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LEHMANN TYPE II GENERALIZED HALF LOGISTIC DISTRIBUTION: PROPERTIES AND APPLICATION

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Abstract

In this study, a new four parameter distribution called the Lehmann type II generalized half logistic distribution was derived. The statistical properties of the Lehmann type II generalized half logistic distribution were studied. Estimates of the parameters of the new distribution under complete and censored observations would be obtained using the maximum likelihood estimation method. Simulation studies were carried out to assess the consistency of the maximum likelihood estimates. Application of the new distribution to a data showed that it performed better than the type I generalized half logistic distribution

Keywords: distribution, censored, estimates, parameters, function, observation, Lehmann type II.

Introduction

The half logistic distribution is gradually getting attention by researcher and few generalizations have been done already. Inferences for the half logistic distribution have been discussed by several authors. Balakrishnan and Puthenpura[1] introduced the best linear unbiased estimators of location and scale parameters of the half logistic distribution through linear functions of order statistics. Balakrishnan and Leung[2], established some recurrence relations for the moments and product moments of order statistics for half-logistic distribution.

Balakrishnan and Puthenpura[1], obtained the best linear unbiased estimates of the location and scale parameters respectively of the half-logistic distribution through linear function of order statistics and they also tabulated the values of the variance and covariance of these estimates.

Olapade[3] stated and proved some theorems that characterized the half logistic distribution. Tobari and Bagheri [4] presented an extended generalized half logistic distribution and studied different methods for estimating its parameter based on complete and censored data. They derived maximum likelihood equations for estimating the parameters based on censored data. Also, the asymptotic confidence intervals of the estimators are presented in which they applied using simulation studies and properties of maximum likelihood of the estimators were given.

Olapade[5] obtain a generalized form of half logistic distribution through a transformation of an exponential random variable called the four-parameter type I generalized half logistic distribution. He also obtained the cumulative distribution function (CDF), the survival function and the hazard function, moments, the 100p-percentage point and the mode of the distribution. Olapade [6], obtained a probability density function of the Type I generalized half logistic distribution as He obtained the cummulative distribution function, moments, median, mode, 100p-percentage point and order statistics of the distribution and estimates the parameter of the distribution using maximum likelihood method.

Awodutire et. al[7] introduced the half logistic distribution derived by Olapade[6] to survival analysis and derived a survival model. Awodutire[8] applied the derived survival model to assess the survival times of breast cancer patients in Nigeria.

The rest of the paper is organized as follows: section II deals with the material and methods which introduces the newly derived distribution, section III presents the mathematical properties, estimation of parameters under complete and censored observations, simulation studies and application of the new distribution to a dataset and section IV concludes the paper.

The generalized form of half logistics distribution obtained by Olapade[6] is

$$f(t) = \frac{\beta 2^{\beta} e^{\frac{t-\mu}{\sigma}}}{\sigma\left(1+e^{\frac{t-\mu}{\sigma}}\right)^{\beta+1}} \qquad \beta, \mu, \sigma > 0 \qquad (1)$$

with cumulative distribution function as

$$F(t) = 1 - \frac{2^{\beta}}{\left(1+e^{\frac{t-\mu}{\sigma}}\right)^{\beta}} \qquad \beta, \mu, \sigma > 0 \qquad (2)$$

where β is the shape parameter

μ is the location parameter

σ is the scale parameter

2. MATERIALS AND METHOD

Lehmann type II distribution is a dual transformation defined by Exp^c(1-F) as being introduced by Cordeiro[9] among others.

Now, let T be a random variable form of the distribution with shape parameter b, we defined the Lehmann Type II generalized distribution using the logit of beta function as defined by Jones[10] as

$$f_{LHL} = \frac{1}{B(1,b)}[1-F(t)]^{b-1} f(t) \qquad (3)$$

In the literature, researchers/authors on Lehmann Type II are very few. Badmus[11] obtained a Lehmann type II weighted Weibull distribution. He studied some statistical properties and applied it to a lifetime data.

Also, Ajitoni et.al [12] had a comparative analysis on the performance of Lehmann type II inverse Gaussian model and the standard inverse Gaussian model in terms of flexibility in which was found that the Lehmann Inverse Gaussian shows more flexibility in modeling skewed data than the inverse Gaussian model.

