

A Generalized Higher Reverse Left (respectively Right) Centralizer on Prime Γ-Rings

Fawaz Ra'ad Jarullah Salah Mehdi Salih
Department of Mathematics , College of Education , Al-Mustansirya University , Iraq

Abstract:
This study introduces the concepts of generalized higher reverse left (respectively right) centralizer , Jordan generalized higher reverse left (respectively right) centralizer and Jordan triple generalized higher reverse left (respectively right) centralizer of Gamma-rings. In this paper we prove the following main results . Every Jordan generalized higher reverse left (respectively right) centralizer of a 2-torsion free prime Γ-ring M into itself is generalized higher reverse left (respectively right) centralizer of M and every Jordan generalized higher reverse left (respectively right) centralizer of a 2-torsion free Γ-ring M, into itself , such that x α y β x = x β y α x is a Jordan triple generalized higher reverse left(respectively right) centralizer of M , for all x , y ∈ M and α , β ∈ Γ

It is noteworthy that $T_0(x) = x$ and $T_0(x\,\alpha\,y) = y\alpha\,x$, for all a , b ∈ M and α ∈ Γ .

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1-INTRODUCTION:

Let M and Γ be two additive a belian groups. Suppose that there is a mapping from M × Γ × M ⟶ M (the image of (x , α , y) denoted by x α y, where x , y ∈ M and α ∈ Γ) satisfying the following properties for all x , y , z ∈ M and α , β ∈ Γ

(i) (x + y) α z = x α z + y α z

x (α + β) z = x α z + x β z

x α (y + c) = x α y + x α z

(ii) (x α y) β z = x α (y β z) .

Then M is called a Γ-ring. [1] .

M is called a prime if x Γ M Γ y = (0) implies that x = 0 or y = 0, where x , y ∈ M [5].

M is called a semiprime if x Γ M Γ x =(0) implies that x = 0, where x ∈ M [5] .

M is called a 2-torsion free if 2 x = 0 implies that x = 0, for all x ∈ M [5] .

If M is a Γ-ring **,** then $[x,y]_\alpha = x\,\alpha\,y - y\,\alpha\,x$, for all x , y ∈ M and α ∈ Γ,is known as a commutator . [5]

An additive mapping d : M ⟶ M is called a derivation if the following holds :

d(x α y) = d(x) α y + x α d(y), for all x , y ∈ M and α ∈ Γ [2]

Additionally, d is called a Jordan derivation if the following property holds :

$d(x \alpha x) = d(x) \alpha x + x \alpha d(x)$, for all $x \in M$ and $\alpha \in \Gamma$. [2]

An additive mapping $F : M \longrightarrow M$ is called a generalized derivation associated with the derivation $d : M \longrightarrow M$ if the following equation holds :

$F(x \alpha y) = F(x) \alpha y + x \alpha d(y)$, for all $x, y \in M$ and $\alpha \in \Gamma$ [2]

In addition, F is called a Jordan generalized derivation associated with the Jordan derivation $d : M \longrightarrow M$ if the following property is satisfied :

$F(x \alpha x) = F(x) \alpha x + x \alpha d(x)$, for all $x \in M$ and $\alpha \in \Gamma$ [2]

A left (respectively right) centralizer of a Γ-ring M, is an additive mapping, $T : M \longrightarrow M$ which satisfies the following equation $T(x \alpha y) = T(x) \alpha y$ (respectively $T(x \alpha y) = x \alpha T(y)$), for all $x, y \in M$ and $\alpha \in \Gamma$ T is called a centralizer of M if it is both a left and right centralizer [4] . A left (respectively right) Jordan centralizer of a Γ-ring M, is an additive mapping, $T : M \longrightarrow M$ which satisfies the following equation $T(x \alpha x) = T(x) \alpha x$ (respectively $T(x \alpha x) = x \alpha T(x)$), for all $x \in M$ and $\alpha \in \Gamma$. T is called a Jordan centralizer of M if it is both a left and right Jordan centralizer [4] .

An additive mapping $F : M \longrightarrow M$ is called a generalized centralizer of M associated with the centralizer $T : M \longrightarrow M$ if the following equation holds :

$F(x \alpha y + y \beta x) = F(x) \alpha y + y \beta T(x)$, for all $x, y \in M$ and $\alpha, \beta \in \Gamma$ [3] .

F is called a Jordan generalized centralizer of M associated with the Jordan centralizer $T : M \longrightarrow M$ if the following equation holds :

$F(x \alpha x + x \alpha x) = F(x) \alpha x + x \alpha T(x)$, for all $x \in M$ and $\alpha, \beta \in \Gamma$ [3] .

