Structural Properties of Marshall-Olkin Extended Inverse Exponential Distribution

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Abstract

This paper introduces a new extension of the Inverse Exponential distribution using the framework of Marshall-Olkin (1997) family of distributions. The new model is capable of modeling various shapes of aging and failure criteria. The statistical properties of the new model are discussed and the maximum likelihood and maximum product spacing's methods are used to estimate the parameters involved. Explicit expressions are derived for the moments and the order statistics are examined for the new proposed model. Finally, the usefulness of the new model for modeling reliability data is illustrated using two real data sets with simulation study.

Keywords: Inverse Exponential distribution, reliability analysis, maximum likelihood estimation, maximum product spacing's estimates.

1.0 Introduction

The inverse exponential (IE) distribution was introduced by Keller and Kamath [9] as special form of the inverse Weibull distribution. IE distribution is a lifetime distribution Lin, et al. [10] and found application in areas such as medicine, Engineering and biology (especially in event that exhibits non-monotone failure rate). More details on the study of IE distribution can be found in the work of Singh, et al. [19], Prakash [16], Bakoban and Abu- Zinadah [2], Oguntunde and Adeyemo [14].

Suppose a random variable Y has an exponential distribution, the variable $X = \frac{1}{Y}$, will have an IE distribution. Random variable X is said to have IE distribution with parameter θ if its cumulative density function (cdf) and the probability density function (pdf) is given respectively as:

$$F(x,\lambda) = e^{-\frac{\lambda}{x}}, \qquad x \geq 0,\ \lambda > 0 \tag{1}$$

$$f(x,\lambda) = \frac{\lambda}{x^2} e^{-\frac{\lambda}{x}}, \qquad x \geq 0,\ \lambda > 0 \tag{2}$$

Where λ is a scale parameter.

2.0 The Generalised Inverse Exponential Distribution

The IE distribution was generalised using the beta generated distribution that was introduced by McDonald (1984). Its probability density function is given as:

$$w_{GB}(x; c, \alpha, \beta) = \frac{f(x)}{B(\alpha, \beta)} c[F(x)]^{c\theta-1}(1 - [F(x)]^c)^{\beta-1} \ , \ \ 0 < x < 1 \tag{3}$$

Two distributions can be obtained from the density above; they are the classical beta distribution ($c = 1$), the Kumaraswamy distribution ($\alpha = 1$).suppose we let $\beta = c = 1$, then we have a new pdf given as

$$w(x) = \alpha f(x)[F(x)]^{\theta-1} \ , \qquad x > 0, \alpha > 0 \tag{4}$$

Since,

$$W(x) = \int_{-\infty}^{\infty} w(x)dx \tag{5}$$

Then, the cdf is given as

$$W(x) = [F(x)]^{\theta} \ , \qquad x > 0, \alpha > 0 \tag{6}$$

Where α is a shape parameter that controls both the skewness and kurtosis of the distribution.

Inserting equation (1) and equation (2) in equation (4) and equation (6) we have the pdf and the cdf of a new generalised IE distribution respectively given as

$$w(x) = \frac{\lambda\theta}{x^2} e^{-\frac{\theta\lambda}{x}} \ , \qquad x > 0, \alpha > 0 \tag{7}$$

$$W(x) = e^{-\frac{\theta\lambda}{x}} \tag{8}$$

3.0 The Generalised Marshall-Olkin Extended Inverse Exponential Distribution

The generalised Marshall-Olkin Extended Inverse Exponential distribution was generated using the Marshall-Olkin generator which was proposed by Marshall-Olkin [11]. The new distribution is flexible to work with and adaptable to various form of survival data.

For any baseline cdf $W(x)$, $x \in \Re$, the cdf of the Marshall-Olkin Extended Generalised MOEG distribution is given by

$$G(x) = \frac{W(x)}{\alpha + \bar{\alpha} W(x)}, \qquad \alpha > 0 \tag{9}$$

Where $\bar{\alpha} = 1 - \alpha$, and α is a tilt parameter

Since, $\frac{dG(x)}{dx} = g(x)$

Therefore,

$$g(x) = \frac{\alpha w(x)}{[\alpha + \bar{\alpha} W(x)]^2} \tag{10}$$

Where $g(x)$ is the pdf of $G(x)$. The MOE-G reverses to baseline distribution when $\alpha = 1$.

