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EFFECTS ON TEMPERATURE ON APPLYING VARIABLEPRESSURE GRADIENT TO A MAGNETOHYDRODYNAMIC FLUID FLOWING BETWEEN PLATES WITH INCLINED MAGNETIC FIELD

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Abstract

In this study the effects on temperature on applying variable pressure gradient to a MHD fluid flowing between two plates in an inclined magnetic field was carried out. It involved an unsteady hydromagnetic fluid flowing through two plates where the upper plate was porous and moving with a constant velocity in a direction that is contrary to fluid flow direction. The lower plate was non-porous and stationary. In the past years, research studies relating to MHD fluid flow through plates have been done. The information obtained from these studies have been implemented in various industrial systems for instance designing of electromagnetic meters and MHD pumps. However these research studies have been carried out when the pressure gradient was a constant and none when the pressure gradient was a variable, hence the reason why we carried out this investigation. The objective of this project was to determine the effect on temperature on applying variable pressure gradient to a MHD flow in inclined magnetic field. Also the outcome of varying Hartmann number, Suction number, Reynolds number and Eckert number on temperature profiles have been determined. The governing equations used to carry out analysis were continuity equation, electromagnetic equations and energy equations. These equations were then non-dimensionalised. The resulting equations were non-linear thus were solved using the finite difference method. MATLAB software was then inculcated in the resulting equations which were expressed in finite difference form and resulting solutions illustrated graphically. The results obtained showed that the application of a variable pressure gradient in the presence of an inclined magnetic field resulted to a decrease in the fluid temperature, increasing the suction number increased the temperature; increasing Reynold's number yielded to a reduction in temperature profiles; increase in Hartmann number resulted to an increase in temperature. These thermal profiles are important in the dyeing industries and modelling of systems that help in cooling of automobile moving parts.

Keywords: Magnetohydrodynamic (MHD), variable pressure gradient, inclined magnetic field

1. Introduction

There are three classes of matter namely solid, liquid and gas. Liquids and gases are termed as fluids. Magnetohydrodynamics (MHD) is the study of motion of fluids in both electric and magnetic field through various channels and with the forces associated with it. The general model used in this study consist of infinitely long channel with consistent cross-section and an invariable magnetic field applied to the axis of channel. The walls of channel maybe conductors, non-conductors or both depending on the application to be used. In these studies, the fluid flowing in the channel is usually electrically conductive and non-magnetic. This limits us to liquid metals, plasmas and strong electrolytes.

Magnetic field applied on fluid flows are of two types namely: transverse and inclined magnetic field. In transverse magnetic field, the line of magnetic force applied is perpendicular to the flow while for the inclined one, the line of magnetic force is tilted such that they form an angle with the direction of flow. When a line of magnetic force is imposed on a moving electrical conductive fluid, it experiences a force acting on it and new currents are induced. The new induced currents in turn induce their own magnetic field which then affects the original magnetic field. These new currents generated have led to designing of MHD power generators and MHD pumps used in chemical energy technology (Jha 2001). The force experienced is called Lorentz force and it arises from the interaction of the total magnetic field (i.e. original imposed magnetic field and induced magnetic field) with new induced currents. Also whenever an electric current is passed through a fluid, new heat energy is generated. This heat energy is called ohmic heat and it affects the temperature profile of the fluid.

Research studies on MHD flows through plates in inclined magnetic field have been done but not when the pressure gradient is variable. Therefore in this project we carried out a study on MHD flow through parallel plates where one of the plates used in the channel of passage was porous and inclined magnetic field was subjected to the flow. The porosity of the upper plate will help in the regulation of temperature inside the channel. This study will get noticeable interest from science communities as detailed information on temperature distributions will be useful in dyeing industries.

An investigation on two layered MHD model for parallel plate haemodialysis under the influence of uniform magnetic field that was applied perpendicularly was carried out and it was found out that a surge in magnetic field intensity resulted to a rise in the uniform velocity of the flow Chaturani *et al* (2001).

Attia (2007) analyzed the effects of heat transmission between porous plates where exponential decaying pressure gradient was applied. He found that both suction velocity and porosity had an effect on velocity and temperature profiles. Increasing the porosity parameter resulted to a decrease in both velocity and temperature. He did another study where he investigated the outcomes of applying Hall current on temperature and velocity profiles of a Couette flow. Here the plates had a constant suction and injection applied across the top surfaces. This was done in presence of exponentially declining pressure gradient and were non-conductors of electricity. He found out that the velocity in z-direction affected the main velocity that was in the x-direction by decreasing it, (Attia (2009)).

