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Unique Fixed Points in $\mathcal{G}$ −Spaces

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Abstract :

This paper is review about fixed points in $\mathcal{G}$ −spaces. We see the conditions where the map has unique fixed points in $\mathcal{G}$ −spaces.

Keywords: unique fixed point, $\mathcal{G}$ −space, complete $\mathcal{G}$ −space.

1. Introduction

fixed point theory is an important concept due to its applications in many branches like optimization, and approximation theory. in metric spaces , The fixed point theorems are using to solve problems in applied mathematics and sciences. Some authors extended them to $\mathcal{G}$ −metric spaces as we see in this paper. Let $\mathfrak{N}$ is $\mathcal{G}-$ space such that $\mathcal{G}: \mathfrak{Y}^3 \to \mathbb{R}_+$. We said that a sequence $(\varkappa_j)$ converges if there is $\varkappa \in \mathfrak{Y}$ such that $\lim_{j,i\to\infty} \mathcal{G}(\varkappa, \varkappa_j, \varkappa_i) = 0$, and it called $\mathcal{G}$ −Cauchy if given $\varsigma > 0$, there exists $\mathfrak{N} \in \mathbb{N}$ such that $\mathcal{G}(\varkappa_j, \varkappa_i, \varkappa_{\mathfrak{k}}) < \varsigma$, for all $j, i, \mathfrak{k} \geq \mathfrak{N}$. If every $\mathcal{G}$ −Cauchy sequence is $\mathcal{G}$ −convergent in $\mathfrak{Y}$ then $\mathfrak{Y}$ is complete. The two self-mappings r and t of $\mathfrak{Y}$ are be satisfying condition $\mathcal{G}-$ (E.A) - property if there is a sequence $(\varkappa_j)$ in $\mathfrak{Y}$ such that $(r(\varkappa_j))$ and $(t(\varkappa_j))$ are $\mathcal{G}-$ convergent to some $t \in \mathfrak{Y}$. The map $\Omega: [0,1) \to [0,1)$ is an altering distance if it is increasing and continuous and $t = 0$ if and only if $\Omega(t) = 0$. Let $\mathcal{F}_{\mathcal{G}}$ be the set of all continuous functions $\mathcal{F}(t_1, \dots, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying : $(\mathcal{F}1) : \mathcal{F}(t, t, 0, t, t, t')0$ implies $t < t'$, for all $t' > 0$; $(\mathcal{F}2) : \mathcal{F}(t, 0, 0, t, 0, t) > 0$, for all $t > 0$. (V. Popa and A. M. Patriciu, 2015).

2. Unique Fixed Points

*2.1 Proposition (*Z. Mustafa, H. Obiedat and F. Awawdeh, 2008)

The mapping $\pounds : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \text{ѵ}, \text{з} \in \mathfrak{N}$:

- $\mathcal{G}(\pounds(\varkappa), \pounds(\text{ѵ}), \pounds(\text{з})) \leq \{\mu\mathcal{G}(\varkappa, \text{ѵ}, \text{з}) + \lambda\mathcal{G}(\varkappa, \Upsilon(\varkappa), \pounds(\varkappa)) + \kappa\mathcal{G}(\text{ѵ}, \pounds(\text{ѵ}), \pounds(\text{ѵ})) + \iota\mathcal{G}(\text{з}, \pounds(\text{з}), \pounds(\text{з}))\}$
- $\mathcal{G}(\pounds(\varkappa), \pounds(\text{ѵ}), \pounds(\text{з})) \leq \{\mu\mathcal{G}(\varkappa, \text{ѵ}, \text{з}) + \lambda\mathcal{G}(\varkappa, \varkappa, \pounds(\varkappa)) + \kappa\mathcal{G}(\text{ѵ}, \text{ѵ}, \pounds(\text{ѵ})) + \iota\mathcal{G}(\text{з}, \text{з}, \pounds(\text{з}))\}$

where $0 \leq \mu + \lambda + \kappa + \iota < 1$.

*2.2 Proposition (*K. Rauf, B. Y. Aiyetan, D. J. Raji and R. U. Kanu, 2017)

The mapping $\pounds : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \text{ѵ}, \text{з} \in \mathfrak{N}$:

- $\mathcal{G}(\pounds(\varkappa), \pounds(\text{ѵ}), \pounds(\text{з})) \leq \mu\mathcal{G}(\varkappa, \pounds(\varkappa), \pounds(\varkappa)) + \lambda\mathcal{G}(\varkappa, \text{ѵ}, \text{з})$
- $\mathcal{G}(\pounds(\varkappa), \pounds(\text{ѵ}), \pounds(\text{з})) \leq \mu\mathcal{G}(\varkappa, \varkappa, \pounds(\varkappa)) + \lambda\mathcal{G}(\varkappa, \text{ѵ}, \text{з})$

,where $0 \leq \mu + \lambda < 1$.