Several new distributions have been proposed and discussed using the Marshall Olkin approach in statistical modelling. Examples, include MOE generalised linear exponential distribution was studied by Okasha et al. [15], Benkhelifa [3], studied the MOE generalised Lindley distribution, the properties of the MOE- G distribution was proposed by Cordeiro et al, [4], MOE pareto distribution was introduced by Alice and Jose [1], the MOE Lomax distribution was proposed by Ghitany et al. [7], and MOE generalised exponential distribution was introduced by Ristic and Kundu [17], MOE normal distribution was proposed by Garcia et al. [6], MOE gamma distribution was studied by Ristic et al. [18].

4.0 Marshall Olkin Extended Generalised Inverse Exponential Distribution

The three parameters Marshall Olkin Extended Generalised Inverse exponential (MOEGIE) distribution was obtained by inserting (8) into (9) and (10), we obtained the cdf of MOEGIE distribution as :

$$G(x;\ \theta, \alpha, \lambda) = \frac{e^{-\frac{\lambda\theta}{x}}}{\alpha + \bar{\alpha} e^{-\frac{\lambda\theta}{x}}} \qquad x > 0; \alpha, \theta, \lambda > 0 \tag{11}$$

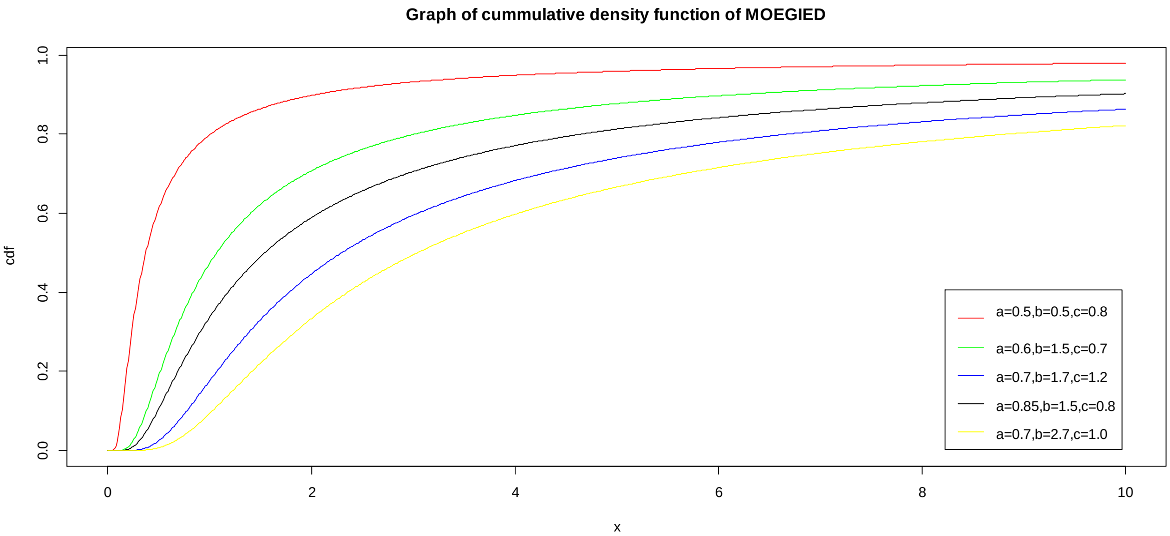

Figure 1.0. The graph of density function of MOEGIE distribution for various values of the parameter of the distribution.

The pdf corresponding to (11) is

$$g(x;\ \theta, \alpha, \lambda) = \frac{\alpha\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left[\alpha+\bar{\alpha}e^{-\frac{\lambda\theta}{x}}\right]^2} \qquad x > 0; \alpha, \theta, \lambda > 0 \tag{12}$$

