APPLICATION OF MARKOV CHAIN MODEL IN CAREER PROGRESSION OF UNIVERSITY ACADEMIC STAFF: *A Case Study of the Moi University-Eldoret, Kenya.*

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Abstract: The use of Mathematical models for manpower planning has increased in recent times for better manpower planning quantitatively both in public and private sectors. In respect of organizational management, numerous previous studies have applied Markov chain models in describing title or level promotions, demotions, recruitment, withdrawals, or changes of different career development paths to confirm the actual manpower needs of an organization or predict the future manpower needs. The movements of staff within the grades or job group levels called transitions are usually the consequences of promotions or transfers between segments or wastage and recruitment into the system. In this study we determined and compared the transition rates of the academic staff of science and art faculties, the expected time taken before one attains the highest academic rank, and the absorption rates in the university. The data was collected from Moi University- Eldoret and the grades or job groups were: Tutorial Fellow, Lecturer, Senior Lecturer, Associate Professor, and full Professor. The study established that the transition rates are high at the Tutorial fellow and lecturer levels in both science and art with 67.09% and 86.31% and 86.00% and 97.53% respectively within the first ten years of employment. But it was low at 50% at senior lecturer and associate professor in the faculty of science and 63.51% and 88.69% for the same ranks in the faculty of arts. . It took academic staff 19.51 years and 22.74 years in science and art respectively to attain the rank full professor.

Keywords: *Jobgroups, transitional rate, markov chain,absorption rate,expectation*

I. Introduction

A Markov chain, named after a Russian Mathematician, Andrey Markov in 1907, is a random process that undergoes transition from one state to another on a state space. A Markov process is a random process in which the future is independent of the past, given the present. Thus, Markov processes are the natural stochastic analogs of the deterministic processes described by differential and difference equations. They form one of the most important classes of random processes.

Markov models are being extensively used for analysis of manpower planning systems. Most of these models concentrate either on estimating the grade wise distribution of future manpower structure, given the existing structure and promotion policies, or on deriving policies towards promotion, given the required future structure. Manpower systems are hierarchical in nature and consists of a finite number of ordered grades for which internal movement or promotion of staff is possible from one grade to another though there is no promotion beyond the highest grade. Members of staff in the same grade have certain common characteristics and attributes (such as rank, trade, or experience) and the grades are mutually exclusive and exhaustive so that any staff must belong to one but only one grade at any time (Georgiou & Tsantas, 2002) Markov chain theory is one of the Mathematical tools used to investigate dynamic behaviours of a system (e.g. workforce system, financial system, health service system) is a special type of discrete-time stochastic process in which the time evolution of the system is described by a set of random variables.

Universities largely exist to perform three main core functions; advancement of knowledge through research, impacting of (acquired) knowledge through teaching (in its various forms) and community service. These functions culminate into national development and competitive for a country and the world at large and need to be rewarded by participants. The achievement of universities objectives, to a large extent, depends on the academic staff because of the critical role they play in the teaching and learning process. Again, assert that the most attractive reward perceived by the staff in any organization is promotion and that promotion will improve the staff objective, performance and aspirations which put academics under pressure. There are many reasons for academics under pressure, but striving for academic promotion is regarded as one of the most influential factors. Academic staff transition within the ranks is understood as a movement from one academic rank to another higher rank. Academic promotions in educational institutions remain a crucial part of the development of both institutions and individuals for many reasons. It seeks to recognize and reward excellent staff. All academic staff, other than those on casual appointments, is eligible and encouraged to apply for academic promotion in universities around the world. In many instances, there are no quotas and all staff meeting the promotion standards can advance their promotion process. Contributions to teaching, research and service are all highly valued. Academic promotions criteria are basically factored on teaching, research and publications and community service, publication being a major factor. Fundamentally, global university rankings are used to compare educational performance and productivity nationally and internationally, and measure educational quality and excellence. Also, global university rankings are often used as an indication of a nation's global competitiveness, given the importance of higher education to social and economic growth and innovation. Promotion brings with it not only financial rewards but also a mark of recognition of the individual performance and acceptance by staff and students. The process of decision-making for academic staff promotion often involves criteria, such as tasks, activities, teaching, supervision, publications, research, consulting, conferencing, administration, and community service. From academic staff appraisal can also be evaluated through items, such as research articles produced, teaching method, presentation style, and involvement in university and community activities which at the end culminate to promotion of the individual.

