subseteq cl(\{x\}^{*x}) \subseteq U$ and $cl(\{y\}) \subseteq cl(\{y\}^{*x}) \subseteq V$. Thus $cl(\{x\}) \subseteq U$ and $cl(\{y\}) \subseteq V$. Therefore (X,T) is R_1 – space.

Theorem 3.15

A s-T_I-space (X, T) is regular space iff for each F closed set and $x \notin F$, then $\{x\}^{*x} \cap F^{*y} = \emptyset$.

Proof

Let F be closed set and $x \notin F$,thus $y \in F$, so that$\{x\}^{*x} \cap F^{*y} = \emptyset$[Proposition 2.6].

Conversely, let F be closed set and $x \notin F$ and $y \in F$ implies$\{x\}^{*x} \cap F^{*y} = \emptyset$. Since (X,T) is a s-$T_I$-space and by definition 3.1, we get that {x},F are a strongly sets, so $\{x\}^{*x}$, F^{*x} an open subsets of X , with $x \in \{x\}^{*x}$ and $F \subseteq F^{*y}$. Thus (X, T) is regular space.

Theorem 3.16

A s-T_I-space (X, T) is normal space iff for each disjoint closed sets F , H, then $F^{*x} \cap H^{*x} = \varnothing$.

Proof

By the same way of proof of above theorem.

Theorem 3.17

For a topological space (X, T) ,then the following properties hold:

1. (X,T) is I^*-T_0–space iff I^{**}-T_0–space.
2. Every I^*-T_1–space is a I^{**}-T_1–space.
3. Every I^*-T_2–space is a I^{**}-T_2–space.

Proof : Straight forward.

Remark 3.18

The converse of part(2),(3) ,need not be true as seen from the following examples.

Example 3.19

Let (X,T) be a topological space such that X={x,y,z} , T={∅,X,{z}}, and I_x={∅,{y},{y,z},{z}}, I_y={∅,{x},{x,z},{z}}.Set A={z} ,B={x,y} ,then $A^{*x}=\{\varnothing\}$, $A^{*y}=\{\varnothing\}$,B^{*y} ={x,y},that means $y \notin A^{*x}$ and $x \notin A^{*y}$ but $y \notin A^{*x}$ and $x \in B^{*y}$. Hence(X,T) is an I^{**}-T_1 –space but not I^*-T_1 -space

Example 3.20

Let (X,T) be a topological space such that X={x,y,z,w} T={∅,X,{x,y},{x,y,z},{z}},and I_z={∅,{y},{x},{w},{x,y},{x,w},{x,y,w},{w,y}},I_y={∅,{x},{z},{w},{x,z},{x,w},{z,w},{x,z,w}}.Set A= {y,z} , B={a,c} then A^{*y}= {x,y,w} and $A^{*z} = B^{*z}$ ={z,w} ,so that $y \notin A^{*y}$ and $x \notin A^{*z}$. But $A^{*y} \cap B^{*z} \neq \varnothing$ .Hence (X,T) is an I^{**}-T_1- space, but not I^*-T_2–space.

Theorem 3.21

A s-T_I-space (X, T) is a T_i-space if and only if it is R_{i-1}-space and I^*-T_j-space, i = 1,2,3,4 ,j = 0,1,2

Proof

By theorem 3.1, remark 3.10 and remark 1.2.

Theorem 3.22

A s-T_I-space (X, T) is a T_i-space if and only if it is R_{i-1}-space and I^{**}-T_j-space, i = 1,2,3,4 ,j = 0,1,2

Proof

By theorem 3.1, theorem 3.17, remark 3.10 and remark 1.2.

4. I^*- map and I^{**}- map

Definition 4.1

A mapping f: (X,T) → (Y,σ) is called I^*- map .If and only if, for every subset A of X, x∈X, $f(A^{*x}) = (f(A))^{*f(x)}$.