Let $T = (t_i)_{i \in N}$ be a family of additive mappings of a ring R, into itself . Then T is called a higher left centralizer, we have that

$$t_n(xy) = \sum_{i=1}^{n} t_i(x) t_{i-1}(y), \text{ for all } x, y \in R \text{ and } n \in N \quad [6].$$

In addition, T is called a Jordan higher left centralizer if the following equation holds :

$$t_n(x^2) = \sum_{i=1}^{n} t_i(x) t_{i-1}(x), \text{ for all } x \in R \text{ and } n \in N \quad [6].$$

Let $F = (f_i)_{i \in N}$ be a family of additive mappings of a ring R, into itself .Then F is called a generalized higher left centralizer associated with the higher left centralizer $T = (t_i)_{i \in N}$ of R, if the following equation is satisfied :

$$f_n(xy) = \sum_{i=1}^{n} f_i(x) t_{i-1}(y), \text{ for all } x, y \in R \text{ and } n \in N \quad [6]$$

Moreover, F is called a Jordan generalized higher left centralizer associated with the Jordan higher left centralizer $T = (t_i)_{i \in N}$ of R, we have that

$$f_n(x^2) = \sum_{i=1}^{n} f_i(x) t_{i-1}(x), \text{ for all } x \in R \text{ and } n \in N \quad [6].$$

Jarullah and Salih introduced the concepts of higher reverse left (resp. right) centralizer and Jordan higher reverse left (respectively right) centralizer a Γ-ring as follows :

Let $t = (t_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into itself .Then t is called a higher reverse left (respectively right) centralizer of M , we have that

$$t_n(x\,\alpha\,y) = \sum_{i=1}^{n} t_i(y)\ \alpha\ t_{i-1}(x)$$

(respectively $t_n(x\,\alpha\,y) = \sum_{i=1}^{n} t_{i-1}(y)\ \alpha\ t_i(x)$) , for all $x, y \in M$, $\alpha \in \Gamma$ and $n \in N$ [7] .

Let $t = (t_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into itself .Then t is called a Jordan higher reverse left (respectively right) centralizer of M if the following equation holds :

$$t_n(x\,\alpha\,x) = \sum_{i=1}^{n} t_i(x)\ \alpha\ t_{i-1}(x)$$

(respectively $t_n(x\,\alpha\,x) = \sum_{i=1}^{n} t_{i-1}(x)\,\alpha\,t_i(x)$), for all $x \in M$, $\alpha \in \Gamma$ and $n \in N$ [7] .

Jarullah and Salih introduced the concepts of higher reverse left (respectively right) centralizer , Jordan higher reverse left (respectively right) centralizer and generalization on Rings as follows :

Let $t = (t_i)_{i\in N}$ be a family of additive mappings of a ring R into itself .Then t is called a higher reverse left (respectively right) centralizer of R , we have that

$$t_n(xy) = \sum_{i=1}^{n} t_i(y)\, t_{i-1}(x)$$

(respectively $t_n(xy) = \sum_{i=1}^{n} t_{i-1}(y)\, t_i(x)$) , for all $x, y \in R$ and $n \in N$ [8] .

Let $t = (t_i)_{i\in N}$ be a family of additive mappings of a ring R into itself .Then t is called a Jordan higher reverse left (respectively right) centralizer of R , if the following equation holds :

$$t_n(x^2) = \sum_{i=1}^{n} t_i(x)\, t_{i-1}(x)$$

(respectively $t_n(x^2) = \sum_{i=1}^{n} t_{i-1}(x)\, t_i(x)$), for all $x \in R$ and $n \in N$ [8] .

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a ring R into itself .Then T is called a generalized higher reverse left (respectively right) centralizer of a ring R into itself associated with the higher reverse left (respectively right) centralizer $t = (t_i)_{i\in N}$ of R , such that

$$T_n(xy) = \sum_{i=1}^{n} T_i(y)\, t_{i-1}(x)$$

(respectively $T_n(xy) = \sum_{i=1}^{n} t_{i-1}(y)\, T_i(x)$) , for all $x, y \in R$ and $n \in N$ [9] .

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a ring R into itself .Then T is called a generalized Jordan higher reverse left (respectively right) centralizer of a ring R into itself associated with Jordan higher reverse left (respectively right) centralizer $t = (t_i)_{i\in N}$ of R , such that

$$T_n(x^2) = \sum_{i=1}^{n} T_i(x)\, t_{i-1}(x)$$

(respectively $T_n(x^2) = \sum_{i=1}^{n} t_{i-1}(x)\, T_i(x)$) , , for all $x \in R$ and $n \in N$ [9] .

2- Generalized Higher Reverse Left (Respectively Right) Centralizer on Prime Γ-Rings

Definition (2.1)

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into itself .Then T is called a generalized higher reverse left (respectively right) centralizer of the Γ-ring M, associated with the higher reverse left (respectively right) centralizer $t = (t_i)_{i\in N}$ of the Γ-ring M, into itself if , for all $x, y \in M$, $\alpha \in \Gamma$ and $n \in N$

$$T_n(x\alpha y) = \sum_{i=1}^{n} T_i(y)\,\alpha\, t_{i-1}(x)$$

(respectively $T_n(x\alpha y) = \sum_{i=1}^{n} t_{i-1}(y)\,\alpha\, T_i(x)$)

Example (2.2)

Let $T = (T_i)_{i\in N}$ be a generalized higher reverse left (respectively right) centralizer of a ring R, into itself associated with the higher reverse left (respectively right) centralizer $t = (t_i)_{i\in N}$ of R such that, for all x , $y \in R$ and $n \in N$:

$$T_n(xy) = \sum_{i=1}^{n} T_i(y)\, t_{i-1}(x)$$

(respectively $T_n(xy) = \sum_{i=1}^{n} t_{i-1}(y)\, T_i(x)$)

Let $M = M_{1\times 2}(R)$ and $\Gamma = \left\{ \begin{pmatrix} n \\ 0 \end{pmatrix}, n \in Z \right\}$. Then M is a Γ-ring .