The Kenyan higher education system has been under transformation since 1963 (post-independence) with the changing system of government. Academic profession in Kenya has also been changing under the impact of various approaches of new public management by the Ministry of Education through the Commission for University

Education (CUE). The Commission for University Education in its harmonized criteria and guidelines for appointment and promotions of academic staff in Universities in Kenya 2014, developed the grading nomenclature as follows; Graduate assistant/research Assistant, Tutorial Fellow/ Junior Research Fellow, Lecturer/Research Fellow, Senior Lecturer/Research Fellow, Associate Professor, Professor, Adjunct Academic Staff, and Visiting Academic Staff. The Grading Nomenclature shall carry the rider "or equivalent". Universities may have equivalents provided for in their chatters (CUE, 2014). Different categories of staff have different promotional criteria and are encouraged to seek for promotions as and when individuals are prepared. Modeling and formulating processes such as academic promotion has taken many dimensions but little is said of statistical application to this vital part of educational institutions. Considering the fact that promotion process is a form of transition that academic staff goes through, and at the end of their tenure, they shall retire on the grade of professor or non-professor, the process of academic promotions can assume a stochastic absorbing process.

II. Related Literature Review

In the respect of organizational management and manpower planning, numerous previous studies have applied the Markov chain in describing title or level promotions, demotions, or changes of different career development paths to confirm the actual manpower needs of an organization or predict the future manpower needs. Optimization of manpower and forecasting manpower needs in modern conglomerates are essential part of the future strategic planning and a very important different nature of business imperatives, (Adisak, 2015). The Markov chain model allows us to answer questions from policy makers. For example, it allows easy computation of various statistics at both individual and aggregate levels. At the individual level, it can be used to describe the probabilistic progression for a staff at a given career stage. At aggregate level, it can be used to derive information on overall continuation rates and separation behavior which are critical inputs in developing retention programs. Markov chain model has been widely used in different fields including Education to study students' enrolment projection both in secondary schools and tertiary institutions. Education system is comparable to a hierarchal organization in which after an academic session, three possibilities arise in the new status of the students; the students may move to the next higher class, may repeat the same class, or leave the system successfully as graduates or drop out of the system before attaining the maximum qualification (Nyandwaki & Kennedy, 2016). Modeling the manpower management mainly concerns the prediction of future behavior of employees, (Rachid & Mohamed, 2013). Trend researchers used Markov chain model associated or integrated to describe the change of the process in light of its historical evolutions, (Bartholomew, 1991). Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making bodies use, Hamedet at, (2013). Manpower planning is the process by which the management determines how an organization should move from its current manpower position to its desired manpower position. Through planning, management strives to have the right number and right kinds of people, at the right places at the right time, doing things which result in both the organization and individual receiving maximum long-run benefits. Collings and Wood (2009) defined manpower planning as the range of philosophies, tools and techniques that any organization should deploy to monitor and manage the movement of staff both in terms of numbers and profiles. These movements of staff called transitions are usually the consequences of promotions, transfer between segments or wastage and recruitment into the system. The approach to manpower policy in most Nigerian universities appears to be guided by the traditional method of putting the right number of people in the right place at the right time or arranging the suitable number of people to be allocated to various jobs usually in a hierarchal structure, (Igboanugo & Onifade, 2011). Manpower planning involves two stages. The first stage is concerned with the detailed planning of manpower requirements for all types and levels of employees throughout the planning period and the second stage is concerned with the planning of manpower supplies to provide the organization with the right types of people from all sources to meet the planning requirements. An adage says, he who