Modified Trapezoidal Rule Based Different Averages for Numerical Integration

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Abstract:

In this paper, A Modified Trapezoidal Rule is presented for the evaluation of numerical integration; the proposed method appears to be efficient modification of trapezoidal rule which is the composition of arithmetic mean of subintervals while the proposed method uses arithmetic mean and Heronian mean at the subintervals. The accuracy of this rule is higher than the original trapezoidal rule .the comparison between the modified trapezoidal rule and original trapezoidal rule is made by using numerical experiments on the basis of local truncation Error.

Keywords: Modified trapezoidal rule, Arithmetic mean, Heronian mean, Numerical examples, Accuracy, Numerical Integration.

1 Introduction

In numerical Analysis, Numerical Integration is the process of computing the value of definite integrals. When analytical methods failed or integrand function given in a data table .it has the several applications in the field of physics and engineering, the problems of calculating an integral numerically occur very often in physics, first subdividing the integrand into equidistant mesh and apply a simple rule of integration then finally required accuracy achived.General quadrature rule for the evaluation of numerical integration is given by

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n} w_i f(x_i) \qquad (1)$$

Subdividing the finite interval $[a,b]$ into large number of subintervals by taking the step size as $h=\frac{b-a}{n}$ by defining $(n+1)$ intermediate points such that $a=x_0<x_1<x_2<x_3<\Lambda<x_n=b, x_i=x_0+ih$ and $(n+1)$ $w_0,w_1,w_2,\Lambda\ w_n$ take the values for $w_i=0,1,2,3...n$.

The well known method for Computation of numerical integration is closed Newton cotes quadrature formula .if we put the value of $n=1$ in Newton cotes formula we get a formula. When $n=1$: Trapezoidal Rule

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$$\int_a^b f(x)dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + \Lambda + y_{n-1}) + y_n\right] \qquad (2)$$

In this formula Arithmetic mean is used at the subintervals so trapezoidal rule is the composition of arithmetic mean.

In this paper, a modified trapezoidal rule is presented, based on arithmetic mean and heronian mean. This modified rule is the composition of two times arithmetic mean and two times heronian mean of subintervals respectively. Numerical examples are discussed and the results show that the accuracy of the new approach is higher than the existing trapezoidal rule.

2 Arithmetic Mean and Heronian Mean –Based Trapezoidal Rule.

In this section, modified trapezoidal rule is derived for the computation of definite integrals $\int_a^b f(x)dx$ over $[a,b]$. First tabulate the function by taking step size $h = \frac{b-a}{n}$

X	x_0	x_1	x_2	…….	x_{n-1}	x_n
Y	y_0	y_1	y_2		y_{n-1}	y_n

Now find Arithmetic mean and Heronian mean two times respectively

$$I_1 = \int_{x_0}^{x_1} f(x)dx = h(\frac{y_0 + y_1}{2})$$

$$I_2 = \int_{x1}^{x_2} f(x)dx = h(\frac{y_1 + y_2}{2})$$

$$I_3 = \int_{x_2}^{x_3} f(x)dx = h(\frac{y_2 + \sqrt{y_2 y_3} + y_3}{3})$$

$$I_4 = \int_{x_3}^{x_4} f(x)dx = h(\frac{y_3 + \sqrt{y_3 y_4} + y_4}{3})$$

$$I_5 = \int_{x_4}^{x_5} f(x)dx = h(\frac{y_4 + y_5}{2})$$

$$I_6 = \int_{x_5}^{x_6} f(x)dx = h(\frac{y_5 + y_6}{2})$$

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Continue the above procedure then add the results.

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$$\int_a^b f(x)dx = h[\frac{(y_0+y_1)}{2}+\frac{(y_1+y_2)}{2}+\frac{(y_2+\sqrt{y_2y_3}+y_3)}{3}+\frac{(y_3+\sqrt{y_3y_4}+y_4)}{3}+\Lambda\ ]$$

3 Numerical examples

In this section, in order to compare the effectiveness of original Trapezoidal rule and Modified Trapezoidal rule Based Arithmetic and heronian mean. The integral $\int_1^2 \frac{1}{1+x}dx$, $\int_1^2 e^x dx$ and $\int_0^1 \frac{\ln(1+x)}{1+x}dx$ are evaluated and the results are compared and are shown in Table 1

We know that

Error = |Exact value-Approximate value|

Example: 3.1 Solve $\int_1^2 \frac{1}{1+x}dx$ and compare the results

Exact value of $\int_1^2 \frac{1}{1+x}dx$ =0.405465

Example: 3.2 solve $\int_1^2 e^x dx$ and compare the results

Exact value of $\int_1^2 e^x dx$ =4.670774

Example: 3.3 solve $\int_0^1 \frac{\ln(1+x)}{1+x}dx$ and compare the results.

Exact Value of $\int_0^1 \frac{\ln(1+x)}{1+x}dx = 0.272198$

Table 1.1

Function	Exact Value	Trapezoidal Rule	Error	Proposed Method	Error
$\int_1^2 \frac{1}{1+x}dx$	0.405465	0.405810	0.000425	0.40585	0.000386
$\int_1^2 e^x dx$	4.670774	4.686327	0.0155530	4.683004	0.012230
$\int_0^1 \frac{\ln(1+x)}{1+x}dx$	0.272198	0.268530	0.003668	0.268443	0.003755

4 Conclusions:

In this paper a new method, modified trapezoidal rule based- arithmetic and heronian mean is presented for the evaluation of definite integrals, this proposed method gives good Accuracy as compare to original trapezoidal rule, and finally numerical examples are solved to show the accuracy of the proposed method.

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