Let $F = (F_i)_{i\in N}$ be a family of additive mappings from a Γ-ring M into itself , such that for all $(x\ \ y) \in M$
$F_n((x\ \ y)) = (T_n(x)\ \ T_n(y))$.

Then, there exists a higher reverse left (respectively right) centralizer $f = (f_i)_{i\in N}$ of a Γ-ring M into itself , such that for all $(x\ \ y) \in M$, the following equation holds :
$f_n((x\ \ y)) = (t_n(x)\ \ t_n(y))$.

Therefore, F_n is a generalized higher reverse left (respectively right) centralizer of M .

Definition (2.3)

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into itself .Then T is called a Jordan generalized higher reverse left (respectively right) centralizer of a Γ-ring M associated with the Jordan higher reverse left(respectively right) centralizer $t = (t_i)_{i\in N}$ of the Γ-ring M into itself , if for all $x \in M$, $\alpha \in \Gamma$ and $n \in N$, the following equation holds :

$$T(x\alpha x) = \sum_{i=1}^{n} T_i(x)\,\alpha\, t_{i-1}(x)$$

(respectively $T_n(x\alpha x) = \sum_{i=1}^{n} t_{i-1}(x)\,\alpha\, T_i(x)$).

Definition (2.4)

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into itself .Then T is called a Jordan triple generalized higher reverse left (respectively right) centralizer of a Γ-ring M, associated with the Jordan triple higher reverse left (respectively right) centralizer $t = (t_i)_{i\in N}$ of the Γ-ring M into itself if , for all $x, y \in M$, $\alpha, \beta \in \Gamma$ and $n \in N$, the following equation holds :

$$T_n(x\alpha y\beta x) = \sum_{i=1}^{n} T_i(x)\beta t_{i-1}(y)\alpha t_{i-1}(x)$$

(respectively resp. $T_n(x\alpha y\beta x) = \sum_{i=1}^{n} t_{i-1}(x)\,\beta\, t_{i-1}(y)\alpha\, T_i(x)$).

Lemma (2.5)

Let$T = (T_i)_{i\in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a Γ-ring M into itself .Then for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$ and $n \in N$, the following equation holds :

(i) $T_n(x\alpha y + y\alpha x) = \sum_{i=1}^{n} T_i(y)\alpha t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\alpha t_{i-1}(y)$

(respectively $T_n(x\alpha y + y\alpha x) = \sum_{i=1}^{n} t_{i-1}(y)\alpha\, T_i(x) + \sum_{i=1}^{n} t_{i-1}(x)\alpha\, T_i(y)$)

(ii) $T_n(x\alpha y\beta x + x\beta y\alpha x) = \sum_{i=1}^{n} T_i(x)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\alpha\, t_{i-1}(y)\,\beta\, t_{i-1}(x)$

(respectively $T_n(x\alpha y\beta x + x\beta y\alpha x) = \sum_{i=1}^{n} t_{i-1}(x)\,\beta\, t_{i-1}(y)\,\alpha\, T_i(x) + \sum_{i=1}^{n} t_{i-1}(x)\,\alpha\, t_{i-1}(y)\,\beta\, T_i(x)$)

(iii) $T_n(x\alpha y\beta z + z\alpha y\beta x) = \sum_{i=1}^{n} T_i(z)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(z)$

(respectively $T_n(x\alpha y\beta z + z\alpha y\beta x) = \sum_{i=1}^{n} t_{i-1}(z)\,\beta\, t_{i-1}(y)\,\alpha\, T_i(x) + \sum_{i=1}^{n} t_{i-1}(x)\,\beta\, t_{i-1}(y)\,\alpha\, T_i(z)$)

(iv) In particular , if M is a 2-torsion free commutative Γ-ring , then

$$T_n(x\alpha y\beta z) = \sum_{i=1}^{n} T_i(z)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(x)$$

(respectively $T_n(x\alpha y\beta z) = \sum_{i=1}^{n} t_{i-1}(z)\,\beta\, t_{i-1}(y)\,\alpha\, T_i(x)$)

(v) $T_n(x\alpha y\alpha z + z\alpha y\alpha x) = \sum_{i=1}^{n} T_i(z)\,\alpha\, t_{i-1}(y)\,\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\alpha\, t_{i-1}(y)\,\alpha\, t_{i-1}(z)$

(respectively $T_n(x\alpha y\alpha z + z\alpha y\alpha x) = \sum_{i=1}^{n} t_{i-1}(z)\,\alpha\, t_{i-1}(y)\,\alpha\, T_i(x) + \sum_{i=1}^{n} t_{i-1}(x)\,\alpha\, t_{i-1}(y)\,\alpha\, T_i(z)$)

Proof:

(i) Since T is Jordan generalized higher reverse left (respectively right) centralizer ,

(ii) we have that

$$T_n((x+y)\,\alpha\,(x+y)) = \sum_{i=1}^{n} T_i(x+y)\,\alpha\,t_{i-1}(x+y)$$

$$= \sum_{i=1}^{n} T_i(x)\,\alpha\,t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\alpha\,t_{i-1}(y) + \sum_{i=1}^{n} T_i(y)\,\alpha\,t_{i-1}(x) + \sum_{i=1}^{n} T_i(y)\,\alpha\,t_{i-1}(y)$$