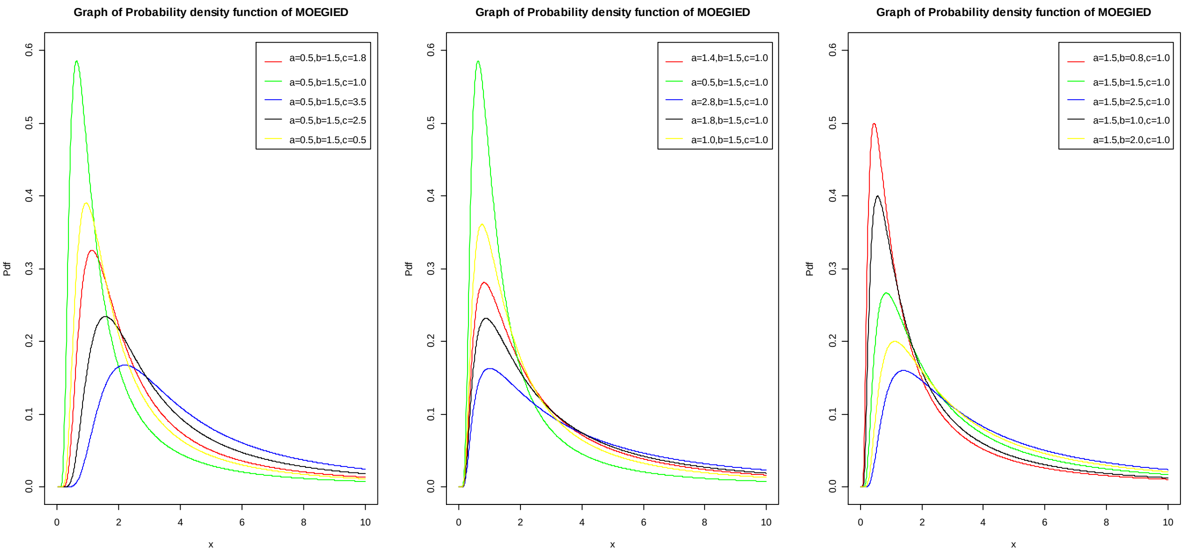

Figure 2.0 The graph of probability density function of MOEGIE distribution.

- ✓ The graph of the pdf of MOEGIE distribution drawn above indicates that the distribution is positively skewed and also mesokurtic, platykurtic and kleptokurtic for various values of the parameters.

Hence, when a random variable X follows a MOEGIE distribution, it will be denoted by $X \sim MOEGIE\ (\alpha, \lambda, \theta)$. Plots of the MOEGIE cdf and pdf for selected parameter values are shown below in figure 1.0 and figure 2.0 respectively has shown below.

5.0 Survival function of MOEGIE distribution

The survival function ($S(x)$) is defined as the probability that a system survive to time t. For any given system the survival function is given as:

$$S(x) = 1 - G(x) \tag{13}$$

By putting (11) in (13), we obtained the survival function of MOEGIE distribution which is

$$S(x) = \frac{\alpha\left\{1 + e^{-\frac{\lambda\theta}{x}}\right\}}{\alpha + \bar{\alpha}e^{-\frac{\lambda\theta}{x}}} \tag{14}$$

The graph of the survival function of MOEGIE distribution is plotted below in figure 3.0.