fails to plan, plans to fail. The planning processes one of the most crucial, complex and continuing managerial functions which embraces organizational, development, managerial development, career planning and succession planning. The process of manpower planning may rightly be regarded as a multi-step process including various issues such as; Deciding goals or objectives, Auditing of internal resources, Formulation of recruitment plan, Estimating future organizational structure and manpower requirements and developing a human resource plan. Effective manpower planning is very crucial which organizations, like large companies, academic system, federal and state administration must carryout, since human resources are considered as the most crucial, volatile and potentially unpredicted resource which an organization utilizes. The prediction of manpower is subject to how current supply of employees will change internally. These changes are observed by analyzing what happened in the past, in terms of staff retention or movement, extrapolating into the future to see what happens with the same trend of the past. Markov chain is a useful tool in prediction and has been used extensively in many areas of human endeavors. Rachid and Mohamed (2013) presented a predictive model of numbers of employees in a hierarchal dependent- time system of human resources, incorporating subsystems that each contains grades of the same family. The proposed model was motivated by the reality of staff development which confirms that the path evolution of each employee is usually in its family of grades. Kwon (2006) used Marko chain model and job coefficient to investigate the difference of manpower status between US and Korean nuclear industry and to predict the future manpower requirements in Korea. The workforce planning, on the basis of established process, requires a good knowledge of those deployed in the establishment, as well as entry, dropout ant promotion of employees in order to reach a future plan fit and desired administration in determining the future policies of the workforce system, Touama, (2015) applied Markovian models and transition probability matrix to analyze the movement of the workforce in Jordan productivity companies. To achieve his aim, he collected secondary data related to workforce movement selected from annual reports of Jordanian productivity companies (potash, phosphate and pharmaceutical) for year 2004. The purpose of manpower planning is to get a better matching between manpower requirements and manpower availability. Parma et al, (2013) considered an optimization model for manpower system where vacancies are filled up by promotion and recruitment in automation system engineering private limited. They proposed a method for the determination of transition probability of promotion and recruitment vector by using Markovian theory with certain assumptions. Rahela, (2015) analyzed manpower data of higher learning institution using Markov chain. His objective was to design a planning model for projecting university faculty employment under alternative policies suggested by the government. Wan-yin and Shou, (2015) focused on the improved gray Markov model in human resource internal supply forecasting, so that enterprises can reasonably predict their internal human resource supply through Markov model and provide important guarantee for enterprises to develop human resources strategic planning. Ekhosuehi, (2013) examined the passage of staff in a faculty using one state absorbing Markov chain. He considered two cases involving regardless of staff leaving intension and staff unwillingness to leave. Babu and Rao, (2013) carried out studies on two graded manpower model with bulk recruitment in both grades. They assumed that the organization is having two grades and recruitment is done in both grades in bulk. Osagiede and Ekhosuehi, (2006) presented the use of Markovian model to project the future enrolment of students in a university where they removed the assumption of certain constant values in the rate of new intake by some previous authors and provided a better method for calculating the constant value of increment in the new intake. The general objective of the study is to determine and compare the transition rates in science and art based schools and identify the area of specialization.

III. Material And Methods

3.1 Research design

In this study we used secondary data collected from Moi University human resources department on academic staff employment for the years of 1987 - 2018 to examine the dynamics of the academic staff within the job groups or grades. We achieved analytically through solving the transition probability matrix of the Markov Chain to evaluate academic staff progression.

3.2 Target population

The target population for the study will be the academic staff employed by Moi University between 1987 and 2018 in science and art faculties of the university.