*2.3 Proposition (*S. K. Mohanta and S. Mohanta, 2012)

The mapping $\pounds : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \text{ѵ} \in \mathfrak{N}$:

- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathfrak{v})) \le \mu\{\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})) + \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa))\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(y)) \le \mu\{\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})) + \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa))\}$

, where $\mu \in [0, \frac{1}{2})$.

2.4 Proposition (Z. Mustafa, H. Obiedat and F. Awawdeh,2008, S. K. Mohanta and S. Mohanta, 2012)

The mapping $£ : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \mathfrak{v}, \mathcal{Z} \in \mathfrak{N}$:

- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\{\mathcal{G}(\varkappa, £(\varkappa), £(\varkappa)), \mathcal{G}(\mathfrak{v}, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\mathcal{Z}, £(\mathcal{Z}), £(\mathcal{Z}))\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\{\mathcal{G}(\varkappa, \varkappa, £(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathfrak{v})), \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\mathcal{Z}))\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa)), \mathcal{G}(\mathfrak{v}, £(\mathfrak{v}), £(\mathfrak{v}))\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathfrak{v}))\}$,
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le$
  $$\rho \max\left\{\begin{matrix}\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\varkappa, £(\mathcal{Z}), £(\mathcal{Z})), \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa)), \mathcal{G}(\mathfrak{v}, £(\mathcal{Z}), £(\mathcal{Z})), \mathcal{G}(\mathcal{Z}, £(\varkappa), £(\varkappa)),\\ \mathcal{G}(\mathcal{Z}, £(\mathfrak{v}), £(\mathfrak{v}))\end{matrix}\right\}$$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\left\{\begin{matrix}\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})), \mathcal{G}(\varkappa, \varkappa, £(\mathcal{Z})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathcal{Z})), \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\varkappa)),\\ \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\mathfrak{v}))\end{matrix}\right\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa))\}$,
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa))\}$,
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\left\{\begin{matrix}\mathcal{G}(\varkappa, \mathfrak{v}, \mathcal{Z}), \mathcal{G}(\varkappa, £(\varkappa), £(\varkappa)), \mathcal{G}(\mathfrak{v}, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\mathcal{Z}, £(\mathcal{Z}), £(\mathcal{Z})),\\ \frac{\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})) + \mathcal{G}(\mathcal{Z}, £(\varkappa), £(\varkappa))}{2}, \frac{\mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})) + \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa))}{2},\\ \frac{\mathcal{G}(\mathfrak{v}, £(\mathcal{Z}), £(\mathcal{Z})) + \mathcal{G}(\mathcal{Z}, £(\mathfrak{v}), £(\mathfrak{v}))}{2}, \frac{\mathcal{G}(\varkappa, £(\mathcal{Z}), £(\mathcal{Z})) + \mathcal{G}(\mathcal{Z}, £(\varkappa), £(\varkappa))}{2}\end{matrix}\right\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\left\{\begin{matrix}\mathcal{G}(\varkappa, \mathfrak{v}, \mathcal{Z}), \mathcal{G}(\varkappa, \varkappa, £(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathfrak{v})), \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\mathcal{Z})),\\ \frac{\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})) + \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\varkappa))}{2}, \frac{\mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})) + \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa))}{2},\\ \frac{\mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathcal{Z})) + \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\mathfrak{v}))}{2}, \frac{\mathcal{G}(\varkappa, \varkappa, £(\mathcal{Z})) + \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\varkappa))}{2}\end{matrix}\right\}$
- $\mathcal{G}(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le$
  $$\rho \max\left\{\begin{matrix}\mathcal{G}(\varkappa, \mathfrak{v}, \mathcal{Z}), \mathcal{G}(\varkappa, £(\varkappa), £(\varkappa)), \mathcal{G}(\mathfrak{v}, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\varkappa, £(\mathfrak{v}), £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, £(\varkappa), £(\varkappa)),\\ \mathcal{G}(\mathcal{Z}, £(\mathcal{Z}), £(\mathcal{Z}))\end{matrix}\right\}$$
- $v(£(\varkappa), £(\mathfrak{v}), £(\mathcal{Z})) \le \rho \max\{\mathcal{G}(\varkappa, \mathfrak{v}, \mathcal{Z}), \mathcal{G}(\varkappa, \varkappa, £(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\mathfrak{v})), \mathcal{G}(\varkappa, \varkappa, £(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, £(\varkappa)), \mathcal{G}(\mathcal{Z}, \mathcal{Z}, £(\mathcal{Z}))\}$

, where $0 \le \rho < 1$.