….(1)

Meanwhile , we have that

$$T_n((x+y)\,\alpha\,(x+y)) = T_n(x\,\alpha\,x + x\,\alpha\,y + y\alpha\,x + y\alpha\,y)$$

$$= T_n(x\,\alpha\,x) + T_n(y\alpha\,y) + T_n(x\,\alpha\,y + y\alpha\,x)$$

$$= \sum_{i=1}^{n} T_i(x)\,\alpha\,t_{i-1}(x) + \sum_{i=1}^{n} T_i(y)\,\alpha\,t_{i-1}(y) + T_n(x\,\alpha\,y + y\alpha\,x) \quad \ldots\ \ldots (2)$$

We obtain the following equation by Comparing equations (1) and (2)

$$T_n(x\,\alpha\,y + y\alpha\,x) = \sum_{i=1}^{n} T_i(y)\alpha\,t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\alpha\,t_{i-1}(y)$$

(ii) By substituting that) , we have i (Lemma (2.5) in $b$for)$a\,\beta\,b + b\,\beta\,a$ (

$$= \sum_{i=1}^{n} T_i(y)\,\beta\,t_{i-1}(x)\alpha\,t_{i-1}(x) + T_i(x)\,\beta\,t_{i-1}(y)\,\alpha\,t_{i-1}(x) +$$

$$T_i(x)\,\alpha\,t_{i-1}(y)\,\beta\,t_{i-1}(x) + T_i(x)\,\alpha\,t_{i-1}(x)\,\beta\,t_{i-1}(y) \ldots (1)$$

In addition , we obtain that

$$T_n(x\,\alpha\,(x\,\beta\,y + y\,\beta\,x) + (x\,\beta\,y + y\,\beta\,x)\,\alpha\,x)$$

$$= T_n(x\,\alpha\,x\,\beta\,y + x\,\alpha\,y\,\beta\,x + x\,\beta\,y\alpha\,x + y\,\beta\,x\,\alpha\,x)$$

$$= T_n(y\,\beta\,x\,\alpha\,x) + T_n(x\,\alpha\,x\,\beta\,y) + T_n(x\,\alpha\,y\,\beta\,x + x\,\beta\,y\alpha\,x)$$

$$= \sum_{i=1}^{n} T_i(y)\,\beta\,t_{i-1}(x)\alpha\,t_{i-1}(x) + T_i(x)\,\alpha\,t_{i-1}(x)\,\beta\,t_{i-1}(y) + T_n(x\,\alpha\,y\,\beta\,x + x\,\beta\,y\alpha\,x) \quad \ldots.(2)$$

We get the following equation by Comparing equations (1) and (2)

$$T_n(x\,\alpha\,y\,\beta\,x + x\,\beta\,y\,\alpha\,x) = \sum_{i=1}^{n} T_i(x)\,\beta\,t_{i-1}(y)\,\alpha\,t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\alpha\,t_{i-1}(y)\,\beta\,t_{i-1}(x)$$

(iii) By substituting $(a + c)$ for a in Definition (2.4) , we have

$$= \sum_{i=1}^{n} T_i(x)\,\beta\,t_{i-1}(y)\alpha\,t_{i-1}(x) + T_i(x)\,\beta\,t_{i-1}(y)\,\alpha\,t_{i-1}(z) +$$

$$T_i(z)\,\beta\,t_{i-1}(y)\,\alpha\,t_{i-1}(x) + T_i(z)\,\beta\,t_{i-1}(y)\,\alpha\,t_{i-1}(z) \ldots (1)$$

Moreover,

$$T_n((x+z)\alpha\,y\,\beta(x+z)) = T_n(x\,\alpha\,y\,\beta\,x + x\,\alpha\,y\,\beta z + z\,\alpha\,y\,\beta\,x + z\,\alpha\,y\,\beta z) =$$

$$T_n(x\,\alpha\,y\,\beta\,x) + T_n(z\,\alpha\,y\,\beta z) + T_n(x\,\alpha\,y\,\beta z + z\,\alpha\,y\,\beta\,x)$$

$$= \sum_{i=1}^{n} T_i(x)\,\beta\, t_{i-1}(y)\alpha\, t_{i-1}(x) + T_i(z)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(z)\;\; T_n(x\,\alpha\, y\,\beta z + z\,\alpha\, y\,\beta\, x)$$

… (2)

The following equation is obtained by comparing equations (1) and (2)

$$T_n(x\,\alpha\, y\,\beta\, z + z\,\alpha\, y\,\beta\, x) = \sum_{i=1}^{n} T_i(z)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(z)$$

(iv) Using Lemma (2.5) (iii) and the fact that M is a commutative Γ-ring , we have that

$$T_n(x\,\alpha\, y\,\beta z + x\,\alpha\, y\,\beta z) = 2\,T_n(x\,\alpha\, y\,\beta z)$$

$$= 2\sum_{i=1}^{n} T_i(z)\,\beta\, t_{i-1}(y)\,\alpha\, t_{i-1}(x)$$

We obtain the required result , by utilizing the fact that M is a 2-torsion free .