3.1 The Lehmann Type II Generalized Half Logistic Distribution And Its Statistical Properties.

Both probability density function and the cumulative density function (p.d.f and c.d.f) of Lehmann type II generalized half logistic distribution is obtained by substituting pdf and cdf of type I generalized half logistic distribution in equations (1) and (2) in equation (3). We then have

$$f_{LHL} = b(\frac{2}{1+e^{\frac{t-\mu}{\sigma}}})^{\beta(b-1)} \frac{2^{\beta}\beta e^{\frac{t-\mu}{\sigma}}}{\sigma(1+e^{\frac{t-\mu}{\sigma}})^{\beta+1}}$$

which is

$$f_{LHL} = b\beta(\frac{2}{1+e^{\frac{t-\mu}{\sigma}}})^{b\beta} \frac{e^{\frac{t-\mu}{\sigma}}}{\sigma(1+e^{\frac{t-\mu}{\sigma}})}$$

And which finally gives

$$f_{LHL} = \frac{b\beta 2^{b\beta}\, e^{\frac{t-\mu}{\sigma}}}{\sigma(1+e^{\frac{t-\mu}{\sigma}})^{b\beta+1}}$$

Therefore, the Lehmann type II generalized half logistic distribution is

$$f_{LHL} = \frac{b\beta 2^{b\beta}\, e^{\frac{t-\mu}{\sigma}}}{\sigma(1+e^{\frac{t-\mu}{\sigma}})^{b\beta+1}} \qquad (4)$$

Proof:

a) Due to the exponent, $f_{LHL} \geq 0$ i.e trivial
b) To prove the second condition, Without loss of generality, let μ=0 and σ=1, then,

$$\int_0^{\infty} f_{LHL}\, dt = b\beta 2^{b\beta} \int_0^{\infty} \frac{e^t}{(1+e^t)^{b\beta+1}} dt$$

Let

$1 + e^t = a$

$e^t = a - 1$

$t = ln\,(a-1)$

$dt = \frac{1}{a-1}\, da$

$when\ x = 0, a = 2$ and when t= ∞, a= ∞, therefore,

$$f_{LHL} = b\beta 2^{b\beta} \int_0^{\infty} \frac{a-1}{a^{b\beta+1}} \frac{da}{a-1}$$

$$= b\beta 2^{b\beta} \int_2^{\infty} \frac{1}{a^{b\beta+1}} \frac{da}{a-1}$$

$$f_{LHL} = b\beta 2^{b\beta} \int_2^{\infty} a^{-b\beta-1}\, da$$

$$f_{LHL} = b\beta 2^{b\beta} [\frac{a^{-b\beta}}{-b\beta}]$$

$$f_{LHL} = 2^{b\beta}[2^{-b\beta}]$$

$$\int_0^{\infty} f_{LHL} = 1$$

The c.d.f (F_{LHL}) is defined as

$$F_{LHL}(t) = P(T \le t) = \int_0^t f(t)dt$$

Thus we have it as

$$F_{LHL}(t) = b\beta 2^{b\beta} \int_0^t \frac{e^t}{(1+e^t)^{b\beta+1}} dt \qquad (5)$$

Let

$1 + e^t = a$

$e^t = a - 1$

$t = ln\,(a-1)$

$$dt = \frac{1}{a-1}\, da$$

$when\ x = 0, a = 2$ and when t=t, a=1+e^t

$$F_{LHL}(t) = b\beta 2^{b\beta} \int_0^{1+e^t} \frac{a-1}{a^{b\beta+1}} \frac{da}{a-1}$$

$$F_{LHL}(t) = b\beta 2^{b\beta} \int_2^{1+e^t} \frac{1}{a^{b\beta+1}} \frac{da}{a-1}$$