Definition 4.2

A mapping f: (X,T) → (Y,σ) is called I^{**}- map. If and only if, for every subset A of Y,y∈Y, $f^{-1}(\mathbf{A}^{*y}) = (f^{-1}(A))^{*f^{-1}(y)}$

Theorem 4.3

If f: (X,T)→ (Y,σ) is one-one I^*- map of an I^*-T_0–space X onto a space Y, then Y is an I^*-T_0–space.

Proof

Let(X,T) be I^*-T_0–space and f: X→Y be onto , one-one and I^*- map. We want to prove that Y is I^*-T_0–space. Let y^1 and y^2 be two distinct points of Y. Since f is one-one and onto, there exists distinct points x_1, x_2 of X such that $f(x_1)= y^1$ and $f(x_2) = y^2$.Since (X,T) is I^*-T_0–space, there exist non-empty subsets A,B of X such that $x_2 \notin A^{*x1}$ or $x_1 \notin B^{*x2}$, so that $f(x_2) \notin (f(A^{*x1}) = (f(A))^{*f(x1)}$ or $f(x_1) \notin (f(B^{*x2}) = (f(B))^{*f(x2)}$ .Thus $y^2 \notin (f(A))^{*f(x1)=y1}$ and $y^1 \notin (f(B))^{*f(x2)=y2}$.Therefore we get that Y is I^*-T_0–space.

Theorem 4.4

If f: (X,T)→ (Y,σ) is one-one I^*- map of an I^*-T_1–space X onto a space Y, then Y is an I^*-T_1–space.

Proof

By the same way of proof of above theorem**.**

Theorem 4.5

If f: (X,T)→ (Y,σ) is I^{**}- map injection of a space X into I^*-T_0–space Y, then X is an I^*-T_0–space.

Proof

Let (Y,σ) be I^*-T_0–space and f: $X\to Y$ be I^{**}-map injection. We want to prove that X is I^*-T_0–space. Let x^1 and x^2 be two distinct points of X. Since f is injection, then $f(x^1)\neq f(x^2)$. Since (Y,σ) is I^*-T_0–space, there exist non-empty subsets C,D of Y such that $f(x^1)\notin C^{*f(x2)}$ or $f(x^2)\notin D^{*f(x1)}$,so that $f^{-1}(f(x^1))\notin f^{-1}(C^{*f(x2)}) =(f^{-1}(C))^{*f-1(f(x2))}$ or $f^{-1}(f(x^2))\notin f^{-1}(D^{*f(x1)}) =(f^{-1}(D))^{*f-1(f(x1))}$.This implies $x^1\notin(f^{-1}(C))^{*x2}$ or $x^2\notin(f^{-1}(D))^{*x1}$.Therefore we get that X is I^*-T_0–space.

Theorem 4.6

If f: $(X,T)\to(Y,\sigma)$ is I^{**}- map injection of a space X into I^*-T_1–space Y, then X is an I^*-T_1–space.

Proof

By the same way of proof of above theorem**.**

Theorem 4.7

If f: $(X,T)\to(Y,\sigma)$ is one-one ,I^*- map of an I^*-T_2–space X onto a space Y, then Y is an I^*-T_2–space.

Proof

Let (X,T) be I^*-T_2–space and f: $X\to Y$ be , one-one onto I^*- map. We want to prove that $f(X) = Y$ is I^*-T_2–space. Let y^1 and y^2 be two distinct points of Y. Since f is onto I^*- map, there exists distinct points x_1, x_2 of X such that $f(x_1) = y^1$ and $f(x_2) = y^2$.Since (X,T) is I^*-T_2–space, there exist non-empty subsets A,B of X such that $A^{*x1}\cap B^{*x2}=\varnothing$,with $x_2\notin A^{*x1}$ and $x_1\notin B^{*x2}$. But f is onto I^*-map, so that $f(A^{*x1})\cap f(B^{*x2})=f(A)^{*f(x1)}\cap f(B)^{*f(x2)}=f(A)^{*y1}\cap f(B)^{*y2}=\varnothing$, with $f(x_2)\notin(f(A))^{*f(x1)}$ and $f(x_1)\notin(f(B))^{*f(x2)}$.Thus there exist non-empty subsets f(A),f(B) of Y such that $f(A)^{*y1}\cap f(B)^{*y2}=\varnothing$, with $y^2\notin(f(A))^{*y1}$ and $y^1\notin(f(B))^{*y2}$. Therefore by definition we get that Y is I^*-T_2–space.