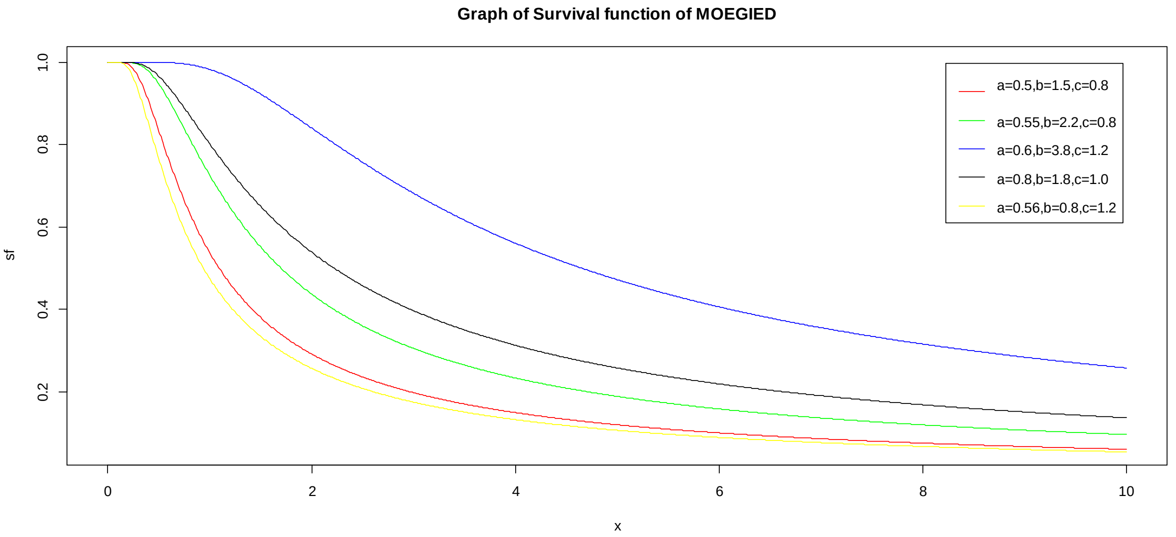

Figure 3.0 The graph of Survival function of MOEGIE distribution.

6.0 Hazard rate function of MOEGIE distribution

The hazard function $h(x)$ is an important quantity that is used to determine the nature of life time of a system. It can be said to be the probability of failure of a system having survived to time t. mathematically it can be defined as

$$h(x) = \frac{g(x)}{S(x)} \tag{15}$$

By inserting (12) and (14) in (15), we obtained the hazard function of MOEGIE distribution as

$$h(x) = \frac{\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left\{\alpha + \bar{\alpha}e^{-\frac{\lambda\theta}{x}}\right\}\left\{1 + e^{-\frac{\lambda\theta}{x}}\right\}} \tag{16}$$

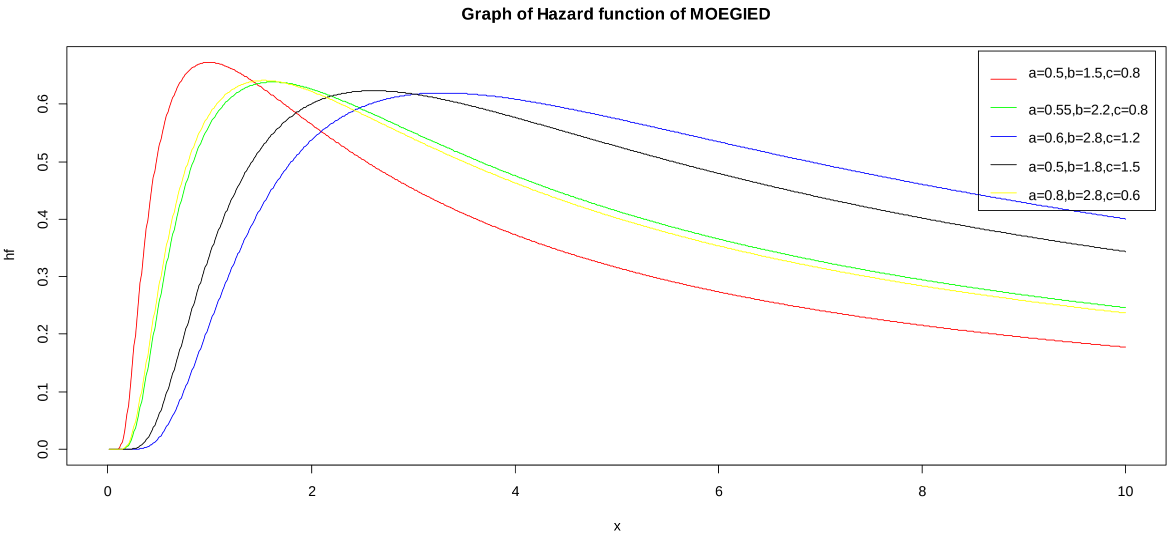

Figure 4.0 The graph of probability density function of MOEGIE distribution.

- ✓ The graph of the hazard function drawn above shows that distribution is increasing, decreasing with inverted bathtub shape which implies that it can be used to model a phenomenon that exhibits non-monotone failure rate or unimodal distribution

It can be deduce from above that $h(x) = \frac{r(x)}{\alpha+\bar{\alpha}G(x)}$, where $r(x)$ is the hazard function of the baseline distribution function. The graph of the hazard function for various values of the parameters is shown below in figure 4.0.