$$F_{LHL}(t) = b\beta 2^{b\beta} \int_2^{1+e^t} a^{-b\beta-1}\, da$$

$$F_{LHL}(t) = b\beta 2^{b\beta} [\frac{a^{-b\beta}}{-b\beta}]$$

$$F_{LHL}(t) = 2^{b\beta}[2^{-b\beta} - (1+e^t)^{-b\beta}]$$

$$F_{LHL}(t) = 1 - \frac{2^{b\beta}}{(1+e^t)^{b\beta}} \qquad (6)$$

3.1.3. Survival Function

The survival function *S(t)* is defined as

$S(t) = 1 - F(t)$

$$S(t) = 1 - (1 - \frac{2^{b\beta}}{(1+e^t)^{b\beta}})$$

$$S(t) = \frac{2^{b\beta}}{(1+e^t)^{b\beta}} \qquad (7)$$

3.1.4. Hazard Function

The Hazard Function *H(t)* is defined as

$$H(t) = \frac{f(t)}{S(t)}$$

$$H(t) = \frac{\frac{b\beta 2^{b\beta}\, e^{t}}{(1+e^{t})^{b\beta+1}}}{\frac{2^{b\beta}}{(1+e^{t})^{b\beta}}}$$

$$H(t) = \frac{b\beta 2^{b\beta}\, e^{t}}{(1+e^{t})^{b\beta+1}} \div \frac{2^{b\beta}}{(1+e^{t})^{b\beta}}$$

$$H(t) = \frac{b\beta e^{t}}{1+e^{t}} \qquad (8)$$

3.1.5. Median

The median of a probability density function is a point x_m which satisfies the equation

$$F(t_m) = \frac{1}{2}$$

This implies

$$1 - \frac{2^{b\beta}}{(1+e^{t_m})^{b\beta}} = \frac{1}{2}$$

$$\frac{2^{b\beta}}{(1+e^{t_m})^{b\beta}} = \frac{1}{2}$$

$$\frac{(1+e^{t_m})}{2} = \sqrt[b\beta]{2}$$

$$(1+e^{t_m}) = 2\ \sqrt[b\beta]{2}$$

$$e^{t_m} = 2\ \sqrt[b\beta]{2} - 1$$

$$t_m = ln\,(2^{\frac{1+b\beta}{b\beta}} - 1) \qquad (9)$$

3.1.6. Moment Generating Function

Cordeiro et al[9] described a series expansion in their paper for $\mu_r{}'$ in terms of r(r,m) =E(Y^r F(Y)^m) where Y follows the parent distribution, then for m=0,1,…

$$\mu_r{}' = b\sum_{i=0}^{\infty}(-1)^{i}\left(\frac{b-1}{i}\right) r(r, i-1)$$

They further discussed another mgf of x for generated beta distribution as

$$M(t) = b\sum_{i=0}^{\infty}(-1)^{i}\left(\frac{b-1}{i}\right) \eta(t, i-1)$$

where $\eta(t,r) = \int_{-\infty}^{\infty} e^{tx}[F(x)]^r f(x)\,dx$

$$M(t) = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_{-\infty}^{\infty} e^{tx}[F(x)]^{i-1} f(x)dx \qquad (10)$$

Substituting both pdf and cdf (F(t) and f(t)) of the type I generalized half logistic distribution into above equation, we obtain

$$M(t) = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_{-\infty}^{\infty} e^{tx}\left[1 - \frac{2^{b\beta}}{(1+e^{\frac{x-\mu}{\sigma}})^{b\beta}}\right]^{i-1} \frac{b\beta 2^{b\beta} exp(\frac{x-\mu}{\sigma})}{\sigma(1+exp(\frac{x-\mu}{\sigma}))^{b\beta+1}}\,dx \qquad (11)$$

By setting b=i=1, we have the moment generating function of the type I generalized half logistic distribution.

From the moment generating function we can estimate the mean, variance, kurtosis and the skewness. The expression is not in closed form. To get estimates of some of the properties, data with sample size 50 was simulated with constant value of β and varying values of b. After simulation the following table was derived

Table 1: Simulation results showing the mean, variance, kurtosis and skewness

n=50,μ=0 ,σ =1,β=2.5

b	Mean	Variance	kurtosis	Skewness
0.25	2.223992	3.215231	0.712069	1.060506
0.30	1.9058	2.3412	0.6141318	1.029402
0.35	1.6738	1.79971	0.5371257	1.005975
0.40	1.4965	1.438008	0.4775134	0.988677
0.45	1.3560	1.1826	0.4318879	0.9761621
0.50	1.2415	0.9945	0.3973554	0.9673455
0.55	1.1462	0.8512	0.371584	0.9613803
0.60	1.0656	0.7390	0.3527372	0.9576152
0.65	0.9963	0.6493	0.3393802	0.9555512
0.70	0.9360	0.5761	0.3303936	0.9548054