2.5 Proposition (Z. Mustafa, H. Obiedat and F. Awawdeh, 2008)

The mapping $£ : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \mathfrak{v}, \mathcal{Z} \in \mathfrak{N}$:

- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le \{\mu\mathcal{G}(\varkappa, \mathfrak{v}, \mathfrak{v}) + \lambda\mathcal{G}(\varkappa, \pounds^m(\varkappa), \pounds^m(\varkappa)) + \kappa\mathcal{G}(\mathfrak{v}, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})) + \iota\mathcal{G}(\mathfrak{z}, \pounds^m(\mathfrak{z}), \pounds^m(\mathfrak{z}))\}$
- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le \{\mu\mathcal{G}(\varkappa, \mathfrak{v}, \mathfrak{v}) + \lambda\mathcal{G}(\varkappa, \varkappa, \pounds^m(\varkappa)) + \kappa\mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\mathfrak{v})) + \iota\mathcal{G}(\mathfrak{z}, \mathfrak{z}, \pounds^m(\mathfrak{z}))\}$

, where $m \in \mathbb{N}$, $0 \le \mu + \lambda + \kappa + \iota < 1$.

*2.6 Proposition (*Z. Mustafa, H. Obiedat and F. Awawdeh, 2008)

The mapping $\pounds : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \mathfrak{v}, \mathfrak{z} \in \mathfrak{N}$:

- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le k \max\{\mathcal{G}(\varkappa, \pounds^m(\varkappa), \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})), \mathcal{G}(\mathfrak{z}, \pounds^m(\mathfrak{z}), \pounds^m(\mathfrak{z}))\}$
- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le k \max\{\mathcal{G}(\varkappa, \varkappa, \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\mathfrak{v})), \mathcal{G}(\mathfrak{z}, \mathfrak{z}, \pounds^m(\mathfrak{z}))\}$

*2.7 Proposition (*Z. Mustafa, H. Obiedat and F. Awawdeh, 2008)

The mapping $\pounds : \mathfrak{N} \to \mathfrak{N}$ on complete $\mathcal{G}$ −metric space $\mathfrak{N}$ has a unique fixed point if one of the following hold for all $\varkappa, \mathfrak{v}, \mathfrak{z} \in \mathfrak{N}$:

- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le \rho \max\left\{\begin{array}{c}\mathcal{G}(\varkappa, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})), \mathcal{G}(\varkappa, \pounds^m(\mathfrak{z}), \pounds^m(\mathfrak{z})), \mathcal{G}(\mathfrak{v}, \pounds^m(\varkappa), \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \pounds^m(\mathfrak{z}), \pounds^m(\mathfrak{z})), \\ \mathcal{G}(\mathfrak{z}, \pounds^m(\varkappa), \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{z}, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v}))\end{array}\right\}$,
- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{z})) \le \rho \max\left\{\begin{array}{c}\mathcal{G}(\varkappa, \varkappa, \pounds^m(\mathfrak{v})), \mathcal{G}(\varkappa, \varkappa, \pounds^m(\mathfrak{z})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\mathfrak{z})), \\ \mathcal{G}(\mathfrak{z}, \mathfrak{z}, \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{z}, \mathfrak{z}, \pounds^m(\mathfrak{v}))\end{array}\right\}$,
- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, \pounds^m(\varkappa), \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v}))\}$,
- $\mathcal{G}(\pounds^m(\varkappa), \pounds^m(\mathfrak{v}), \pounds^m(\mathfrak{v})) \le \rho \max\{\mathcal{G}(\varkappa, \varkappa, \pounds^m(\mathfrak{v})), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\varkappa)), \mathcal{G}(\mathfrak{v}, \mathfrak{v}, \pounds^m(\mathfrak{v}))\}$,

, for some $m \in \mathbb{N}$, where $\rho \in [0, 1)$.