7.0 Mathematical Properties of MOEGIE distribution

Here, we examine the some mathematical properties of the MOEGIE distribution.

7.1 Useful expansions

We provide the some expansion that facilitates the study of the properties of MOEGIE distribution. If $|\kappa| < 1$ *and* $\gamma > 0$ is a real non-integer, the following expansion exist

$$(1-\kappa)^{-\gamma} = \sum_{i=0}^{\infty}\binom{\gamma+i-1}{i}\kappa^i \,, \tag{17}$$

If γ is an integer, index i in the previous sum stops at $\gamma - 1$. Using this we can rewrite the pdf of MOEGIE distribution given in (12) as

$$g(x;\ \theta,\alpha,\lambda) = \frac{\alpha\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left[1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]^2}$$

Since $\alpha \in (0,1)$, by applying (17) to the expression above we obtained

$$\left[1-\overline{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]^{-2}=\sum_{j=0}^{\infty}(j+1)\overline{\alpha}^{j}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}^{j}=\sum_{j=0}^{\infty}(j+1)\overline{\alpha}^{j}\sum_{i=0}^{j}(-1)^{i}\binom{j}{i}\left\{e^{-\frac{\lambda\theta}{x}}\right\}^{i}$$

Finally we have,

$$g(x;\ \theta,\alpha,\lambda)=\frac{\alpha\theta\lambda}{x^{2}}\sum_{j=o}^{\infty}\sum_{i=0}^{j}(j+1)\,\overline{\alpha}^{j}(-1)^{i}\binom{j}{i}e^{-\frac{\lambda\theta}{x}(i+1)} \tag{18}$$

7.2 r^{th} Moment of MOEGIE distribution

The r^{th} moment of a distribution can be obtained using the relation

$$E(X^{r})=\mu_{r}^{'}=\int_{-\infty}^{\infty}x^{r}f(x,\alpha,\lambda,\theta)dx \tag{19}$$

Inserting equation (18) in (19), we have

$$E(X^{r})=\alpha\lambda\theta\sum_{j=0}^{\infty}\sum_{i=0}^{j}\overline{\alpha}^{j}(j+1)\,(-1)^{i}\binom{j}{i}\int_{0}^{\infty}x^{r-2}e^{-\frac{\theta\lambda}{x}(1+i)}dx \tag{20}$$

Letting, $m=\frac{\theta\lambda}{x}(1+i),dm=-\theta\lambda(i+1)x^{-2}dx$

$$\int_{0}^{\infty}x^{r-2}e^{-\frac{\theta\lambda}{x}(1+i)}dx=-\{\theta\lambda(i+1)\}^{r-1}\int_{\frac{\theta\lambda}{x}(1+i)}^{\infty}m^{-r}e^{-m}dm \tag{21}$$

Since X can only take values on the positive real line we can introduce the exponential integral defined by

$$Ei(-X)=-\int_{x}^{\infty}t^{-1}e^{-t}dt \tag{22}$$

For further study see Chapter 5 of Abramowitz and Stegun [8] and Equation (6.2.6) of Oliver et al. [7]): and applying it to equation (14) will transform to

$$\int_{0}^{\infty}x^{r-2}e^{-\frac{\theta\lambda}{x}(1+i)}dx=\{\theta\lambda(i+1)\}^{r-1}\int_{\frac{\theta\lambda}{x}(1+i)}^{\infty}t^{r}e^{-t}dt \tag{23}$$

From generalised gamma given as

$$\Gamma(\eta, v) = \int_{v}^{\infty} w^{\eta-1} e^{-w}\, dw$$

Then we obtain,

$$\int_{0}^{\infty} x^{r-2} e^{-\frac{\theta\lambda}{x}(1+i)} dx = \{\theta\lambda(i+1)\}^{r-1} \Gamma\left\{r+1, \frac{\theta\lambda}{x}(1+i)\right\}$$

Finally,

$$E(X^r) = \alpha\lambda\theta \sum_{j=0}^{\infty} \sum_{i=0}^{j} \bar{\alpha}^j (j+1) (-1)^i \binom{j}{i} \{\theta\lambda(i+1)\}^{r-1} \Gamma\left\{r+1, \frac{\theta\lambda}{x}(1+i)\right\} \quad (24)$$