A study on unsteady couette flow was done by Gunakala *et al* (2014).The lower plate was the only one taken to be porous and not moving while upper plate moved with a uniform velocity. Inclined magnetic field was then imposed to the flow. He concluded that as magnetic field was increased the fluid velocity dropped. The above analysis was conducted again but when a constant pressure gradient was applied by Singh *et al (*2008).They used equation of continuity and equation of motion as the governing equations. They concluded that velocity profiles decreased as the pressure gradient applied was a constant. Manyonge *et al*. (2015) discussed MHD poiseuille flow passing through two plates where lower plate was porous in inclined magnetic field. He found out that velocity of the fluid is influenced by the factors namely magnetic inclination, suction/injection rates and Hartmann number. Increase in these factors decreased the velocity.

Heat transfer between two parallel porous plates in a couette flow was investigated. The lines of magnetic forces were applied perpendicularly to direction of fluid flow and pressure gradient used was constant throughout the flow. Both the plates were porous. Their observation was that with increasing porosity number the temperature decreased. Also there was a reduction in velocity and temperature when the Hartmann number was increased, Attia *et al.* (2015). Sharmilla and Gayathri (2015) analyzed the effects of Pelcet and Nusselt numbers on temperature on an unsteady MHD stokes flow. The fluid used in this analysis was viscous and was been drawn out through both plates and transverse magnetic field was used. His conclusions were that as temperature was increasing, the value of Pelcet number went up and so did Nusselt number.

2. Mathematical formulation

In this study, a 2-dimensional fluid flow flowing through two parallel plates under a variable pressure gradient was considered. The plates used were situated at a distance of 2b apart. The upper plate was porous and had constant suction. The magnetic field applied was inclined at an angle α to the fluid flow. At time t=0 the upper and lower plate along with the fluid were stationary while at time t>0, the lower plate remained stationary as the upper plate started moving at a velocity u in the direction opposite to the fluid flow direction.

The following assumptions were made for this research study;

a) The fluid flow was of 2-dimensions.
b) The fluid was incompressible and unsteady.
c) The direction of fluid flow was along the x-direction.
d) Gravitational forces were taken to be negligible.
e) Both the upper and lower plates were of immeasurable length in x- and z-axis.
f) There was no potential difference applied externally.

The equations governing the flow are continuity equation and energy equation and are expressed in tensor form as follows;

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (1)$$

$$\rho\frac{\partial r}{\partial t} + \frac{\partial}{x_j}\left(\rho u_j r\right) = \frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j}\left(u_j P\right) - \frac{\partial q_j}{\partial x_j} + \varphi \quad (2)$$

where

$$\varphi = \mu\left\{\left[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial w}{\partial z})^2\right] + \left[(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2 + (\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x})^2 + (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2\right]\right\} \quad (2a)$$

This flow is incompressible flow thus $\frac{\partial \rho}{\partial t} = 0$ and the plates are of infinite length in x and z-directions thus $\frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial z} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial z} = 0, \frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial z} = 0$. Equation (1) becomes

$$\frac{\partial v}{\partial y} = 0 \tag{3}$$

$$v = v_o \tag{3a}$$

Using equation (3) and the assumption of infinite plates, equation (2a) reduces to

$$\varphi = \mu \left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

Using Fourier's law, the heat produced is expressed as

$$q_j = -K \frac{\partial T}{\partial x_j} \tag{5}$$

Equation (2) was simplified using definition of specific enthalpy r given as

$$r = e + \frac{P}{\rho} \tag{6}$$

Differentiating equation (6) we have

$$dr = de + \frac{1}{\rho} dP + Pd\left(\frac{1}{\rho}\right) \tag{7}$$

Applying 1st and 2nd law of thermodynamics (Hatsopolous and Keenan, 1965), change in specific internal energy e was given as

$$de = TdS - Pd\left(\frac{1}{\rho}\right) \tag{8}$$

Substituting equation (8) in equation (7) we get

$$dr = TdS + \frac{1}{\rho} dP \tag{9}$$

Entropy is a property of fluid and can be expressed in terms of pressure and temperature and hence can be written as

$$S = S(P, T) \tag{10}$$

Differentiating equation (10)

$$dS = \left(\frac{\partial S}{\partial P}\right)_T dP + \left(\frac{\partial S}{\partial T}\right)_P dT \tag{11}$$

We then used the following generalized thermodynamic relations

$$\left(\frac{\partial S}{\partial P}\right)_T = -\frac{G}{\rho}, \left(\frac{\partial S}{\partial T}\right)_P = \frac{C_p}{T} \quad (12)$$

Where G is volumetric coefficient and C_p is specific heat capacity

On substituting eqn. (12) on eqn. (11) we get

$$dS = -\frac{G}{\rho}dP + \frac{C_p}{T}dT \quad (13)$$

Substituting equation (13) on equation (9) and multiply it by T we get

$$dr = C_p dT + \frac{1}{\rho}(1 - GT)dP \quad (14)$$

Making use of equations (14), (4), (5) and further substituting on (2) gives the energy equation which is expressed as

$$\frac{\partial}{\partial t}\left(\rho C_\rho T\right) + \frac{\partial}{\partial x_j}\left(\rho C_p\, u_j T\right) = \frac{\partial}{\partial x_j}\left(K\frac{\partial T}{\partial x_j}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (15)$$

