

Deteriorating Inventory Model For Two Parameter Weibull Demand With Shortages

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Abstract

In this paper a deteriorating inventory model have been developed for two parameter Weibull demand rate. Shortages are allowed and are completely backlogged .This inventory system follows an two-parameter exponnential distribution deterioration rate in which the holding cost is constant .The results are described with the numerical example and sensitivity analysis.

Keywords: Deterioration, Exponential distribution, holding cost, Inventory, shortages, Weibull demand rate.

1. Introduction

Many Researchers have developed inventory models to maximize the profit (or) to minimize the total cost for deteriorating items with respect to time. Deterioration arises due to some changes in the products which makes the product value dull. Deterioration in each product cannot be completely avoided and the rate of deterioration for each product will vary. Azizul Baten and Abdulbasah developed an inventory model in which the shortages not allowed with constant demand and deterioration rate. Many Researchers were interested in taking weibull deteriorating rate (in two (or) three). Azizul Baten and Abdulbasah also presented a review for Weibull distributed distribution .C.K Tripathy and U.Mishra developed an inventory model with time-varying holding cost with shortages which are completely backlogged.C.K.Tripathy, L.M.Pradhan improved their model for not only power demand but also partially backlogged .C.K.Tripathy,U.Mishra gave an ordering policy for Quadratic demand with permissible delay in payments. Kun-Shan Wu presented an ordering policy for items with Weibulll deteriorating rate and permissible delay in payments. Shanghi,P.R.China developed an inventory model for Weibull distribution deterioration rate with ramp type demand .The assumption of selling price as demand rate was studied by Ajantha Roy who used deterioration rate as time proportional and shortages were completely backlogged. P.K.Tripathy and S.Pradhan developed an model for time proportional deterioration rate with two parameter Weibull distribution demand rate and partial backlogging . Vijay P. Goel and S.P Aggarwal developed an algorithm for determining optimal pricing and ordering policy for 3-parameter Weibull rate of deterioration. Also, this algorithm was done for with-shortages and without – shortages. Peter Chu and Patrick S.Chen gave a note for "On an inventory model for deteriorating items and time –varying demand". Aggoun.L., L.Benkherouf, and L.Tadj suggested a new inventory continuous stochastic model for deteriorating items. Researchers also concentrated in production based inventory models also. Few are [Vinoth Kumar , Gede Agus Widyadana, Huimuijwee Babu Krishnaraj .R and Ramasamy .K.presented an inventory model with power demand pattern for Weibull deterioration rate without shortages. Nita H. Shah and Nidhi Raykundaliya gave a Retailer's pricing and ordering strategy for Weibull distribution deterioration under trade credit in declining market. They also conclude that the changes in shape Parameter automatically increases cycle time and decrease the profit

In our paper we've developed an inventory model for deteriorating items with demand as two parameter Weibull distribution rate and constant holding cost .In this model shortage are allowed and are completely backlogged.

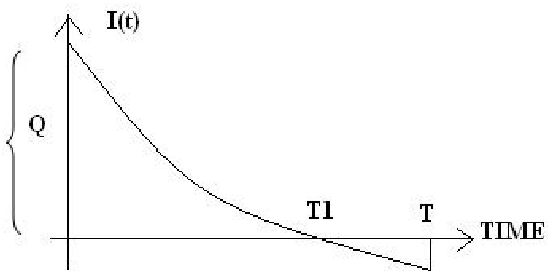
2. Assumptions and Notations

- The inventory system involves only one item.
- Lead time is zero.

- The demand rate of any time is $\alpha\beta t^{(\beta-1)}$ two parameter weibull distribution, where $0<\alpha \leq 1, \beta > 0$ are called scale and shape parameter respectively.
- $\theta(t) = \frac{1}{\theta}$ deterioration rate follows an two parameter exponential distribution.
- Shortages are allowed and are completely backlogged.
- A: Setup Cost
- C_1: Deterioration Cost
- C_2: Shortages Cost
- I(t) : Inventory level at time t=0
- Q(t):Order quantity at time t=0
- T: Duration of a cycle
- T_1: the time at which the inventory level reaches zero
- K(T) : The total cost per unit time

3. Mathematical Model:

Let I(t) be the inventory level at time $t(0 \leq t \leq T)$. The differential equations for the instantaneous state over (0, T) are given by



$$\frac{dI(t)}{dt} + \frac{1}{\theta} I(t) = -\alpha\beta t^{\beta-1}, \quad 0 \leq t \leq T_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1}, \quad T_1 \leq t \leq T \qquad (2)$$

With boundary conditions I (T_1) = 0 and I (0) = Q

Solving equations (1) and (2) we get

$$I(t) = \alpha\beta\left(\frac{T_1^{\beta}-t^{\beta}}{\beta} + \frac{T_1^{\beta+1}-t^{\beta+1}}{\theta(\beta+1)} + \frac{T_1^{\beta+2}-t^{\beta+2}}{2\theta^2(\beta+2)}\right) - \frac{\alpha\beta}{\theta}\left(\frac{T_1^{\beta}t-t^{\beta+1}}{\beta} + \frac{T_1^{\beta+1}t-t^{\beta+2}}{\theta(\beta+1)} + \frac{T_1^{\beta+2}t-t^{\beta+3}}{2\theta^2(\beta+2)}\right) + \frac{\alpha\beta}{2\theta^2}\left(\frac{T_1^{\beta}t^2-t^{\beta+2}}{\beta} + \frac{t^2T_1^{\beta+1}-t^{\beta+3}}{\theta(\beta+1)} + \frac{t^2T_1^{\beta+2}-t^{\beta+4}}{2\theta^2(\beta+2)}\right) \qquad (3)$$

$$I(t) = \alpha\,(T_1^{\beta}-t^{\beta}) \qquad (4)$$

Deteriorating Cost

$$DC = \frac{c_1}{T}\left[Q-\int_0^{T_1} D(t)\,dt\right]$$

$$= \frac{\alpha\beta C_1 T_1^{(\beta+1)}}{T\theta(\beta+1)} + \frac{T_1^{(\beta+2)}\alpha\beta C_1}{2T\theta^2(\beta+2)} \quad (5)$$