Suppose we let, $\xi_{ij} = \alpha\lambda\theta \sum_{j=0}^{\infty} \sum_{i=0}^{j} \bar{\alpha}^j (j+1) (-1)^i \binom{j}{i}$. Then equation (24) we transform to

$$E(X^r) = \xi_{ij} \{\theta\lambda(i+1)\}^{r-1} \Gamma\left\{r+1, \frac{\theta\lambda}{x}(1+i)\right\} \quad (25)$$

7.4 Mean of the MOEGIE distribution

Setting $r = 1$ in equation (25) leads to the mean of the MOEGIE distribution, which is given by

$$\mu_1' = \xi_{ij} \Gamma\left\{3, \frac{\theta\lambda}{x}(1+i)\right\} \quad (26)$$

7.5 Second moment of the MOEGIE distribution

Setting $r = 2$ in equation (25)

$$\mu_2' = \xi_{ij} \{\theta\lambda(i+1)\} \Gamma\left\{2, \frac{\theta\lambda}{x}(1+i)\right\} \quad (27)$$

7.6 Variance of the MOEGIE distribution

The variance of the of the MOEGIE distribution can be obtained using the relation

$$V(x) = \mu_2 = \mu_2' - (\mu_1')^2 \quad (28)$$

$$\mu_2 = \xi_{ij} \{\theta\lambda(i+1)\} \Gamma\left\{3, \frac{\theta\lambda}{x}(1+i)\right\} - \left[\xi_{ij} \Gamma\left\{3, \frac{\theta\lambda}{x}(1+i)\right\}\right]^2 \quad (29)$$

7.7 The third and the fourth moments of the MOEGIE distribution

Setting $r = 3$ in equation (25),

$$\mu_3' = \xi_{ij}\{\theta\lambda(i+1)\}^2\Gamma\left\{4,\frac{\theta\lambda}{x}(1+i)\right\} \quad (30)$$

Then

$$\mu_3 = \xi_{ij}\{\theta\lambda(i+1)\}^2\Gamma\left\{4,\frac{\theta\lambda}{x}(1+i)\right\} - 3\Gamma\xi_{ij}\left\{2,\frac{\theta\lambda}{x}(1+i)\right\}\{\theta\lambda(i+1)\}\Gamma\left\{2,\frac{\theta\lambda}{x}(1+i)\right\} + 3\left(\xi_{ij}\right)^3\Gamma^3\left\{2,\frac{\theta\lambda}{x}(1+i)\right\} \quad (31)$$

Also for, $r = 4$ in equation (24), we have

$$\mu_4' = \xi_{ij}\{\theta\lambda(i+1)\}^3\Gamma\left\{5,\frac{\theta\lambda}{x}(1+i)\right\} \quad (32)$$

Thus,

$$\mu_4 = \xi_{ij}\{\theta\lambda(i+1)\}^3\Gamma\left\{5,\frac{\theta\lambda}{x}(1+i)\right\} - 4\left(\xi_{ij}\right)^2\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\}\xi_{ij}\{\theta\lambda(i+1)\}^2\Gamma\left\{4,\frac{\theta\lambda}{x}(1+i)\right\} \quad (33)$$

7.5 Standard deviation of MOEGIE distribution

The standard deviation is defined as the positive square root of the variance. It is represented as, $\sigma = \sqrt{\sigma^2}$. From equation (25), the variance of MOEGIE distribution is given as

$$\sigma^2 = \xi_{ij}\{\theta\lambda(i+1)\}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\} - \left[\xi_{ij}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\}\right]^2$$

$$\sigma = \sqrt{\xi_{ij}\{\theta\lambda(i+1)\}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\} - \left[\xi_{ij}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\}\right]^2}$$

$$\Rightarrow \quad \sigma = \sqrt{\sigma_1 - \sigma_1^2}$$

Where

$$\sigma_k = \xi_{ij}\{\theta\lambda(i+1)\}^{k-1}\Gamma\left\{k+1,\frac{\theta\lambda}{x}(1+i)\right\} \quad (34)$$