*2.8 Proposition (*A. Branciari, 2002)

Let $\pounds : \mathfrak{N} \to \mathfrak{N}$ be a mapping on complete $\mathcal{G}$ −metric space $\mathfrak{N}$, and a Lebesgue measurable $\eth$ be mapping with finite integral on each compact subset of $[0, \infty)$, such that for $\varsigma > 0$, $\int_0^{\varsigma} \eth(t)dt > 0$. ϒ has a unique fixed point if, for all $\varkappa, \mathfrak{v} \in \mathfrak{N}$, it is satisfying

$$\int_0^{d(\pounds(\varkappa),\pounds(\mathfrak{v}))} \eth(t)dt \le c \int_0^{d(\varkappa,\mathfrak{v})} \eth(t)dt$$

, where $c \in (0,1)$.

*2.9 Proposition (*V. Popa and A. M. Patriciu, 2015)

Let $r, t : \mathfrak{N} \to \mathfrak{N}$ be weakly compatible mapping on complete $\mathcal{G}$ −metric space $\mathfrak{N}$, and a Lebesgue measurable $\eth$ be mapping with finite integral on each compact subset of $[0, \infty)$, such that for $\varsigma > 0$, $\int_0^{\varsigma} \eth(t)dt > 0$. Then r and t have a unique fixed point if, for all $\varkappa, \mathfrak{v} \in \mathfrak{N}$, $\mathcal{F} \in \mathfrak{F}_{\mathcal{G}}$, the following hold :

- $\mathcal{F}\begin{pmatrix} \int_0^{\mathcal{G}(r(x),r(v),r(v))} \eth(t)dt, \int_0^{\mathcal{G}(t(x),t(v),t(v))} \eth(t)dt, \int_0^{\mathcal{G}(t(v),r(v),r(v))} \eth(t)dt, \\ \int_0^{\mathcal{G}(r(x),t(v),t(v))} \eth(t)dt, \int_0^{\mathcal{G}(r(v),t(x),t(v))} \eth(t)dt, \int_0^{\mathcal{G}(r(x),t(x),t(v))} \eth(t)dt \end{pmatrix} \leq 0$
- The two maps r and t are satisfying $\mathcal{G}-$ (E:A) - property,
- $t(\mathfrak{N})$ is a subspace and closed in $\mathfrak{N}$.

*2.10 Proposition (*V. Popa and A. M. Patriciu, 2015)

Let $r, t : \mathfrak{N} \to \mathfrak{N}$ be weakly compatible mapping on complete $\mathcal{G}$ −metric space $\mathfrak{N}$, $\mathcal{F} \in \mathfrak{F}_{\mathcal{G}}$ and let Ω be an altering distance. Then the two maps r and t have unique fixed point if the following hold for all $x, v \in \mathfrak{N}$:

- $\mathcal{F}(\Omega\left(\mathcal{G}(r(x), r(v)\, r(\mathfrak{y}))\right),\ \Omega\left(\mathcal{G}(t(x), t(v), t(v))\right),$

  $\Omega\left(\mathcal{G}(t(v), r(v), r(v))\right), \Omega\left(\mathcal{G}(r(x), t(v), t(v))\right), \Omega\left(\mathcal{G}(t(x), r(v), t(v))\right),$

  $\Omega(\mathcal{G}(r(x), t(x), t(v)))) \leq 0$
- The two maps r and t are satisfying $\mathcal{G}-$ (E:A) - property,
- $t(\mathfrak{N})$ is a subspace and closed in $\mathfrak{N}$.

*2.11 Proposition (*V. Popa and A. M. Patriciu, 2015)

Let $r, t : \mathfrak{N} \to \mathfrak{N}$ be weakly compatible mapping on complete $\mathcal{G}$ −metric space $\mathfrak{N}$, $\mathcal{F} \in \mathfrak{F}_{\mathcal{G}}$. Then r and t have a unique fixed point if , for all $x, v \in \mathfrak{N}$, the following hold :

- $\mathcal{F}(\mathcal{G}(r(x), r(v), r(v)), \mathcal{G}(t(x), t(v), t(v)), \mathcal{G}(t(v), r(v), r(v)), \mathcal{G}(r(x), t(v), t(v)), \mathcal{G}(r(v), t(x), t(v)),$

  $\mathcal{G}(r(x), t(x), t(v))) \leq 0$
- The two maps r and t are satisfying $\mathcal{G}-$ (E:A) - property,
- $t(\mathfrak{N})$ is a subspace and closed in $\mathfrak{N}$,

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