In present study the assumption of plates being infinite in length in x and z and hence was imposed on equation (15) and using the result of equation (3) then equation (15) resulted to

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_o\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (16)$$

Ohmic heating which is expressed as $\frac{J^2}{\sigma}$ was considered due to the electrical resistance of the fluid and hence equation (16) became

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_o\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{J^2}{\sigma} \quad (17)$$

In this study, the lines of magnetic force were inclined at an angle α to the y-axis. Thus

$$\vec{B} = \vec{B}(0, Bsin\alpha, 0) \quad (18)$$

The moving plate was moving along x-axis with a velocity of u thus

$$\vec{V} = \vec{V}(u, 0,0) \quad (19)$$

On using eqn (18) and (19) $\vec{J}$ becomes

$$\vec{J} = \sigma(\vec{V} \times \vec{B}) = \sigma \begin{vmatrix} \bar{\iota} & \bar{J} & \bar{k} \\ u & 0 & 0 \\ 0 & Bsin\alpha & 0 \end{vmatrix} = \sigma uBsin\alpha\,\bar{k} \qquad (20)$$

When magnetic field is imposed on a substance, it penetrates through the substance to its inside. This phenomenon is called magnetic permeability and is expressed as

$$\mu_e = \frac{\vec{B}}{\vec{H}} \qquad (21)$$

Substituting equation (21) on (20) we have

$$\vec{J} = \sigma u \mu_e \vec{H} sin\alpha \qquad (22)$$

On substituting equation (3a) and equation (22) on equation (17) the following equation was obtained

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_o\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma u \mu_e^2 H^2 sin^2\alpha \qquad (23)$$

Equation (23) is the final equation that describes this research problem.

The initial and boundary conditions used were

$t = 0, u = 0 \quad T = 0$ at $-b \le y \le b$

$t > 0,\ u = 0 \quad T = T_w\ at\ y = -b$

$t > 0\ , u = U \quad T = T_\infty\ at\ y = b$

Using the following non-dimensional quantities

$$y^* = \frac{y}{L}, u^* = \frac{u}{U}, T^* = \frac{T - T_\infty}{T_w - T_\infty},\ t^* = \frac{tU}{L},\ s_0 = \frac{v_0}{U}, E_c = \frac{U^2}{c_p \Delta T},\ Pr = \frac{c_p\,\mu}{K},\ Ha = L\mu_e H\sqrt{\frac{\sigma}{\mu}},\ Re = \frac{\rho UL}{\mu}$$

Equation (23) becomes

$$\frac{\partial T^*}{\partial t^*} + s_0\frac{\partial T^*}{\partial y^*} = \frac{1}{Re\,Pr}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{E_c}{Re}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{EcHa^2}{Re} sin^2\alpha u^{2*} \qquad (24)$$

The non-dimensionalised conditions of the equations became

$t* = 0 \quad u* = 0$ at $-1 \leqslant y* \leqslant 1$

$t* > 0 \quad u* = 0$ at $y* = -1$

$t* > 0 \quad u* = 0$ at $y* = 1$

3. Method of solution

The equation was solved numerically using finite difference technique. The finite difference forms of are

$$\frac{\partial u^*}{\partial y^*} = \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y}$$

$$\frac{\partial T^*}{\partial y^*} = \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta y}$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1} + T_{j-1}^{k} - 2T_j^{k} + T_{j+1}^{k}}{2(\Delta y)^2}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{T_j^{k+1} - T_j^{k}}{\Delta t}$$

The above finite difference forms are substituted in equation (24) to get

$$\frac{T_j^{k+1} - T_j^{k}}{\Delta t} = -s_0\left(\frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^{k} - T_{j-1}^{k}}{2\Delta y}\right) + \frac{1}{RePr}\left(\frac{T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1} + T_{j-1}^{k} - 2T_j^{k} + T_{j+1}^{k}}{2(\Delta y)^2}\right) + \frac{Ec}{Re}\left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^{k} - U_{j-1}^{k}}{2\Delta y}\right)^2 + \frac{EcHa^2}{Re}\sin^2\alpha\left(U_j^{k}\right)^2 \quad (25)$$

Multiplying by Δt on both sides of equation (25) and rearranging it resulted to

$$T_j^{k+1} = T_j^{k} - s_0\Delta t\left(\frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^{k} - T_{j-1}^{k}}{2\Delta y}\right) + \frac{\Delta t}{RePr}\left(\frac{T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1} + T_{j-1}^{k} - 2T_j^{k} + T_{j+1}^{k}}{2(\Delta y)^2}\right) + \frac{Ec\Delta t}{Re}\left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^{k} - U_{j-1}^{k}}{2\Delta y}\right)^2 + \frac{EcHa^2}{Re}\Delta t\sin^2\alpha\left(U_j^{k}\right)^2 \quad (26)$$