(v) The substitution β for α in Lemma (2.5) (iii) , gives that

$$T_n(x\,\alpha\, y\,\alpha\, z + z\,\alpha\, y\,\alpha\, x) = \sum_{i=1}^{n} T_i(z)\,\alpha\, t_{i-1}(y)\,\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\,\alpha\, t_{i-1}(y)\,\alpha\, t_{i-1}(z)$$

Definition (2.6)

Let T = (T_i)$_{i\in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a Γ-ring M ring into itself .Then for all x , y ∈ M , α ∈ Γ and n ∈ N , we define

$\delta_n(x, y)_\alpha = T_n(x\,\alpha\, y) - \sum_{i=1}^{n} T_i(y)\alpha\, t_{i-1}(x)$

(respectively $\delta_n(x, y)_\alpha = T_n(x\,\alpha\, y) - \sum_{i=1}^{n} t_{i-1}(y)\alpha\, T_i(x)$)

Lemma (2.7)

Let T = (T_i)$_{i\in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a Γ-ring M ring into itself .Then for all x , y , z ∈ M , α , β ∈ Γ and n ∈ N , we have that the following equations hold :

(i) $\delta_n(x, y)_\alpha = -\delta_n(y, x)_\alpha$

(ii) $\delta_n(x + y, z)_\alpha = \delta_n(x, z)_\alpha + \delta_n(y, z)_\alpha$

(iii) $\delta_n(x, y + z)_\alpha = \delta_n(x, y)_\alpha + \delta_n(x, z)_\alpha$

(iv) $\delta_n(x, y)_{\alpha+\beta} = \delta_n(x, y)_\alpha + \delta_n(x, y)_\beta$

Proof:

(i) By applying Lemma (2.5) (i) , we have that

$$T_n(x\,\alpha\, y + y\alpha\, x) = \sum_{i=1}^{n} T_i(y)\alpha\, t_{i-1}(x) + \sum_{i=1}^{n} T_i(x)\alpha\, t_{i-1}(y)$$

$$T_n(x\,\alpha\, y) - \sum_{i=1}^{n} T_i(y)\alpha\, t_{i-1}(x) = -(T_n(y\,\alpha\, x) - \sum_{i=1}^{n} T_i(x)\alpha\, t_{i-1}(y))$$

Thus, $\delta_n(x, y)_\alpha = -\delta_n(y, x)_\alpha$

(ii) $\delta_n(x+y,z)_\alpha = T_n((x+y)\alpha z) = \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(x+y)$

$$= T_n(x\alpha z + y\alpha z) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(x) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(y)$$

$$= T_n(x\alpha z) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(x) + T_n(y\alpha z) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(y)$$

$= \delta_n(x,z)_\alpha + \delta_n(y,z)_\alpha$

(iii) $\delta_n(x,y+z)_\alpha = T_n(x\alpha(y+z)) = \sum_{i=1}^{n} T_i(y+z)\alpha t_{i-1}(x)$

$$= T_n(x\alpha y + x\alpha z) - \sum_{i=1}^{n} T_i(y)\alpha t_{i-1}(x) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(x)$$

$$= T_n(x\alpha y) - \sum_{i=1}^{n} T_i(y)\alpha t_{i-1}(x) + T_n(x\alpha z) - \sum_{i=1}^{n} T_i(z)\alpha t_{i-1}(x)$$

$= \delta_n(x,y)_\alpha + \delta_n(x,z)_\alpha$

(iv) $\delta_n(x,y)_{\alpha+\beta} = T_n(x(\alpha+\beta)y) - \sum_{i=1}^{n} T_i(y)(\alpha+\beta)\, t_{i-1}(x)$

$$= T_n(x\alpha y) - \sum_{i=1}^{n} T_i(y)\alpha t_{i-1}(x) + T_n(x\beta y) - \sum_{i=1}^{n} T_i(y)\beta\, t_{i-1}(x)$$

$= \delta_n(x,y)_\alpha + \delta_n(x,y)_\beta$

Remark (2.8)

It is noteworthy that $T = (T_i)_{i\in N}$ is a generalized higher reverse left (respectively right) centralizer of a Γ-ring M, into itself if and only if $\delta_n(x,y)_\alpha = 0$, for all $x, y \in M$, $\alpha \in \Gamma$ and $n \in N$.

Lemma (2.9)

Let $T = (T_i)_{i\in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a prime Γ-ring M, into itself. Then for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$ and $n \in N$, the following equations hold :

(i) $\delta_n(x,y)_\alpha\ \beta\ t_{n-1}(z)\ \beta\ [t_{n-1}(x), t_{n-1}(y)]_\alpha = 0$

(ii) $\delta_n(x,y)_\alpha\ \alpha\ t_{n-1}(z)\ \alpha\ [t_{n-1}(x), t_{n-1}(y)]_\alpha = 0$

(iii) $\delta_n(x,y)_\beta\ \alpha\ t_{n-1}(z)\ \alpha\ [t_{n-1}(x), t_{n-1}(y)]_\beta = 0$

Proof:

(i) The Proof is utilizing induction on $n \in N$

If n = 1,

let let $w = x\alpha y\beta z\beta y\alpha x + y\alpha x\beta z\beta x\alpha y$

Then , we obtain that

$T(w) = T(x\alpha(y\beta z\beta y)\alpha x + y\alpha(x\beta z\beta x)\alpha y)$

$= T(x)\alpha y\beta z\beta y\alpha x + t(y)\alpha x\beta z\beta x\alpha y$ …(1)

Moreover , we have that

$T(w) = T((x\alpha y)\beta z\beta(y\alpha x) + (y\alpha x)\beta z\beta(x\alpha y))$

$= T(y\alpha x)\beta z\beta y\alpha x + t(x\alpha y)\beta z\beta x\alpha y$ … (2)

The Comparison of equations (1) and (2) yields that

$0 = (T(y \alpha x) - t (x) \alpha y) \beta z \beta y \alpha x + (T(x \alpha y) - t (y) \alpha x) \beta z \beta x \alpha y$

$0 = \delta(y , x)_{\alpha} \beta z \beta y \alpha x + \delta(x , y)_{\alpha} \beta z \beta x \alpha y$

$0 = - \delta(x , y)_{\alpha} \beta z \beta y \alpha x + \delta(x , y)_{\alpha} \beta z \beta x \alpha y$

$0 = \delta(x , y)_{\alpha} \beta z \beta (x \alpha y - y \alpha x)$

Thus, $\delta(x , y)_{\alpha} \beta z \beta [x , y]_{\alpha} = 0$, for all $x , y , z \in M$ and $\alpha , \beta \in \Gamma$.

Now , we can ssume the following :

$\delta_s (x , y)_{\alpha} \beta t_{s-1}(z) \beta [t_{s-1} (x) , t_{s-1} (y)]_{\alpha} = 0$, for all $x , y , z \in M$, and s , $n \in N$, $s < n$.

$T_n (w) = T_n(x \alpha (y \beta z \beta y) \alpha x + y \alpha (x \beta z \beta x) \alpha y)$

$= \sum_{i=1}^{n} T_i (x) \alpha t_{i-1} (y \beta z \beta y) \alpha t_{i-1}(x) + \sum_{i=1}^{n} T_i (y) \alpha t_{i-1} (x \beta z \beta x) \alpha t_{i-1}(y)$

$= \sum_{i=1}^{n} T_i (x) \alpha t_{i-1}(y) \beta t_{i-1}(z) \beta t_{i-1} (y) \alpha t_{i-1}(x) +$

$_i t \alpha (y) _i T \sum_{i=1}^{n}$

$_{-1}(x) \beta t_{i-1}(z) \beta t_{i-1} (x) \alpha t_{i-1}(y)$

$= (\sum_{i=1}^{n} T_i (x) \alpha t_{i-1}(y)) \beta t_{n-1}(z) \beta t_{n-1}(y) \alpha t_{n-1}(x) +_{1} {}_{-i} t \alpha (x)_i T \sum_{i=1}^{n-1}$

$(y) \beta t_{i-1}(z) \beta t_{i-1}(y) \alpha t_{i-1}(x) +$

$(\sum_{i=1}^{n} T_i (y) \alpha t_{i-1} (x)) \beta t_{n-1}(z) \beta t_{n-1}(x) \alpha t_{n-1}(y) + \alpha (y)_i T \sum_{i=1}^{n-1}$

$t_{i-1}(x) \beta t_{i-1} (z) \beta t_{i-1}(x) \alpha t_{i-1}(y)$ … (3)

Thus ,

$T_n (w) = T_n ((x \alpha y) \beta z \beta (y \alpha x) + (y \alpha x) \beta z \beta (x \alpha y))$

$= \sum_{i=1}^{n} T_i (y \alpha x) \beta t_{i-1} (z) \beta t_{i-1} (x \alpha y) + \sum_{i=1}^{n} T_i(x \alpha y) \beta t_{i-1} (z) \beta t_{i-1} (y \alpha x)$

$= T_n (y \alpha x) \beta t_{n-1} (z) \beta t_{n-1}(y) \alpha t_{n-1}(x) + \sum_{i=1}^{n-1} T_i (y \alpha x) \beta t_{i-1} (z) \beta t_{i-1}(y)$

$\alpha t_{i-1} (x) + T_n(x \alpha y) \beta t_{n-1} (z) \beta t_{n-1} (x) \alpha t_{n-1} (y)$ +

$\sum_{i=1}^{n-1} T_i (x \alpha y) \beta t_{i-1} (z) \beta t_{i-1} (x) \alpha t_{i-1} (y)$ … .(4)

By comparing equations (3) and (4) , we have that

$0 = (T_n(y \alpha x) - \sum_{i=1}^{n} T_i(x) \alpha t_{i-1} (y)) \beta t_{n-1}(z) \beta t_{n-1} (y) \alpha t_{n-1}(x) +$

$$(T_n(x\,\alpha\,y) - \sum_{i=1}^{n} T_i(y)\,\alpha\,t_{i-1}(x)\,)\,\beta\,t_{n-1}(z)\,\beta\,t_{n-1}(x)\,\alpha\,t_{n-1}(y) +$$

$$+)(x_{1-i}\,t\,\alpha\,(y)_{1-i}\,t\,\beta\,(z)_{1-i}\,t\,\beta)\ y)(_{1-i}\,t\,\alpha\,(x)_i T -)\,x\,\alpha y\,(_i T(\sum_{i=1}^{n-1}$$

$$\sum_{i=1}^{n-1} (T_i(x\,\alpha\,y) - T_i(y)\,\alpha\,t_{i-1}(x)\,)\,\beta\,t_{i-1}(z)\,\beta\,t_{i-1}(x)\,\alpha\,t_{i-1}(y)\,.$$