3.1.7. **Some Related Distributions**

i. When b=1,β=1, μ=0, σ=1, the Lehmann type II generalized half logistic reduces to standardized half logistic distribution
ii. When b=1, , the Lehmann type II generalized half logistic reduces to the type I generalized half logistic of Olapade[6]

3.1.8 **Parameter Estimation Of Lehmann Type II Generalized Half Logistic Distribution Under Complete Data**

Given a sample $T_1, T_2, T_3,\ldots,T_n$ of size n from the Lehmann Type II generalized half logistic distribution with probability density function

$$f_{LHL} = \frac{b\beta 2^{b\beta}\, e^{\frac{t-\mu}{\sigma}}}{(1+e^{\frac{t-\mu}{\sigma}})^{b\beta+1}}$$

where μ is the location parameter and σ is the scale parameter, b and β are the shape parameters, the likelihood function of the distribution is obtained as

$$L(t{:}\,\mu,\beta,b,\sigma) = \frac{b^n\beta^n 2^{nb\beta}\prod_{i=1}^{n} e^{\frac{t_i-\mu}{\sigma}}}{\sigma^n\prod_{i=1}^{n}(1+e^{\frac{t_i-\mu}{\sigma}})^{b\beta+1}} \qquad (12)$$

$$lnL(x{:}\,\mu,\beta,b,\sigma) = nlnb + nln\beta + n\beta bln2 + \sum_{i=1}^{n}\frac{x_i-\mu}{\sigma} - nln\sigma - (b\beta + 1)\sum_{i=1}^{n} ln\,(1+e^{\frac{x_i-\mu}{\sigma}})$$

To obtain the estimates of each parameter, that maximizes the likelihood function, we differentiate with respect to each of the parameters and equate the derivatives to zero to solve for the parameters

$$\frac{dlnL(t{:}\,\mu,\beta,b,\sigma)}{db} = \frac{n}{b} + n\beta ln2 - \beta\sum_{i=1}^{n} ln\,(1+e^{\frac{t_i-\mu}{\sigma}}) \qquad (14)$$

$$\frac{dlnL(t{:}\,\mu,\beta,b,\sigma)}{d\beta} = \frac{n}{\beta} + nbln2 - b\sum_{i=1}^{n} ln\,(1+e^{\frac{t_i-\mu}{\sigma}}) \qquad (15)$$

$$\frac{dlnL(t{:}\,\mu,\beta,b,\sigma)}{d\sigma} = -\frac{n}{\sigma} - \frac{\sum_{i=1}^{n} t_i - \mu}{\sigma^2} + \frac{b\beta+1}{\sigma^2}\sum_{i=1}^{n}(t_i-\mu)\,\frac{e^{\frac{t_i-\mu}{\sigma}}}{(1+e^{\frac{t_i-\mu}{\sigma}})} \qquad (16)$$

$$\frac{dlnL(x{:}\,\mu,\beta,b,\sigma)}{d\mu} = -\frac{n}{\sigma} - \frac{\sum_{i=1}^{n}(t_i-\mu)}{\sigma^2} + \frac{b\beta+1}{\sigma^2}\sum_{i=1}^{n}\frac{e^{\frac{t_i-\mu}{\sigma}}}{(1+e^{\frac{t_i-\mu}{\sigma}})} \qquad (17)$$

From equation we can solve for b, β, μ, σ with the aid of computer programming using a sample data. For interval estimation and hypothesis test on the model parameters, the information matrix needed can be derived by differentiating with respect to the parameters mentioned earlier.

3.1.9 Parameter Estimation Of Lehmann Type II Generalized Half Logistic Distribution Under Complete Data

There are times in life testing experiments in which the data is censored due to withdrawal of subjects under study the subject not experiencing the event or loss to follow up. Such data can be Right censored, left censored and Interval Censored. In a bid to assess the contribution of covariates(factors) to survival times of such subject, we have the regression model called

Accelerated Failure Time Model (AFT).