From equation (26) T_j^{k+1} was made the subject

$$T_j^{k+1} = \left\{T_j^{k} - s_0\Delta t\left(\frac{T_j^{k} - T_{j-1}^{k+1} - T_{j-1}^{k}}{2\Delta y}\right) + \frac{\Delta t}{RePr}\left(\frac{T_{j-1}^{k+1} + T_{j+1}^{k+1} + T_{j-1}^{k} - 2T_j^{k} + T_{j+1}^{k}}{2(\Delta y)^2}\right) + \frac{Ec\Delta t}{Re}\left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^{k} - U_{j-1}^{k}}{2\Delta y}\right)^2 + \frac{EcHa^2}{Re}\Delta t\sin^2\alpha\left(U_j^{k}\right)^2\right\} \div \left(1 + \frac{s_0\Delta t}{2\Delta y} + \frac{\Delta t}{RePr(\Delta y)^2}\right) \quad (27)$$

Equation (27) was solved using a computer code in MATLAB version 9.4.0(R2018a) computer program.

4. Results

The results obtained after running matlab are presented graphically.

Since the problem has been non-dimensionalised, default values are chosen so that they are used to establish the changes which will affect the fluid flow. The default values were chosen in accordance to the surrounding temperature at 20℃ .In this study we adopted the following default parameter values:

$$Pr = 0.5, Ec = 0.05, Pg = -450, S = 150\ , \propto = 30°, Ha = 6, Re = 6$$

Using equation (27) we analyzed the effects of various parameters on temperature. The analysis is illustrated using graphs below

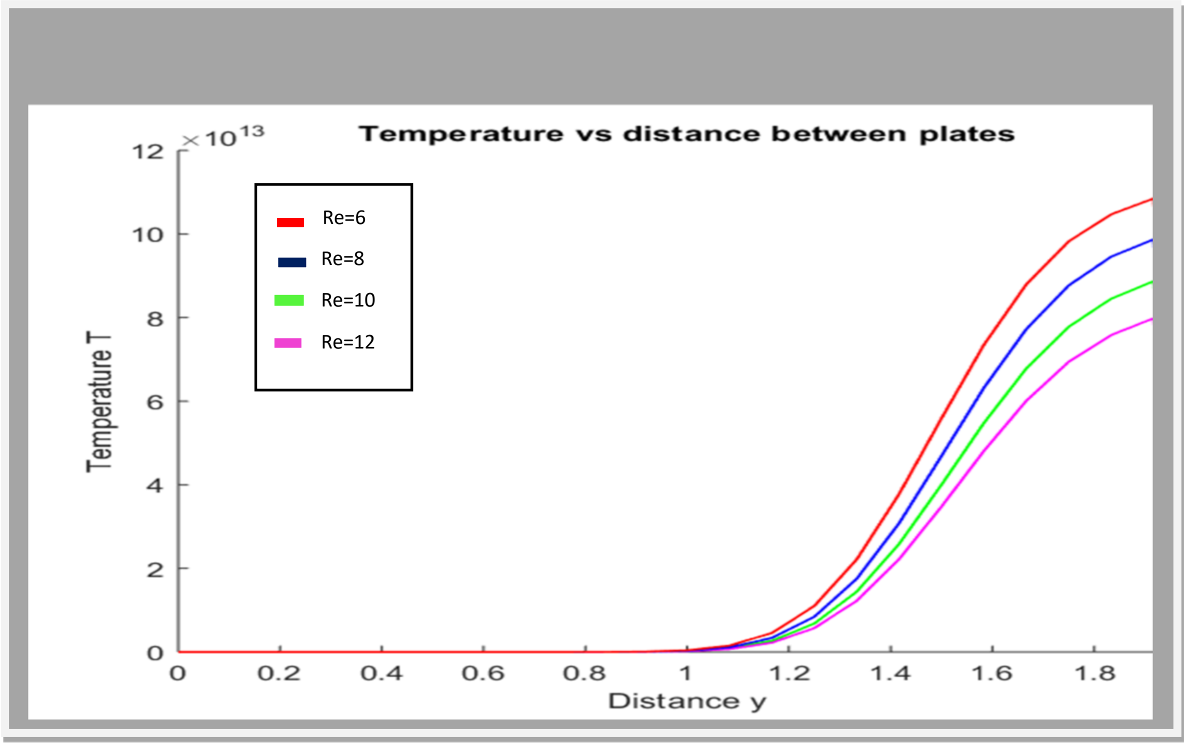

Fig. 1. Temperature profiles for different values of Reynold's number (Re)