It follows that

$0 = \delta_n(y , x)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, t_{n-1}(y)\, \alpha\, t_{n-1}(x) +$

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, t_{n-1}(x)\, \alpha\, t_{n-1}(y)$

$0 =+ \qquad x)(_{1-n}\,t\ \alpha\ (y)_{1-n}\,t\ \beta\ (z)_{1-n}\,t\ \beta\ _\alpha(x ,y)_n\delta -$

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, t_{n-1}(x)\, \alpha\, t_{n-1}(y)$

$0 = \delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, (t_{n-1}(x)\, \alpha\, t_{n-1}(y) - t_{n-1}(y)\, \alpha\, t_{n-1}(x))$

Thus , $\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(x) , t_{n-1}(y)\,]_\alpha = 0$, for all $x , y , z \in M$,

$\alpha , \beta \in \Gamma$ and $n \in N$

(ii) By substituting β for α in Lemma (2.9) (i) and applying similar arguments as in the proof of Lemma (2.9) (i) , we obtain Lemma (2.9) (ii) .

(iii) We get Lemma (2.9) (iii), by Interchanging α and β in Lemma (2.9) (i) .

Lemma (2.10)

Let $T = (T_i)_{i\in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a prime Γ-ring M into itself .Then for all $x , y , z , u , v \in M$, $\alpha , \beta \in \Gamma$ and $n \in N$

(i) $\delta_n(x , y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(v)]_\alpha = 0$

(ii) $\delta_n(x , y)_\alpha\, \alpha\, t_{n-1}(z)\, \alpha\, [\, t_{n-1}(u) , t_{n-1}(v)]_\alpha = 0$

(iii) $\delta_n(x , y)_\alpha\, \alpha\, t_{n-1}(z)\, \alpha\, [\, t_{n-1}(u) , t_{n-1}(v)]_\beta = 0$

Proof:

(i) By substituting $(a + c)$ for a in Lemma (2.9) (i) , we have that

$\delta_n(x + u , y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(x + u) , t_{n-1}(y)\,]_\alpha = 0$. Thus

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(x) , t_{n-1}(y)\,]_\alpha +$

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(y)\,]_\alpha +$

$\delta_n(u ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(x) , t_{n-1}(y)\,]_\alpha +$

$\delta_n(u ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(y)\,]_\alpha = 0$

By applying Lemma (2.9) (i) , we obtain that

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(y)\,]_\alpha +$

$\delta_n(u , y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(x) , t_{n-1}(y)\,]_\alpha = 0$

Therefore, we get that

$\delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(y)\,]_\alpha\, \beta\, t_{n-1}(z)\, \beta\, \delta_n(x ,y)_\alpha$

$\beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) ,t_{n-1}(y)\,]_\alpha = 0$

This implies that

$0 = - \delta_n(x ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta\, [\, t_{n-1}(u) , t_{n-1}(y)]_\alpha\, \beta\, t_{n-1}(z)\, \beta\, \delta_n(u ,y)_\alpha\, \beta\, t_{n-1}(z)\, \beta$

1 $[t_{n-1}(x), t_{n-1}(y)]_\alpha$

Since M is a prime Γ-ring , we have that

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(u), t_{n-1}(y)]_\alpha = 0$ …(1)

The substitution of (y + v) for y in Lemma (2.9) (i) , gives that

$\delta_n(x, y+v)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y+v)]_\alpha = 0$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y)]_\alpha +$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha +$

$\delta_n(x,v)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y)]_\alpha +$

$\delta_n(x,v)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha = 0$

By utilizing Lemma (2.9) (i) , we obtain that

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha +$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y)]_\alpha = 0$

Consequently , we have

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha \beta t_{n-1}(z) \beta \delta_n(x,y)_\alpha$

$\beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha = 0$

This implies that

$0 = \delta -_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha \beta t_{n-1}(z) \beta \delta_n(x,v)_\alpha$

$\beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y)]_\alpha$

The fact that M is a prime number yields that

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha = 0$ … (2)

Now , $\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x+u), t_{n-1}(y+v)]_\alpha = 0$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(y)]_\alpha +$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(x), t_{n-1}(v)]_\alpha +$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(u), t_{n-1}(y)]_\alpha +$

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(u), t_{n-1}(v)]_\alpha = 0$

By employing equations (1) , (2) and Lemma (2.9) (i) , we get

$\delta_n(x,y)_\alpha \beta t_{n-1}(z) \beta [t_{n-1}(u), t_{n-1}(v)]_\alpha = 0$

(ii) We can obtain Lemma (2.10) (ii) by substituting β in Lemma (2.10) (i) .