Generally, the AFT model is of the form

$lnT = \delta + x'\lambda + \varepsilon$

where ε is said to follow a distribution

x is sets of covariates with parameters λ

δ is the constant

Given that survival times $T_1, T_2, ..., T_n$ of size n is assumed to follow the Lehman Type II generalized half-logistic distribution. We estimate the parameters of the AFT model using the maximum likelihood method. The maximum likelihood function is given as

$$L = \prod_{i=1}^{r} f(t_i/x, \lambda) \prod_{r+1}^{n} s(t_i^+/x, \lambda)$$

which equivalently is

$$L = \prod_{i=1}^{r} g(x/\lambda) f_o(Z(t_i)) \prod_{r+1}^{n} s_o(Z(t_i))$$

where $g(x/\lambda)=exp(-x\lambda)$ and $Z(t) = texp(-x\lambda)$.

Also $f_o(Z(t))$ and $s_o(Z(t))$ are the baseline probability density function and baseline survival function respectively. This now gives

$$L = \prod_{i=1}^{r} g(x/\lambda) \frac{b\beta 2^{b\beta} exp\,(\frac{Z(t_i) - \mu}{\sigma})}{\sigma(1 + exp\,(\frac{Z(t_i) - \mu}{\sigma}))^{b\beta+1}} \cdot \prod_{i=r+1}^{n} \frac{2^{b\beta}}{(1 + exp\,(\frac{Z(t_i{}^+) - \mu}{\sigma}))^{b\beta}}$$

$$= g(x/\lambda)^r \frac{(b\beta)^r 2^{b\beta} exp\, \sum_{i=1}^{r} (\frac{Z(t_i) - \mu}{\sigma})}{\sigma^r \prod_{i=1}^{r} (1 + exp\,(\frac{Z(t_i) - \mu}{\sigma}))^{b\beta+1}} \cdot \frac{(2)^{b\beta(n-r)}}{\prod_{i=r+1}^{n} (1 + exp\,(\frac{Z(t_i{}^+) - \mu}{\sigma}))^{b\beta}} \quad (18)$$

Taking logarithm of both sides,

$$lnL = l = r(-x'\lambda) + rln(b\beta) + rb\beta ln2 + \sum_{i=1}^{r} 1 + exp\,(\frac{Z(t_i) - \mu}{\sigma})\,)\,(\,-rln\sigma - (b\beta + 1)\sum_{i=1}^{r} ln(1 + exp\,(\frac{Z(t_i) - \mu}{\sigma})\,) + b\beta(n-r)ln\,(2) - b\beta \sum_{i=r+1}^{n} ln(1 + exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})\,)$$

we obtain the maximum likelihood estimate of the shape parameter b,μ,σ and β as

$$\frac{dl}{d\eta|\eta = b,\mu,\sigma,\beta} = 0\ for$$

$$\frac{dl}{db} = \frac{r}{b} + r\beta ln(2) - \beta \sum_{i=1}^{r}(1 + exp\,(\frac{Z(t_i) - \mu}{\sigma})\,)\ + \beta(n-r)ln\,2 - \beta \sum_{i=r+1}^{n} ln(1 + exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})\,) \quad (19)$$

$$\frac{dL}{d\mu} = -\frac{r}{\sigma} + \frac{b\beta + 1}{\sigma}\sum_{i=1}^{r}\frac{exp\,(\frac{Z(t_i) - \mu}{\sigma})}{(1 + exp\,(\frac{Z(t_i) - \mu}{\sigma})\,)} + \frac{b\beta}{\sigma}\sum_{i=r+1}^{n}\frac{exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})}{(1 + exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})\,)} \quad (20)$$

$$\frac{dL}{d\sigma} = -\frac{r}{\sigma} - \frac{\sum_{i=1}^{r} Z(t_i) - \mu}{\sigma^2} + \frac{b\beta + 1}{\sigma^2}\sum_{i=r+1}^{n}(Z(t_i^{+}) - \mu)\ \frac{exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})}{(1 + exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})\,)} \quad (21)$$

$$\frac{dl}{d\beta} = \frac{r}{\beta} + rbln(2) - b\sum_{i=1}^{r}(1 + exp\,(\frac{Z(t_i) - \mu}{\sigma})\,)\ + b(n-r)ln\,2 - b\sum_{i=r+1}^{n} ln(1 + exp\,(\frac{Z(t_i^{+}) - \mu}{\sigma})\,) \quad (22)$$