Shortage Cost

$$SC = -\frac{C_2}{T}\left[\int_{T_1}^{T} \alpha(T_1^{\beta} - t^{\beta})dt\right]$$

$$= \frac{C_2\alpha\beta T_1^{(\beta+1)}}{T} + \frac{C_2\alpha T^{\beta}}{(\beta+1)} - \alpha C_2 T_1^{\beta} \quad (6)$$

Inventory Holding Cost

$$HC = \frac{h}{T}\int_0^{T_1} I(t)\,dt$$

$$= \frac{\alpha\beta h T_1^{\beta+1}}{T(\beta+1)} + \frac{\alpha\beta h T_1^{\beta+2}}{2\theta T(\beta+2)} + \frac{\alpha\beta h T_1^{\beta+3}}{6\theta^2 T(\beta+3)} - \frac{\alpha\beta h T_1^{\beta+4}}{12\theta^3 T(\beta+4)} + \frac{\alpha\beta h T_1^{\beta+5}}{12\theta^4 T(\beta+5)} \quad (7)$$

Setup Cost

$$SC = \frac{A}{T} \quad (8)$$

Order Quantity

$$Q = \alpha\beta\left(\frac{T_1^{\beta}}{\beta} + \frac{T_1^{(\beta+1)}}{\theta(\beta+1)} + \frac{T_1^{(\beta+2)}}{2(\beta+2)\theta^2}\right) \quad (9)$$

Total cost per unit time is

K (T) = $\frac{1}{T}${Setup Cost+ Deterioration Cost + Holding Cost + Shortages Cost}

$$K(T) = \frac{A}{T} + \frac{\alpha\beta h T_1^{\beta+1}}{T(\beta+1)} + \frac{\alpha\beta h T_1^{\beta+2}}{2\theta T(\beta+2)} + \frac{\alpha\beta h T_1^{\beta+3}}{6\theta^2 T(\beta+3)} - \frac{\alpha\beta h T_1^{\beta+4}}{12\theta^3 T(\beta+4)} + \frac{\alpha\beta h T_1^{\beta+5}}{12\theta^4 T(\beta+5)} + \frac{C_2\alpha\beta T_1^{\beta+1}}{T} + \frac{T^{\beta}\alpha C_2}{\beta+1} - \alpha T_1^{\beta} C_2 + \frac{\alpha\beta C_1 T_1^{(\beta+1)}}{T\theta(\beta+1)} + \frac{T_1^{(\beta+2)}\alpha\beta C_1}{2T\theta^2(\beta+2)} \quad (10)$$

Our objective is to minimize the total Cost. The necessary conditions for minimize the total cost

$$\frac{\partial K(T)}{\partial T_1} = \frac{\alpha\beta C_1 T_1^{\beta}}{\theta T} + \frac{C_1\alpha\beta T_1^{\beta+1}}{2T\theta^2} + \frac{C2\alpha\beta(\beta+1)T_1^{\beta}}{T} - \beta\alpha C2 T_1^{\beta-1} + \frac{h\alpha\beta T_1^{\beta}}{T} + \frac{h\alpha\beta T_1^{\beta+1}}{2\theta T} + \frac{h\alpha\beta T_1^{\beta+2}}{6\theta^2 T} - \frac{h\alpha\beta T_1^{\beta+3}}{12T\theta^3} + \frac{h\alpha\beta T_1^{\beta+4}}{12T\theta^4} = 0. \quad (11)$$

and

$$\frac{\partial^2 K(T)}{\partial T_1^2} = \frac{C_1\alpha\beta^2 T_1^{\beta-1}}{\theta T} + \frac{C_1\alpha\beta(\beta+1)T_1^{\beta}}{2T\theta^2} + \frac{C_2\alpha\beta^2(\beta+1)T_1^{\beta-1}}{T} - \alpha\beta(\beta-1)C_2 T_1^{\beta-2} + \frac{h\alpha\beta^2 T_1^{\beta-1}}{T} + \frac{h\alpha\beta(\beta+1)T_1^{\beta}}{2T\theta} + \frac{h\alpha\beta(\beta+2)T_1^{\beta+2}}{6T\theta^2} - \frac{h\alpha\beta(\beta+3)T_1^{\beta+2}}{12T\theta^3} > 0. \quad (12)$$

4. Numerical example

Consider an inventory system with following parameter in proper unit A = 50, h = 2, α = 0.002, β = 0.8, θ = 0.01, C_1= 0.8, C_2 = 2 we get T_1 = 0.3564 and TC = 50.0019

5. Sensitivity Analysis

β	T_1	Q	TC
1.0	0.3327	0.4434	50.0020
1.5	0.2853	0.3264	50.0019
2.0	0.2497	0.1870	50.0016
2.5	0.2220	0.0903	50.0013
3.0	0.1998	0.0383	50.0010
3.5	0.1887	0.0146	50.0009

Increase in one of the demand rate β decreases procurement quantity and total cost per time unit of an inventory system.

6. CONCLUDING REMARK

In this model we have developed an inventory model for deteriorating items with two parameter Weibull demand and two parameter exponential distribution rate with constant holding cost. This model shows that when one of the demand rate (β) increases the total cost decreases**.**

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