7.6 Coefficient of Variation of MOEGIE distribution

This is the ratio of standard deviation to the mean. Usually, it ids denoted by C.V and is given by

$$C.V = \frac{\sigma}{\mu}$$

This implies that

$$C.V = \frac{\sqrt{\xi_{ij}\{\theta\lambda(i+1)\}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\} - \left[\xi_{ij}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\}\right]^2}}{\xi_{ij}\Gamma\left\{3,\frac{\theta\lambda}{x}(1+i)\right\}} \quad (35)$$

7.7 Skewness of MOEGIE distribution

Ratio of moment is a popular way to measure the skewness and kurtosis of a distribution. Lack of tails (about mean) of frequency distribution curve is known as skewness. The measure of skewness as given by Karl Pearson in terms of moments of frequency distribution is given by

$$\gamma_1 = \frac{\mu_3^2}{\mu_2^3}$$

Putting equation (29) and (31) in the equation above, we will obtain an expression for the skewness of MOEGIE distribution.

8.0 Quantile function

The u^{th} quantile function of the distribution which is defined as the inverse of the distribution function $F(x_u) = u$, given by

$$x = -\left\{\frac{\theta\lambda}{ln\left[\frac{\alpha u}{1-\bar{\alpha}u}\right]}\right\} \quad (36)$$

The equation (36) can be used to obtain a random number that can be used to access the asymptotic properties of the MOEGIE distribution.

When $u = \frac{1}{2}$, we obtained the quantile which represent the median of the distribution given by

$$median = -\left\{\frac{\theta\lambda}{ln\left[\frac{\alpha}{1-\bar{\alpha}}\right]}\right\}$$

For mode of this distribution can be found by solving $\frac{\delta I}{\delta x} = 0\left(\frac{d^2 I}{dx^2} < 0\right)$

9.0 Moment generating function (mgf) of MOEGIE Distribution

The mgf of the (MOEGIE) distribution is given by

$$M_x(t) = E(e^{tx}) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(T^r) \tag{37}$$

Putting equation (25) into equation (37), we have

$$M_x(t) = \alpha\lambda\theta \sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\sum_{i=0}^{j} \frac{t^r}{r!} \bar{\alpha}^j (j+1)\,(-1)^i \binom{j}{i} \{\theta\lambda(i+1)\}^{r-1} \Gamma\left\{r+1, \frac{\theta\lambda}{x}(1+i)\right\} \tag{38}$$

This is the required mgf of Marshall Olkin Extended Generalised inverse Exponential distribution.

10.0 Renyi entropy

The Renyi entropy of a random variable T represents a measure of uncertainty. A large value of entropy indicates the greater uncertainty in the data. The measure have been shown to be effective in comparing the tails and shapes of various standard distributions, Song [20]. The Renyi, A. (1961), Ref. [17] introduced the Renyi entropy defined as

$$H_\alpha(T) = \frac{1}{1-\phi} log \int_{-\infty}^{\infty} f(x)^\phi dx\,, \qquad \phi > 0 \text{ and } \phi \neq 1 \tag{39}$$

Then substituting equation (12) in (39) we have

$$H_\alpha(T) = \frac{1}{1-\phi} log \left\{ \int_{-\infty}^{\infty} \left(\frac{\alpha\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left[1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]^2} \right)^\phi dx \right\} \tag{40}$$

Then we have

$$= \frac{1}{1-\phi} log \left\{ (\alpha\theta\lambda)^\phi \binom{\phi+j-1}{j} \bar{\alpha}^j (-1)^j \binom{i}{j} \int_{-\infty}^{\infty} x^{-2} e^{-\frac{\lambda\theta}{x}(\phi+1)} dx \right\} \tag{41}$$

If we let $m = \frac{\lambda\theta}{x}(\phi+1)$, finally we have

$$H_\alpha(T) = \frac{-1}{1-\phi} log \left\{ (\alpha\theta\lambda)^\phi \binom{\phi+j-1}{j} \bar{\alpha}^j (-1)^j \binom{i}{j} [\theta\lambda(\phi+i)]^{-(2\phi+3)} \Gamma\langle 2\phi + 1; \frac{\lambda\theta}{x}(\phi+1)\rangle \right\} \tag{42}$$