3.1.10 Simulation Studies

In order to assess the performance of the estimates of the distribution a simulation study was carried out. Sample sizes of 50,100,150,200,250,300,350,400,450,500 of data that follows this distribution were simulated. The simulated data was replicated 10000 times. The Mean Square Error (M.S.E) of the estimates was obtained. The simulation results are summarized in table 2. From the table the mean square error reduces as **n** is increased. This indicates that the maximum likelihood estimates provide consistent estimators for the parameters

Table 2: Simulated Results with varying sample sizes

N	b β	MSE(b) MSE(β)
50	3.2768 0.1746	1.83236e-05 1.081487e-05
100	3.1809 0.2108	8.590519e-01 3.240198e-01
150	5.7667 0.1071	2.028968e-02 2.817043e-01
200	3.1745 0.2051	4.27645e-01 2.633235e-01
250	3.3115 0.1818	3.749316e-01 2.149483e-01
300	3.1929 0.1994	2.887034e-01 1.764199e-01
350	3.1812 0.2004	2.454878e-01 1.510792e-01
400	3.1363 0.2284	2.08275e-01 1.289985e-01
450	3.2041 0.1975	1.939373e-01 1.178056e-01
500	3.2115 0.1950	1.754084e-01 1.06258e-01

3.1.11 Application of Lehmann Type II Generalized Half Logistic Distribution

Here we used a data set studied by Olapade[6] on participant in a race to compare the Lehmann type II generalized half logistic of Olapade[6] and the type I generalized half logistic distribution. In a cross-country race of 200 kilometer, a total of 1000 participants started the race and many stopped on the way without completing the race. The distance covered by each participant before stopping was recorded and a frequency distribution table was constructed to reflect the number of participants that are able to cover an interval of the distance. A probability model that will model the data is needed.

Table 3: Data on Cross Country Race

Class of Intervals	Class Midpoint	Class Frequency
1-20	10.5	228
21-40	30.5	203
41-60	50.5	166
61-80	70.5	129
81-100	90.5	95
101-120	110.5	66
121-140	130.5	46
141-160	150.5	32
161-180	170.5	22
181-200	190.5	13

Source: (Olapade[6])

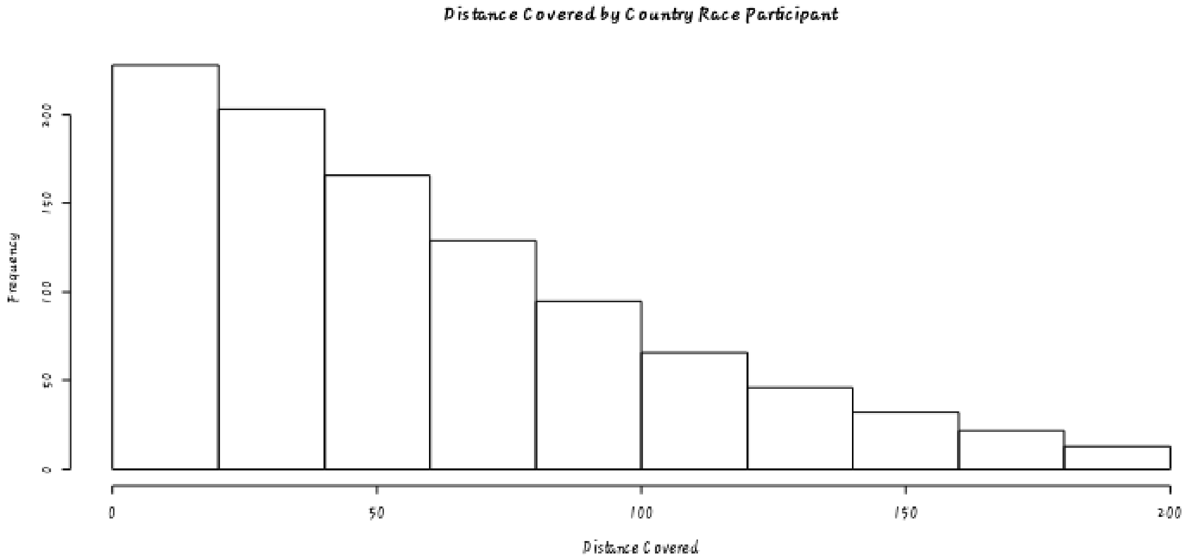

Fig1: Histogram of Distance Covered by Country Race Participants

Using R, we estimated the maximum likelihood estimates for the Lehmann Type II generalized half logistic are b=27.80, μ= 70.45, σ= 44.79, β= 34.867 with loglikelihood value of -1272.696 and the type I generalized half logistic distribution of Olapade as μ=0.5, σ =51.4 and β=1.188 with loglikelihood of -2432.56

Table 2: $-2\ln L$, AIC and BIC of the fitted distributions of the data set.