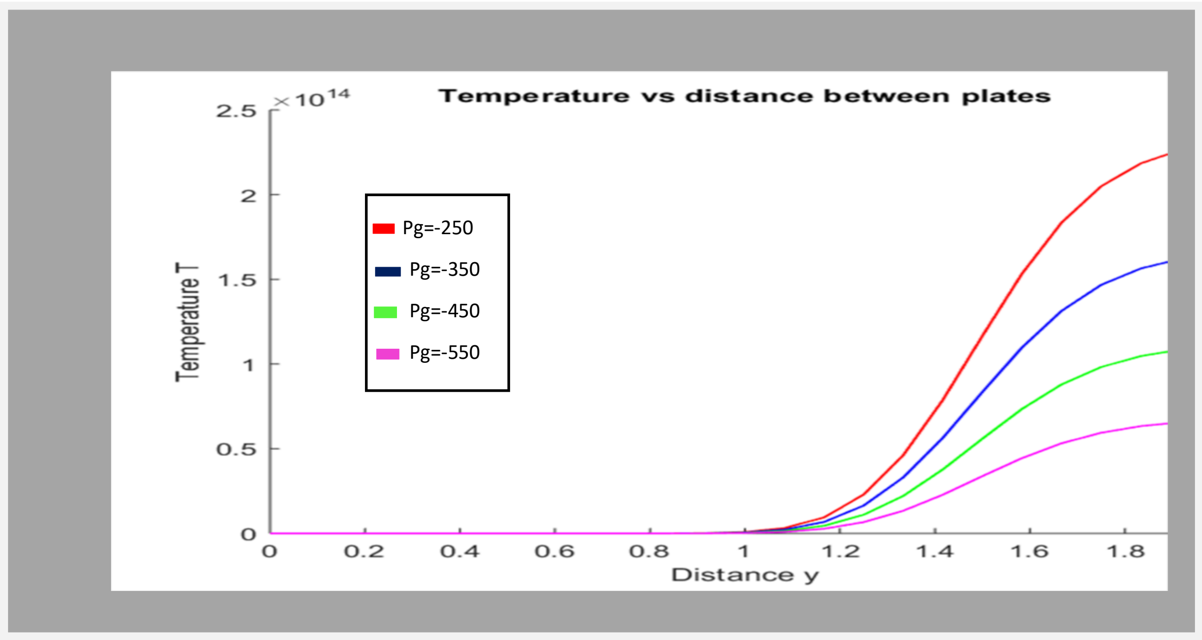

Fig. 2. Temperature profiles for different values of Pressure Gradient (Pg)

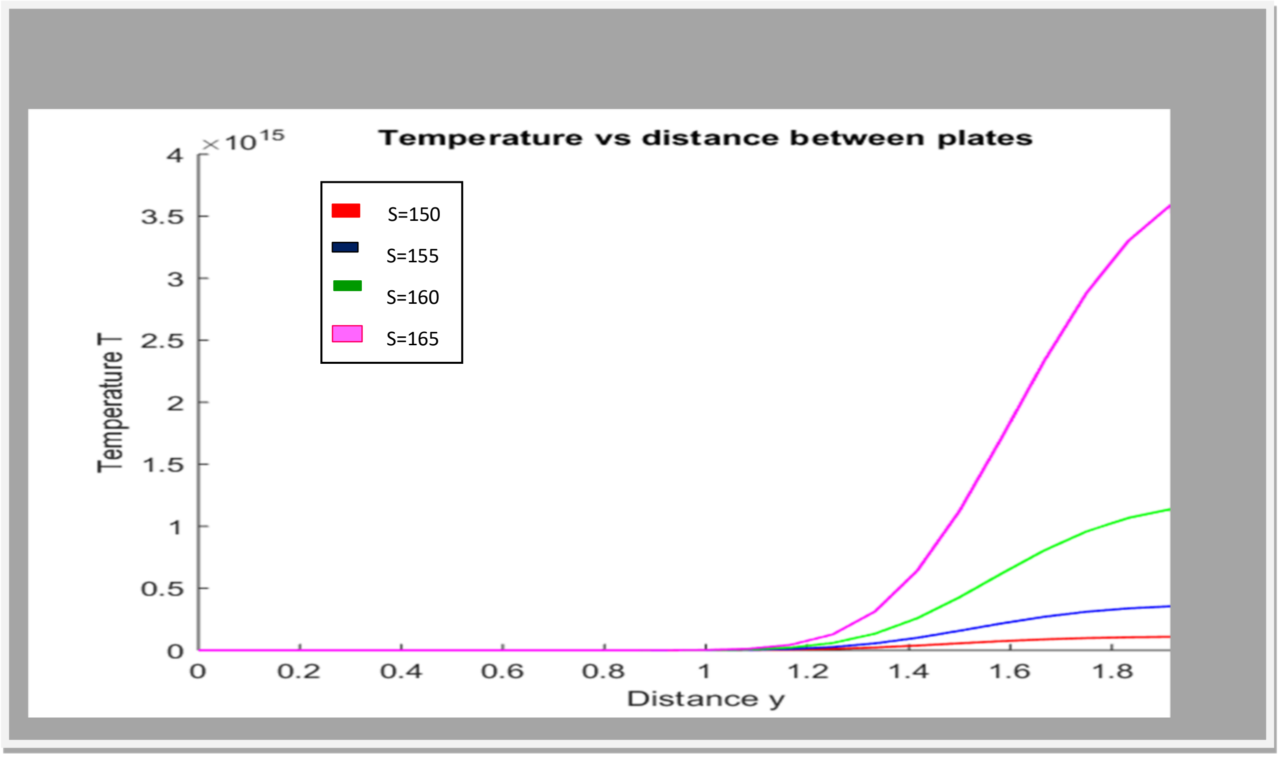

Figure 3 Temperature profiles for different values of suction parameter (S)