3.3 Sample frame

The study will use a purposive sampling where the population of the academic staff will be grouped in five academic grades or job groups in respective schools (science and art) namely; Tutorial Fellow, Lecturer, Senior Lecturer, Associate Professor, Full Professor. The data was collected from Moi University

3.4 Model Development

3.4.1 Grade or Job Group Transition

The variable $p_{i,j}$ represent job group progression of the academic staff in the university. In this study, a Markov Chain model on transition will be employed to compare and determine the career progression of the academic staff in science and art based schools in Moi University. A Markov chain is a process with finite number of states or outcomes, or events in which the probability of being in a particular state at step $n+1$ depends on the state occupied at step n.Let $S = \{S_1, S_2, S_3, \ldots, S_r\}$ be the possible states and

$$P_n = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_r \end{pmatrix}$$

The vector of probabilities of each state at step $\boldsymbol{n}$ that is, i^{th} entry of p_n is the probability that the process is in state S_i at step $\boldsymbol{n}$**.** For such a probability vector,

$$p_1 + p_2 + \cdots + p_r = 1$$

Let $P_{ij} = \Pr\,(state\ at\ step(n+1) = S_j \,|\, state\ at\ step\ n = S_i$

A homogeneous DTMC is described by its transition matrix $P = [p_{i,j}]i,j \in E$. $p_{i,j}$is the transition probability; probability of moving from state i to state$j$. In the transition probability matrix, $p_{i,j}$ where i is the initial state (job group) and j is the next state (job group). i.e.

$\boldsymbol{p_{12}}$**:** Probability of a Tutorial Fellow progressing to Lecturer

$\boldsymbol{p_{23}}$**:** Probability of a Lecturer progressing to Senior Lecturer

$\boldsymbol{p_{34}}$**:** Probability of a Senior Lecturer progressing to an Associate Professor

$\boldsymbol{p_{45}}$**:** Probability of an Associate Professor progressing to Full Professor

$\boldsymbol{p_{i6}}$**:** Probability of attrition or exit of service ;($i = 1,2,3,4,5,6$)

The canonical form of the transition probability matrix is;

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ . & \cdots & \cdots & \cdots \\ \vdots & \cdots & \cdots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{bmatrix}$$

$\boldsymbol{p_{ij}}$ is the conditional probability of being in state $\boldsymbol{S_i}$at step$\boldsymbol{n+1}$given that the process was in state S_i at step n. $P$is called transition matrix. Pcontains all conditional probabilities of the Markov Chain. Pcan labeled as below.

$$State\ \ n;\ where\ \ n = 6\ \ , P = \begin{cases} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{cases} \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{bmatrix}$$

3.4.2 Absorbing Markov Chain

An absorbing state is one in which the probability that the process remains in that state once it enters the state is 1. A Markov chain is absorbing if it has at least one absorbing state, and if from every state it is possible to go to an absorbing state (not necessarily in one step).A Markov Chain which is not absorbing is called transient; a state i is said to be transient if, given that we start in state i there is a non-zero probability that we will never return to that state. Markov chain with t transient states and r absorbing states, the transition probability matrix P, will take the following canonical form;

$$P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$$

Where;

Q is a t x t matrix, q_{ij} is the probability of an element being in state$i$ at time $t-1$ will be in state$j$ at time t; $i, j = 1,2, \dots, t$;

R is a non-zero t x r matrix, r_{ik} being the probability that an element in state i at time $t-1$ will be in state$k$ at time t; $i, j = 1,2, \dots, t$, and $k = 1,2, \dots, t$;

O is r x t zero matrix and I is a r x r identity matrix

IV. Results and Discussion

4.1 Transition rates for the combined faculties (Science and Arts)

The transition probabilities as at 2018 of the combined faculties of 32 academic staff employed as tutorial fellows in the year 1987 is given below;

$$P_{SA} = \begin{bmatrix} 0.0300 & 0.9700 & 0.0000 & 0.000 & 0.0000 & 0.000 \\ 0.0000 & 0.4200 & 0.3900 & 0.0000 & 0.0000 & 0.1900 \\ 0.0000 & 0.0000 & 0.0800 & 0.9200 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2800 & 0.7200 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

