Deteriorating Inventory Model For Two Parameter Weibull Demand With Shortages

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Abstract

In this paper a deteriorating inventory model have been developed for two parameter Weibull demand rate. Shortages are allowed and are completely backlogged .This inventory system follows an two-parameter exponential distribution deterioration rate in which the holding cost is constant .The results are described with the numerical example and sensitivity analysis.

Keywords: Deterioration, Exponential distribution, holding cost, Inventory, shortages, Weibull demand rate.

1. Introduction

Many Researchers have developed inventory models to maximize the profit (or) to minimize the total cost for deteriorating items with respect to time. Deterioration arises due to some changes in the products which makes the product value dull. Deterioration in each product cannot be completely avoided and the rate of deterioration for each product will vary. Azizul Baten and Abdulbasah developed an inventory model in which the shortages not allowed with constant demand and deterioration rate. Many Researchers were interested in taking weibull deteriorating rate (in two (or) three). Azizul Baten and Abdulbasah also presented a review for Weibull distributed distribution .C.K Tripathy and U.Mishra developed an inventory model with time-varying holding cost with shortages which are completely backlogged.C.K.Tripathy, L.M.Pradhan improved their model for not only power demand but also partially backlogged .C.K.Tripathy,U.Mishra gave an ordering policy for Quadratic demand with permissible delay in payments. Kun-Shan Wu presented an ordering policy for items with Weibull deteriorating rate and permissible delay in payments.

2. Assumptions and Notations

• The inventory system involves only one item.
• Lead time is zero.
The demand rate of any time is \( \alpha \beta t^{\beta-1} \) two parameter Weibull distribution, where \( 0 < \alpha \leq 1, \beta > 0 \) are called scale and shape parameter respectively.

- \( \theta'(t) = \frac{1}{\theta} \) deterioration rate follows a two parameter exponential distribution.
- Shortages are allowed and are completely backlogged.
- \( A \): Setup Cost
- \( C_1 \): Deterioration Cost
- \( C_2 \): Shortages Cost
- \( I(t) \): Inventory level at time \( t=0 \)
- \( Q(t) \): Order quantity at time \( t=0 \)
- \( T \): Duration of a cycle
- \( T_1 \): the time at which the inventory level reaches zero
- \( K(T) \): The total cost per unit time

3. Mathematical Model:

Let \( I(t) \) be the inventory level at time \( t(0 \leq t \leq T) \). The differential equations for the instantaneous state over \((0, T)\) are given by

\[
\begin{align*}
\frac{dI(t)}{dt} + \frac{1}{\theta} I(t) &= -\alpha \beta t^{\beta-1}, \quad 0 \leq t \leq T_1 \\
\frac{dI(t)}{dt} &= -\alpha \beta t^{\beta-1}, \quad T_1 \leq t \leq T
\end{align*}
\]

With boundary conditions \( I(T_1) = 0 \) and \( I(0) = Q \)

Solving equations (1) and (2) we get

\[
I(t) = a \beta \left( \frac{t^{\beta+1}}{\beta} + \frac{t^{\beta+2}}{(\beta+1)} + \frac{t^{\beta+3}}{(\beta+2)(\beta+1)} + \frac{t^{\beta+4}}{(\beta+3)(\beta+2)(\beta+1)} \right) - \frac{a \beta}{\delta} \left( \frac{t^{\beta+1}}{\beta} + \frac{t^{\beta+2}}{(\beta+1)} + \frac{t^{\beta+3}}{(\beta+2)(\beta+1)} + \frac{t^{\beta+4}}{(\beta+3)(\beta+2)(\beta+1)} \right) + \frac{a \beta}{\delta^{2}} \left( \frac{t^{\beta+1}}{\beta} + \frac{t^{\beta+2}}{(\beta+1)} + \frac{t^{\beta+3}}{(\beta+2)(\beta+1)} + \frac{t^{\beta+4}}{(\beta+3)(\beta+2)(\beta+1)} \right)
\]

\[
I(t) = \alpha T_1^{\beta} t^{\beta} 
\]

Deteriorating Cost

\[
DC = \int_{T_1}^{T} Q \cdot D(t) \, dt 
\]
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\[ \text{SC} = \frac{a \sigma T \Gamma^{(3)+\alpha}}{\tau(\beta+1)} + \frac{b \sigma T \Gamma^{(3)+\beta}}{2\tau^2(\beta+2)} \]  