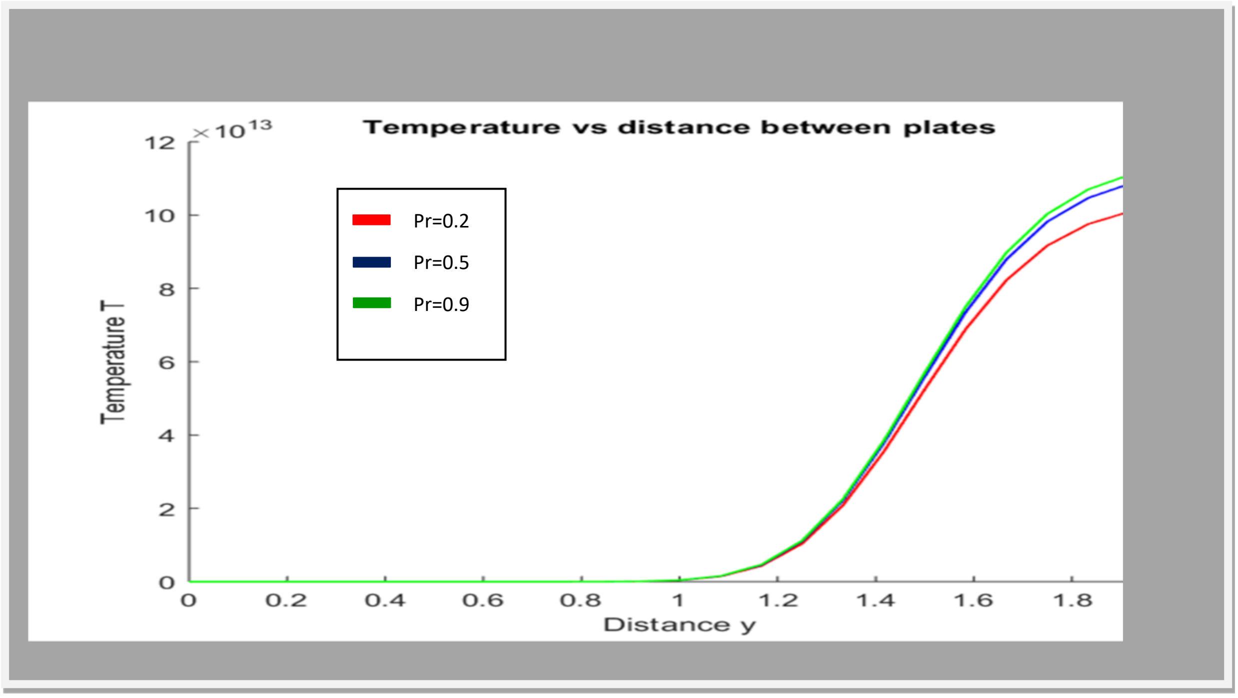

Figure 4 Temperature profiles for different values of Prandtl numbers (Pr)