From P_{SA} we obtain Q_{SA} and R_{SA} as follows;

$$Q_{SA} = \begin{bmatrix} 0.0300 & 0.9700 & 0.0000 & 0.0000 \\ 0.0000 & 0.4200 & 0.3900 & 0.0000 \\ 0.0000 & 0.0000 & 0.0800 & 0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 0.2800 \end{bmatrix} \qquad R_{SA} = \begin{bmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.1900 \\ 0.0000 & 0.0000 \\ 0.7200 & 0.000 \end{bmatrix}$$

After the second period (10 years) the transition rate is given by;

$$P^2 = \begin{pmatrix} Q_{SA}^2 & R_{SA}(I_{SA} + Q_{SA}) \\ 0_{SA} & I_{SA} \end{pmatrix}$$

$$= (I_{SA} + Q_{SA})\ R_{SA}$$

$$\begin{bmatrix} 1.0300 & 0.9700 & 0.0000 & 0.000 \\ 0.0000 & 1.4200 & 0.3900 & 0.0000 \\ 0.0000 & 0.0000 & 1.0800 & 0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 1.2800 \end{bmatrix} \begin{bmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.1900 \\ 0.0000 & 0.0000 \\ 0.7200 & 0.0000 \end{bmatrix} = \begin{bmatrix} 0.0000 & 0.18430 \\ 0.0000 & 0.26980 \\ 0.66240 & 0.00000 \\ 0.92160 & 0.00000 \end{bmatrix}$$

By using the absorbing states model, it is clear that 18.43% of those recruited as tutorial fellows exited or remained in the same academic rank the service while 81.57% transited to the next academic rank and none transited to professor rank after a period of ten years. For those at the lecturer level 26.98% either exited or remained in the same rank while 73.02% proceeded to the next academic rank and none to the rank of professor. For those in senior lecturer rank 33.76% exited or remained in the same rank while 66.24% moved to the next academic rank of professor. For those in associate professor level 7.84% remained or exited while 92.16% moved to the next academic rank of professor.

4.1.1 Transition rates for the faculty of science

The transition rate for the academic staff in the faculty of science can be obtained as follows;

After ten years of service as academic staff, the transition rate will be

$$P^2 = \begin{pmatrix} Q_S^2 & R_S(I_S + Q_S) \\ 0_S & I_S \end{pmatrix}$$

$$(I_S + Q_S)\ R_S$$

$$\begin{pmatrix} 1.1100 & 0.8900 & 0.0000 & 0.0000 \\ 0.0000 & 0.3700 & 0.2500 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \begin{pmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.3700 \\ 0.0000 & 0.0000 \\ 0.5000 & 0.0000 \end{pmatrix} = \begin{pmatrix} 0.00000 & 0.3293 \\ 0.0000 & 0.1369 \\ 0.5000 & 0.0000 \\ 0.5000 & 0.0000 \end{pmatrix}$$

From the matrix calculation above, 32.93% of those engaged as tutorial fellows either remained in the same rank or exited the service while 67.07% transited to the next level and none transited to the rank of professor. For those in the lecturer level 37.00% never progressed to the next level while 63.00% proceeded to the next level. For those in senior lecturer position none exited and therefore all transited to the next level while none transited to the rank of professor. And 50% of those at the associate professor level transited to professor rank while 50% of them remained in the same position.