(iii) By substituting α + β for α in Lemma (2.10) (ii) , we have

$\delta_n(x,y)_{\alpha+\beta} \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_{\alpha+\beta} = 0$

This implies that

$\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\alpha +$

$\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta +$

$\delta_n(x,y)_\beta \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\alpha +$

$\delta_n(x,y)_\beta \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta = 0$

By applying equations (i) and (ii) , we get that

$\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta +$

$\delta_n(x,y)_\beta \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\alpha = 0$

Therefore, we get that

$\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta \alpha t_{n-1}(z) \alpha \delta_n(x,y)_\alpha$

$\alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta = 0$

This yields that

$0 = -\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta \alpha t_{n-1}(z) \alpha$

$\delta_n(x,y)_\beta \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\alpha$

The fact that M is a prime Γ-ring gives that

$\delta_n(x,y)_\alpha \alpha t_{n-1}(z) \alpha [t_{n-1}(u), t_{n-1}(v)]_\beta = 0$

Theorem (2.11)

Every Jordan generalized higher reverse left (respectively right) centralizer of a 2-torsion free prime Γ-ring M into itself is a generalized higher reverse left (respectively right) centralizer of M .

Proof:

Let $T = (T_i)_{i \in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a prime Γ-ring M into itself . Since M is a prime Γ-ring , then by employing Lemma (2.10) (i) , we have that either

$\delta_n(x,y)_\alpha = 0$ or $[t_{n-1}(u), t_{n-1}(v)]_\alpha = 0$, for all $x, y, u, v \in M$,

$\alpha \in \Gamma$ and $n \in N$.

If $[t_{n-1}(u), t_{n-1}(v)]_\alpha \neq 0$, for all $u, v \in M$, $\alpha \in \Gamma$,then $\delta_n(x,y)_\alpha = 0$,

for all $x, y \in M$ and $n \in N$. Hence , using Remark (2.8) , we obtain that

T is a generalized higher reverse left (respectively right) centralizer of M .

If $[t_{n-1}(u), t_{n-1}(v)]_\alpha = 0$, for all $u, v \in M$ and $n \in N$, then

M is commutative .

By utilizing Lemma (2.5) (i) , we have that

$$T_n(x\alpha y + x\alpha y) = T_n(2x\alpha y)$$

$$= 2T_n(x\alpha y)$$

$$= 2\sum_{i=1}^{n} T_i(y)\alpha\, t_{i-1}(x)$$

Since M is a 2-torsion free Γ-ring , we get that

$$T_n(x\alpha y) = \sum_{i=1}^{n} T_i(y)\alpha\, t_{i-1}(x).$$

Then T is a generalized higher reverse left (respectively right) centralizer of M .

Proposition (2.11)

Let $T = (T_i)_{i \in N}$ be a Jordan generalized higher reverse left (respectively right) centralizer of a 2-torsion free Γ-ring M into itself , such that $x\alpha y\beta x = x\beta y\alpha x$, for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Then T is a Jordan triple generalized higher reverse left (respectively right) centralizer of M .

Proof:

The substitution of b for $(x\alpha y + y\alpha x)$ in Lemma (2.5) (i) , we have that

$$= \sum_{i=1}^{n} T_i(y)\ \beta\ t_{i-1}(x)\alpha\ t_{i-1}(x) + T_i(x)\ \beta\ t_{i-1}(y)\ \alpha\ t_{i-1}(x) +$$
$$T_i(x)\ \alpha\ t_{i-1}(y)\ \beta\ t_{i-1}(x) + T_i(x)\ \alpha\ t_{i-1}(x)\ \beta\ t_{i-1}(y) \ldots (1)$$

Moreover , we get that

$$T_n(x\,\alpha\,(x\,\beta\,y + y\,\beta\,x) + (x\,\beta\,y + y\,\beta\,x)\,\alpha\,x)$$
$$= t_n(x\,\alpha\,x\,\beta\,y + x\,\alpha\,y\,\beta\,x + x\,\beta\,y\,\alpha\,x + y\,\beta\,x\,\alpha\,x)$$
$$= T_n(y\,\beta\,x\,\alpha\,x) + T_n(x\,\alpha\,x\,\beta\,y) + T_n(x\,\alpha\,y\,\beta\,x + x\,\beta\,y\,\alpha\,x)$$
$$= \sum_{i=1}^{n} T_i(y)\ \beta\ t_{i-1}(x)\alpha\, t_{i-1}(x) + T_i(x)\ \alpha\ t_{i-1}(x)\ \beta\, t_{i-1}(y) +$$
$$T_n(x\,\alpha\,y\,\beta\,x + x\,\beta\,y\,\alpha\,x) \ldots (2)$$

By comparing equations (1) , (2) and the fact that $x\ \alpha\ y\ \beta\ x = x\ \beta\ y\ \alpha\ x$,

for all $x , y \in M$ and $\alpha , \beta \in \Gamma$, we have that

$$T_n(x\,\alpha\,y\,\beta\,x + x\,\alpha\,y\,\beta\,x) = 2T_n(x\,\alpha\,y\,\beta\,x)$$
$$= 2\sum_{i=1}^{n} T_i(x)\beta\, t_{i-1}(y)\alpha\, t_{i-1}(x)$$

Since M is a 2-torsion free Γ-ring, we obtain that F is a Jordan triple generalized higher reverse left (respectively right) centralizer of M .

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