Theorem 4.8

If f: $(X,T)\to(Y,\sigma)$ is I^{**}- map injection of a space X into I^*-T_2–space Y, then X is an I^*-T_2–space.

Proof

Let (Y,σ) be I^*-T_2–space and f: $X\to Y$ be I^{**}- map continuous injection. We want to prove that X is I^*-T_2–space. Let x^1 and x^2 be two distinct points of X. Since f is injection, then $f(x^1)\neq f(x^2)$.Since (Y,σ) is I^*-T_2–space, there exist non-empty subsets C,D of Y such such $C^{*f(x1)}\cap D^{*f(x2)}=\varnothing$, with $f(x_2)\notin(f(C))^{*f(x1)}$ and $f(x_1)\notin(f(D))^{*f(x2)}$.
But f is I^{**}- map injection, so that $(f^{-1}(C))^{*(x1)}\cap(f^{-1}(D))^{*(x2)} = f^{-1}(C^{*f(x1)})\cap f^{-1}(D^{*f(x2)})=f^{-1}(C^{*f(x1)}\cap D^{*f(x2)})= f^{-1}(\{\varnothing\})=\varnothing$ .Thus there exist non-empty subsets $f^{-1}(C)$, $f^{-1}(C)$ of X such that $(f^{-1}(C))^{*(x2)}\cap(f^{-1}(D))^{*(x1)}$, with $x^2\notin(f^{-1}(C))^{*x1}$ and $x^1\notin(f^{-1}(D))^{*x2}=\varnothing$,for each x^1 and x^2 be two distinct points of X. Therefore by definition we get that X is I^*-T_2–space.

Corollary 4.9

If f: $(X,T)\to(Y,\sigma)$ is I^{**}- map injection of a s-T_I-space X into I^*-T_2–space Y, then X is T_0–space.

Corollary 4.10

If f: $(X,T)\to(Y,\sigma)$ is I^{**}- map injection of a s-T_I-space X into I^*-T_2–space Y, then X is an T_1–space.

Theorem 4.11

If f: $(X,T)\to(Y,\sigma)$ is continuous, injection function of a space X into T_2–space Y, then X is an I^*-T_1–space.

Proof

Let (Y,σ) be T_2–space and f: $X\to Y$ be continuous, injection function. We want to prove that X is I^*-T_1–space. Let x^1 and x^2 be two distinct points of X. Since f is injection, then $f(x^1)\neq f(x^2)$.Since (Y,σ) is T_2–space, then there exist V_1 and $V_2\in T_Y$ such that $f(x^1)\in V_1$, $f(x^2)\in V_2$ and$V_1\cap V_2=\varnothing$.This implies $x^1\in f^{-1}(V_1)$ and $x^2\in f^{-1}(V_2)$. So that $f^{-1}(V_1)\cap\{x^2\}=\varnothing\in I_{x2}$ and $f^{-1}(V_2)\cap\{x^1\}=\varnothing\in I_{x1}$.Put $A=\{x^2\}$, $B=\{x^1\}$.It is follows that $x^1\notin A^{*x2}$ and $x^2\notin A^{*x1}$. Therefore by definition we get that X is I^*-T_1–space**.**

Corollary 4.12

If f: $(X,T)\to(Y,\sigma)$ is injection function of a space X into T_2–space Y, then X is an I^*-T_o–space.

Proof It is clear [Since every I^*-T_1–space is I^*-T_o–space]**.**

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