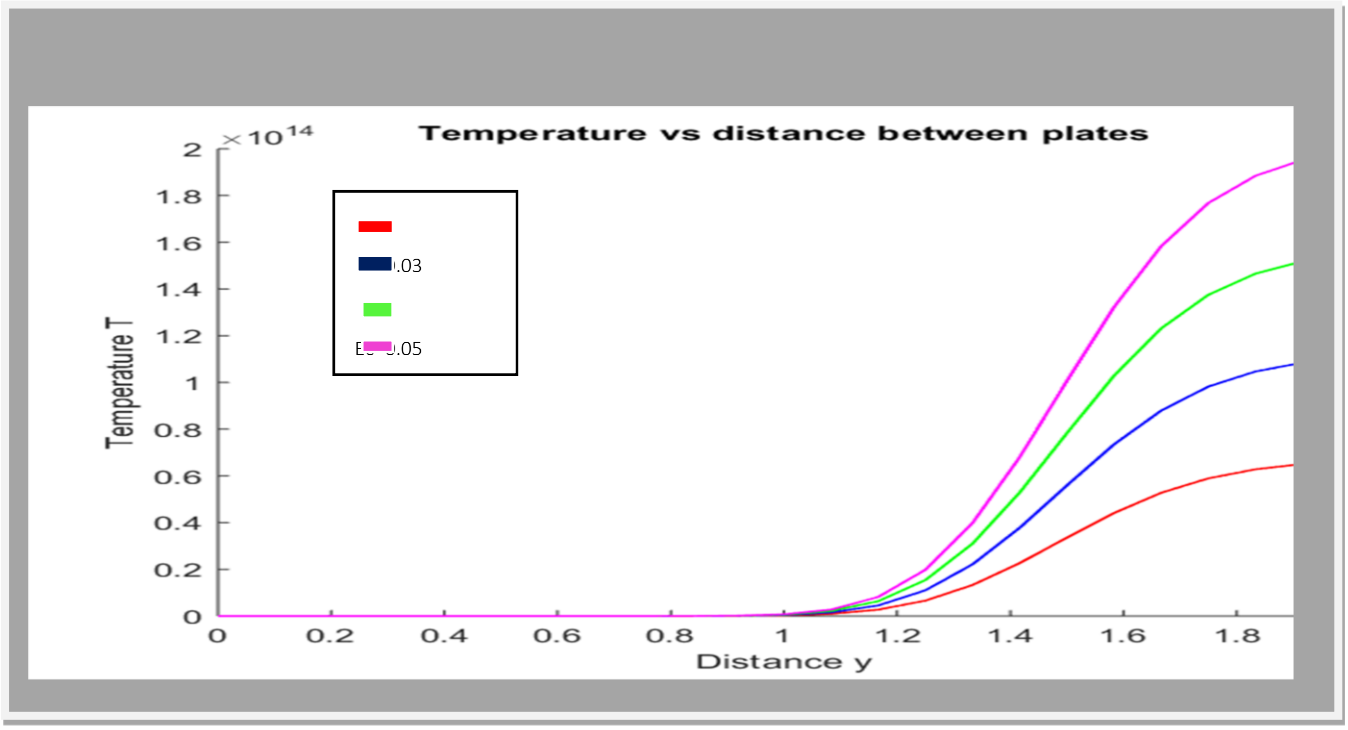

Figure 5 Temperature profiles for different values of Eckert number (Ec)

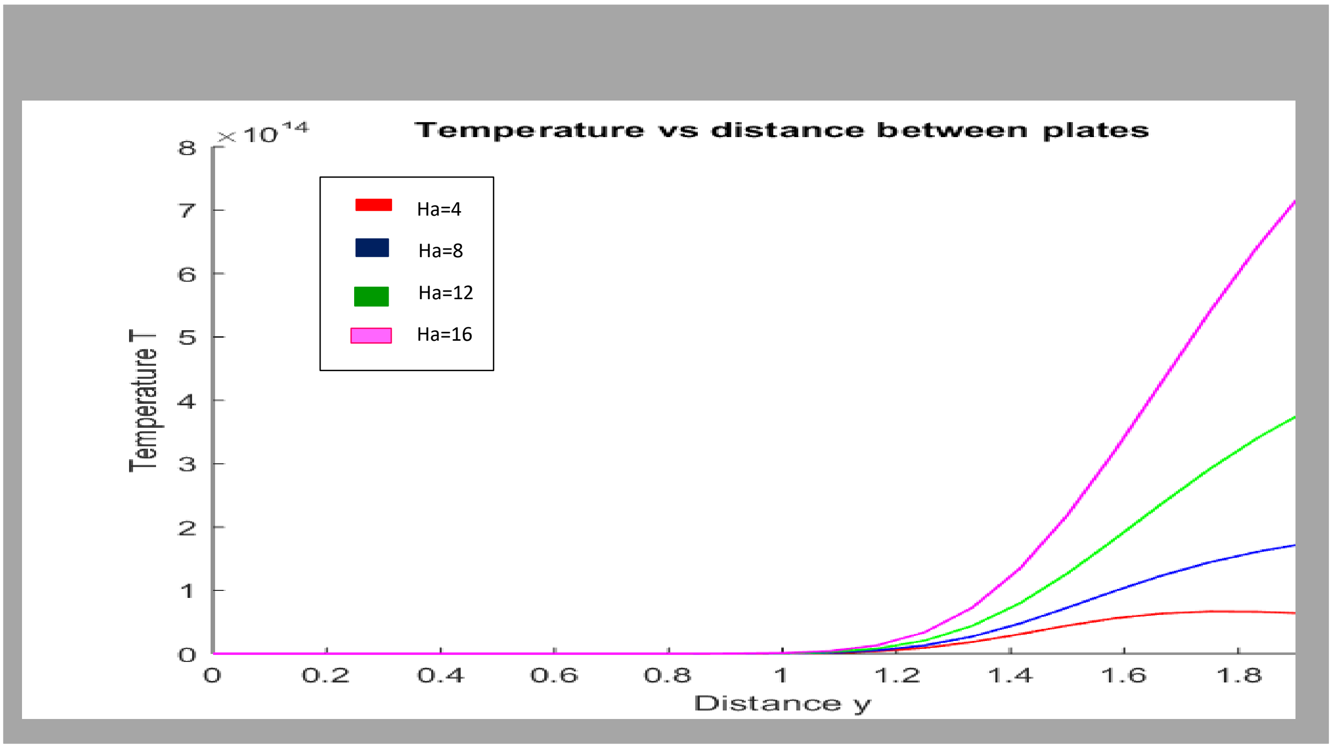

Figure 6 Temperature profiles for different values of Hartmann number (Ha)