Distribution	Estimates	-loglikelihood	AIC	BIC
Type I Generalized Half Logistic Distribution	μ 0.5 σ 51.4 β 1.188	-2432.56	4871.12	4885.84
Lehmann Type II Generalized Half Logistic Distribution	b 27.80 μ 70.45 σ 44.79 β 34.867	-1272.696	2553.392	2573.02
Weibull Distribution	σ 64.3606 λ 1.3316	-5020	10044.78	10054.6
Lognormal Distribution	μ 42.3302 σ 2.4454	-5042.613	10089.23	10099.04

The distribution with lower Akaike Criterion Information (AIC) and Bayesan information Criterion (BIC) gives a better fit, thus the Lehmann Type II generalized half logistic fits the data best when compared to the other distributions.

4. CONCLUSION

In this work, we derived a new flexible model for lifetimes called Lehmann Type II generalized half logistic distribution compounding the Lehmann distribution with the type I generalized half logistic distribution. We explored the analytical characteristics and statistical properties. The maximum likelihood estimates of the model parameters are discussed and the usefulness of the new model is illustrated by means of simulation and a typical data on country race. Finally, we made comparison of the present model with that of type I generalized half logistic distribution in which this new model performs considerably better.

References

1. Balakrishnan, N. and Puthenpura, S. (1986), Best Linear Unbiased Estimators of Location and Scale Parameters of the Half Logistic Distribution. Journal of Statistical Computation and Simulation, **25**, 193-204.

2. Balakrishnan, N. and Leung, M. Y. (1988), Order statistics from the Type I generalized Logistic Distribution. Communications in Statistics Simulation and Computation, **17**(1), (1988), 25-50.

3.Olapade, A. K. (2003), On Characterizations of the Half Logistic Distribution. InterStat, February Issue,**2**, http://interstat.stat.vt.edu/InterStat/ARTICLES/2003articles/F06002.pdf

4.Torabi, H. and Bagheri, F. K. (2010). Estimation of parameter for an extended generalized half logistic distribution based on complete and censored data, JIRSS. Vol 9. pp171-195.

5. Olapade, A. K. (2011). On a four-parameter type I generalized half logistic distribution.

Proceeding of the Jangjeon Mathematical KOREA. Vol 14. pp189-198. do
6. Olapade A.K. (2014) The type I generalized half logistic distribution JIRSS. Vol13 pp 69-82.
7. Awodutire P.O, Olapade A.K, Kolawole O.A. (2016).The type I generalized half logistic survival model. International Journal of Theoretical and Applied Mathematics, vol 2, No 2, pp 74-78. doi: 10.11648/j.ijtam.20160202.17
8. Awodutire P.O, Olapade A.K, Kolawole O. A., Ilori O. R.(2018) Assessing Survival Times Of Breast Cancer Patients Using Type I Generalized Half Logistic Survival Model**.** Journal of Advances in Medicine and Medical Research. 25(2), 1-7. 2018
9. Cordeiro GM, Alexandra C, Ortega MM, Edwin Sarabia JM (2011). Generalized Beta Generated distributions. ICMA Centre. Discussion Papers in Finance DP 2011-05. pp. 1-29.
10. Jones M.C. (2004). Families of distributions arising from distributions of order statistics test 13:1-43.
11. Badmus N.I., Bamiduro T.I., Ogunobi S.G. Lehmann Type II weighted weibull distribution. International Journal of Physical Science, Vol. 9(4), pp. 71-78, 28. DOI:10.5897/IJPS2013.4
12. Ajitoni S. Amusan, Zarina M. Khalid (2016) A comparative analysis on the performance of Lehmann type II Inverse Gaussian Model and Standard Inverse Gaussian Model in terms of Flexibility. Global Journal of Pure and Applied Mathematics,Vol 12, No 5,pp 4535-4552