4.1.2 Transition rates for the faculty of Arts

The transition rate for the academic staff in the faculty of arts can be obtained as follows;

After ten years of service as academic staff, the transition rate will be

$$P^2 = \begin{pmatrix} Q_A^2 & R_A(I_A + Q_A) \\ 0_A & I_A \end{pmatrix}$$

$$(I_A + Q_A)\ R_A$$

$$\begin{pmatrix} 1.300 & 0.700 & 0.000 & 0.000 \\ 0.000 & 0.130 & 0.680 & 0.00 \\ 0.000 & 0.000 & 0.270 & 0.730 \\ 0.000 & 0.000 & 0.000 & 0.130 \end{pmatrix} \begin{pmatrix} 0.000 & 0.000 \\ 0.000 & 0.190 \\ 0.000 & 0.000 \\ 0.870 & 0.000 \end{pmatrix} = \begin{pmatrix} 0.0000 & 0.1330 \\ 0.0000 & 0.0247 \\ 0.6351 & 0.000 \\ 0.1131 & 0.000 \end{pmatrix}$$

From the matrix calculation above, 13.30% of those engaged as tutorial fellows either remained in the same rank or exited the service while 86.70% transited to the next level and none transited to rank of professor. For those in the lecturer level 2.47% never move to the next level while 97.53% proceeded to the next level with none making it to the rank of professor. For those in senior lecturer position only 63.51% transited to the next level while 36.49% remained in the same position and none exited the service. Again 88.69% of those at the associate professor level transited to professor position, 11.31% of them remained in the same position.

4.2 Expected duration in service before attaining the highest academic rank

4.2.1 The expected duration before attaining the highest academic rank for the combined faculties

The fundamental matrix N will be considered. The matrix gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state. The fundamental matrix N is given as; $N = I + Q + Q^2 + \cdots + Q^{n-1}$For the combined faculties, it is obtained by $(I_{SA} - Q_{SA}) =$

$$\begin{bmatrix} 0.9700 & -0.9700 & 0.0000 & 0.0000 \\ 0.0000 & 0.5800 & -.03900 & 0.0000 \\ 0.0000 & 0.0000 & 0.9200 & -0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 0.7200 \end{bmatrix}$$

$$(I_{SA} - Q_{SA})^{-1} = \begin{bmatrix} 1.0300 & 1.7200 & 0.7300 & 0.9300 \\ 0.0000 & 1.7200 & 0.7300 & 0.9800 \\ 0.0000 & 0.0000 & 1.0900 & 1.3900 \\ 0.0000 & 0.0000 & 0.0000 & 1.3900 \end{bmatrix}$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_{SA} - Q_{SA})^{-1}\ (1111)^T$ which gives:

$$\begin{bmatrix} 1.0300 & 1.7200 & 0.7300 & 0.9300 \\ 0.0000 & 1.7200 & 0.7300 & 0.9800 \\ 0.0000 & 0.0000 & 1.0900 & 1.3900 \\ 0.0000 & 0.0000 & 0.0000 & 1.3900 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [5] \begin{bmatrix} 4.41 \\ 3.43 \\ 2.48 \\ 1.39 \end{bmatrix} = \begin{bmatrix} 22.0500 \\ 17.1500 \\ 12.4000 \\ 6.9500 \end{bmatrix}$$

This result gives the total expected duration of the academic staff in service before attaining the highest academic rank. From the result, the expected duration of the academic staff in Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 22.05, 17.15, 12.40, and 6.95 years respectively.

4.2.2 The expected duration before attaining the highest academic rank for the faculty of science

The expected duration is obtained as below;

$$(I_S - Q_S) = \begin{bmatrix} 0.8900 & -0.8900 & 0.0000 & 0.0000 \\ 0.0000 & 0.6300 & -0.2500 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & -0.1000 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 \end{bmatrix}$$

$$(I_S - Q_S)^{-1} = \begin{bmatrix} 1.1240 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 0.0000 & 1.0000 & 2.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.0000 \end{bmatrix}$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_S - Q_S)^{-1}\,(1111)^T$ which gives;

$$\begin{bmatrix} 1.1240 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 0.0000 & 1.0000 & 2.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.0000 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [5] \begin{bmatrix} 3.9020 \\ 2.7780 \\ 3.0000 \\ 2.0000 \end{bmatrix} = \begin{bmatrix} 19.5100 \\ 13.8900 \\ 15.0000 \\ 10.0000 \end{bmatrix}$$

From the result, the expected duration before one attains the highest academic rank for the Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 19.510, 13.890, 15.000, and 10.000 years respectively.