5. Discussion

In figure 1,it was observed that an increase in Reynold's number resulted to a decrease in temperature of the fluid. Reynold's number gives the ratio of inertial force to viscous forces. In terms of this ratio, an increase in Reynold's number means that inertial forces increase while viscous forces decrease. When viscous forces reduce, friction or drag force exerted on the fluid reduces and thus the temperature of the fluid decreases.

In figure 2, the pressure gradient is negative because it acts in the same direction as the fluid flow. It illustrates that increasing the pressure gradient resulted to a decrease in temperature. When pressure gradient was increased there was less resistance to the fluid flow as the pressure gradient counteracted most of the forces that prevented the forward motion of the flow. This less resistance to the fluid resulted to a drop in temperature thus the decrease in temperature.

Figure 3 shows the effect of suction parameter on temperature. It was observed that an increase in suction parameter led to an increase in temperature of the fluid. This is because increasing the suction parameter resulted to decrease in the thickness of the boundary layer hence increasing the temperature gradient of fluid at the surface of the fluid.

From figure 4, it was observed that increasing Prandtl number resulted to an increase in temperature. When Prandtl number is high then the viscous/momentum diffusion governs the flow and the thermal boundary layer thickness decreases. The decrease in the thermal boundary layer thickness is the reason behind the decrease in temperature.

From figure 5, it was noted that increasing the Eckert number resulted to an increase in temperature. This is because an increase in Eckert number meant that the kinetic energy of the fluid flowing increased. A high kinetic energy results to particles of the fluid flowing to have very high velocities and this increases the vibrations of the molecules hence increased collisions of molecules. This increased collision results to increased heat dissipation in the boundary layer region thus increased temperature.

From figure 6, it was observed that an increase in Hartmann number resulted to an increase in temperature. This is due to the fact that an increase in Ha increases the Joule dissipation which is directly proportional to Ha. As a result the temperature increases as the Joule dissipation increased.

6. Conclusion

From this research study the following conclusions were made;

a) The application of variable pressure gradient has significant effects on the velocity and temperature profiles of a fluid.
b) The fluid temperature can be decreased by increasing the Reynold's number and Prandtl number.
c) The temperature distributions of a fluid can be increased by increasing the Eckert number, Hartmann number or the suction parameter.

7. Recommendations

In this research, the study of MHD flows between plates in presence of inclined magnetic field under a variable pressure gradient was not exhausted. It's recommended that this work be extended by considering any of the following:

a) Both plates are porous
b) A three dimensional flow
c) Variable suction
d) Variable inclined magnetic field

References

Attia, H. A. (2007). On the effectiveness of porosity on unsteady flow and heat transfer between parallel porous plates with exponential decaying pressure gradient. *Journal of Porous Media*, *10*(3).

Attia, H. A. (2009). Unsteady MHD Couette flow with heat transfer in the presence of uniform suction and injection. *International Journal for Engineering Modelling*, *22*(1-4).

Attia, H. A., Abbas, W., Abdeen, M. A., & Said, A. A. (2015). Heat transfer between two parallel porous plates for couette flow under pressure gradient and hall current. *Sadhana*, *40*(1), 183-197.

Chaturani, P., and Bharatiya, S. S. (2001). Two layered Magnetohydrodynamic flow through parallel plates with applications. *Indian journal of pure and applied mathematics*, *32*(1), 55-68.

Gunakala, S.R., &Job, V.M., (2014).Unsteady MHD free convection couette flow between two vertical permeable plates in presence of thermal radiation using Galerkins finite element method. *International Journal of heat & mass transfer*, 11(3), 491-508.

Hatsopolous, G. N. and Keenan J. H. (1965), Principle of General Thermodynamic, John Wiley and Sons Inc., 123-190.

Jha, B. K. (2001). Natural convection in unsteady MHD couette flow. *Heat and mass transfer*, *37*(4-5), 329-331.

Manyonge, W. A., Kiema, D. W., & Iyaya, C. C. W. (2012). Steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. *Int J Pure Appl Math*, *76*(5), 661-668.

Singh, C. B., & James, M. O. (2008). Couette flow between two parallel infinite plates in the presence of transverse magnetic field. *Kenya Meteorol. Soc*, *2*(2), 90-94.

Sharmilla, K., & Gayathri, C. (2015).Unsteady MHD stokes flow of viscous fluid with radiative heat transfer.*IOSR Journal of mathematics,* 11(2), 6-11.