(5)

Shortage Cost

\[ \text{SC} = -\frac{2}{\tau} \int_0^T \alpha(T_1^\beta - 1) \, dt \]

\[ = \frac{C_1 \sigma T \Gamma^{(\beta+1)}}{\beta} + C_2 \sigma T \Gamma^{(\beta+2)} \]

(6)

Inventory Holding Cost

\[ \text{HC} = \frac{h}{\tau} \int_0^T T_1 \, dt \]

\[ = \frac{a \sigma T \Gamma^{(3+\alpha)}}{\tau(\beta+1)} + \frac{b \sigma T \Gamma^{(3+\beta)}}{2\tau^2(\beta+2)} + \frac{a \sigma T \Gamma^{(3+\beta+1)}}{12\tau^3(\beta+3)} + \frac{b \sigma T \Gamma^{(3+\beta+2)}}{21\tau^4(\beta+4)} \]

(7)

Setup Cost

\[ \text{SC} = \frac{d}{\tau} \]

(8)

Order Quantity

\[ Q = \alpha \beta \left( \frac{T_1^{(\beta+1)}}{\beta} + \frac{T_1^{(\beta+2)}}{2(\beta+2)} \right) \]

(9)

Total cost per unit time is

\[ K(T) = \frac{1}{\tau} \left( \text{Setup Cost} + \text{Deterioration Cost} + \text{Holding Cost} + \text{Shortages Cost} \right) \]

\[ K(T) = \frac{a \sigma T \Gamma^{(3+\alpha)}}{\tau(\beta+1)} + \frac{b \sigma T \Gamma^{(3+\beta)}}{2\tau^2(\beta+2)} + \frac{a \sigma T \Gamma^{(3+\beta+1)}}{12\tau^3(\beta+3)} + \frac{b \sigma T \Gamma^{(3+\beta+2)}}{21\tau^4(\beta+4)} + \frac{C_1 \sigma T_1^{\beta+1}}{\beta} + \frac{C_2 \sigma T_1^{\beta+2}}{2(\beta+2)} \]

(10)

Our objective is to minimize the total Cost. The necessary conditions for minimizing the total cost

\[ \frac{\partial K(T)}{\partial T_1} = \frac{a \sigma C_2 T_1^{\beta-1}}{\beta} + \frac{C_2 a \sigma T_1^{\beta-1}}{2\beta^2} + \frac{C_2 a \sigma T_1^{(\beta-1)2}}{\beta} - \beta a C_2 T_1^{\beta-1} - \frac{h a \sigma T_1^{\beta-1}}{2\beta^2} - \frac{h a \sigma T_1^{\beta-1}}{6\beta^3} - \frac{h a \sigma T_1^{\beta-2}}{12\beta^4} = 0. \]

(11)

and

\[ \frac{\partial^2 K(T)}{\partial T_1^2} = \frac{C_1 a \sigma T_1^{(\beta+1)2}}{\beta^2} + \frac{C_1 a \sigma T_1^{(\beta+2)}}{2\beta^3} + \frac{C_1 a \sigma T_1^{(\beta+1)2}}{\beta} - \alpha \beta (\beta-1) C_2 T_1^{\beta-2} - \frac{h a \sigma T_1^{\beta-1}}{2\beta^2} + \frac{h a \sigma T_1^{\beta-2}}{2\beta^3} + \frac{h a \sigma T_1^{\beta-2}}{4\beta^4}. \]

(12)

4. Numerical example

Consider an inventory system with following parameter in proper unit A = 50, h = 2, a = 0.002, \( \beta = 0.8 \), \( \theta = 0.01 \), \( C_1 = 0.8 \), \( C_2 = 2 \) we get \( T_1 = 0.3564 \) and \( TC = 50.0019 \)
5. Sensitivity Analysis

<table>
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<tr>
<th>β</th>
<th>T₁</th>
<th>Q</th>
<th>TC</th>
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<tr>
<td>3.5</td>
<td>0.1887</td>
<td>0.0146</td>
<td>50.0009</td>
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</tbody>
</table>

Increase in one of the demand rate β decreases procurement quantity and total cost per time unit of an inventory system.

6. CONCLUDING REMARK

In this model we have developed an inventory model for deteriorating items with two parameter Weibull demand and two parameter exponential distribution rate with constant holding cost. This model shows that when one of the demand rate (β) increases the total cost decreases.

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35

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