11.0 Order Statistics

Order statistics are among the most fundamental tools in non-parametric statistics and inference. The pdf $f_{i:n}(x)$ of the ith order statistic for a random sample $x_1, x_2, \ldots x_n$ from the MOEGIE distribution id given by

$$f_{i:n}(x) = \psi f(x) \sum_{i=1}^{n-i} (-1)^j \binom{n-i}{j} F^{i+j-1}(x) \tag{43}$$

Where

$\psi = \frac{n!}{(i-1)!(n-i)!}$;

$$f_{i:n}(x) = \psi \frac{\alpha\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left[1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]^2} \sum_{i=1}^{n-i} (-1)^j \binom{n-i}{j} \left[\frac{e^{-\frac{\lambda\theta}{x}}}{1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}}\right]^{i+j-1}$$

$$f_{i:n}(x) = \psi\alpha\theta\lambda \sum_{i=i}^{n-i} \sum_{l}^{\infty} \sum_{k=0}^{l} \binom{i+j+l}{l} \bar{\alpha}^l (-1)^k \binom{l}{k} e^{-\frac{\lambda\theta}{x}(i+j+k)} \tag{44}$$

12.0 Estimation of the parameters

In this section method of maximum likelihood is used to estimate the parameters and also we construct a confidence interval for the unknown parameters. Here we find the estimators for the $MOEGIE$ distribution. Let $T_1, T_2, \ldots, T_n$ be a random sample from $T \sim MOEGIE(\alpha, \lambda, \theta)$ with observed values $t_1, t_2, \ldots, t_n$ then the likelihood function $L \equiv L(:t_i)$ can be written as

$$L = \prod_{i=1}^{n} \left\{ \frac{\alpha\theta\lambda e^{-\frac{\lambda\theta}{x}}}{x^2\left[1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]^2} \right\}$$

The log- likelihood is given as

$$l = nlog(\alpha\theta\lambda) - \theta\lambda \sum_{i=1}^{n} x_i^{-1} - 2\sum_{i=1}^{n} \log(x_i) - 2\sum_{i=1}^{n} log\left[1-\bar{\alpha}\left\{1-e^{-\frac{\lambda\theta}{x}}\right\}\right]$$

To obtain numerical solution for the values of the estimates of MOEGIE distribution we may employ software such as R, Maple, OX Program etc.

13.0 Applications

In this section, we illustrate the usefulness and application of the MOEGIE distribution to real data sets. We fit the density function of Marshall Olkin Extended Generalised inverse Exponential (MOEGIE), Extended Generalised inverse Exponential (EIW) and Inverse Exponential (IE) distributions.

The data set from Bjerkedal (1960) represents the survival times, in days of guinea pigs injected with different doses of tubercle bacilli. The data set consists of 72 observations and are listed below: 12, 15, 22, 24 ,24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68,70, 70, 72, 73, 75, 76,76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376.

Exploratory Data Analysis of the first data was given in table 1.0, table 2.0 gives the estimates of the parameters of MOEGIE distribution (standard error in parentheses), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (AICC), Bayesian Information Criterion (BIC) and the Hanan Quinin Information Criteria (HQIC)

Table 1.0 Exploratory Data Analysis of Data

min	Q_1	$median$	$mean$	Q_3	max	$skewness$	$kurtosis$	$range$
12.0	54.75	70.0	99.82	112.80	376	1.84	2.89	364

Table2. MLEs(standard error in parenthesis) and the statistics $l(\hat{\theta})$, AIC, BIC and $HQIC$

$Model$	$Estimates$			$-l$	AIC	$HQIC$	$CAIC$
$MOEGIE$ $(\alpha, \lambda, \theta)$	0.6448 (0.07735)	0.6278 (5.5670)	0.6135 (05.4406	443.18	892.36	897.96	892.39
EIE (λ, θ)	- -	0.4950 (4.4171)	0.6059 (5.4056)	449.96	903.91	907.64	903.93
IE (λ)	- -	0.2999 (0.0095)	- -	449.96	901.91	903.78	900.92

13.0 Discursion/ Conclusion

Since the Marshall-Olkin Extended generalised Inverse Exponential distribution possess the Minimum likelihood, AIC, CAIC and HQIC, it can be considered to be a better model in modeling real life data than the Generalised Inverse Exponential and the Inverse Exponential distribution.

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