4.2.3 The expected duration before attaining the highest academic rank for the faculty of art

The expected duration is obtained as below;

$$(I_A - Q_A) = \begin{bmatrix} 0.7000 & -0.7000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8700 & -0.6800 & 0.0000 \\ 0.0000 & 0.0000 & 0.7300 & -0.7300 \\ 0.0000 & 0.0000 & 0.0000 & 0.8700 \end{bmatrix}$$

$$(I_A - Q_A)^{-1} = \begin{bmatrix} 1.4290 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 0.0000 & 1.3700 & 1.1490 \\ 0.0000 & 0.0000 & 0.0000 & 1.1490 \end{bmatrix}$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_A - Q_A)^{-1}\,(1111)^T$ which gives;

$$\begin{bmatrix} 1.4290 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 0.0000 & 1.3700 & 1.1490 \\ 0.0000 & 0.0000 & 0.0000 & 1.1490 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [5] \begin{bmatrix} 4.5470 \\ 3.1180 \\ 2.51900 \\ 1.1490 \end{bmatrix} = \begin{bmatrix} 22.7350 \\ 15.5900 \\ 12.5980 \\ 5.7450 \end{bmatrix}$$

From the result, the expected duration before one attains the highest academic rank for the Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 22.735, 15.590, 12.598, and 5.745 years respectively.

V. Research Findings ,Conclusion and Recommendation

5.1 Research findings

On the assumption that the academic staff can only stays for only five years in a given academic rank, a Markov chain has been used to study and analyze transition rates of academic staff in the university .From the findings, it was established that the transition rates are high at the Tutorial fellow and lecturer levels in both science and art with 67.09% and 86.31% and 86.00% and 97.53% respectively at the first five years of employment. But it was low at 50% at senior lecturer and associate professor in the faculty of science whereas in the faculty of arts the transition rates were 63.51% and 88.69% for the levels of senior lecturer and associate professor respectively.

There were exits at the level of tutorial fellow and lecturer in both science and art faculties. Science recorded the highest number of exits.

It was established that those academic staff in the faculty of arts stays longer before attaining the rank of professor than their colleagues in the faculty of science. The academic staff in art took 22.74 years to become full professors while those in science took 19.51 years to become full professor.

5.2 Conclusion

- The transition rates in the academic staff in the faculty of science and the faculty arts are not significantly different and therefore it can be concluded that the transition rates are the same.
- The expected duration in service before an academic staff attains the rank of professor is longer in the faculty of arts than in the faculty of science.
- The exit rates are higher in the academic ranks of tutorial fellows and lecturer in both faculties
- Transition rates are higher at the ranks of senior lecturer and associate professor than at the ranks of tutorial fellow and lecturer
- The study has shown that transition records of promotions can be modeled as a stochastic process and the time estimates and proportion of staff at various transition levels estimated.

5.3 Recommendations

From the above conclusions, the following recommendations were made;

- The university administration should device mechanisms on how to get hold of those recruited as tutorial fellows and lecturers to reduce the rate of exits recorded.
- The university administration should develop ways of encouraging the academic staff to work of their progression within the stipulated duration

5.4 Further Research

1. Research should be done to get into the root cause of low transition of academic staff at the tutorial fellow and lecturer levels
2. Research should be carried out to determine the cause of the longer time taken before attaining the rank of professor

Acknowledgements

We wish to express our sincere gratitude to Prof. Fredrick Onyango and Dr. Egar Otumba for their invaluable time, guidance and steadfast support throughout the period of writing this project. We are also grateful to Mr. Chris Okech of Moi University for the unwavering support during data collection at Moi University, thanks for your guidance and support.

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