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A Predictive Model for Monthly Currency in Circulation in Ghana

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Abstract

The Currency in Circulation is the outstanding amount of notes and coins circulated in the economy and are the most liquid monetary aggregate. In this study, secondary data on monthly Currency in Circulation obtained from the Bank of Ghana database was modelled using the Seasonal Autoregressive Integrated Moving Average model. The result revealed that ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ model was appropriate for modelling the Currency in Circulation. This model has the least AIC of -372.16, AICc of -371.97, and BIC of -363.53. Diagnostic test of the model residuals with the Ljung-Box and ARCH-LM test revealed that the model is free from higher order autocorrelation and conditional heteroscedasticity respectively. Thus, we proposed ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ model for predicting the Currency in Circulation in Ghana. However, continues monitoring of the forecasting performance of this model is necessary to make the use of this model more realistic.

Keywords: Currency in Circulation, Liquidity, Monetary aggregate

1. Introduction

The Currency in Circulation is simply the outstanding amount of notes and coins circulated in the economy and they are the most liquid monetary aggregate. The Currency in Circulation forms an important component of the Reserve money growth of a country (suleman and Sarpong, 2012). The key determinant of the Currency in Circulation is the cash demand of both the public and the banking system. The variations in the Currency in Circulation are vital indicators for monetisation and demonetisation of the economy. Also, the variation in the Currency in Circulation could exert inflationary pressure and decrease the availability of loan funds for investments. The share of the Currency in Circulation in money supply and its ratio to nominal Gross Domestic Product reveals its relative importance in any economy (Simwaka, 2006; Stavreski, 1998).

Umpteen of researches have been carried out all over the world using time series models to model the pattern of Currency in Circulation. For instance, Hlavacek *et al*., (2005) used both linear (ARIMA) and non-linear technique to model the Currency in Circulation for Czech Republic. Also, Dheerasinghe (2006) modelled the currency in demand in Sri-Lanka with monthly, weekly and daily data set using time series models. This study, thus aims to model the monthly volume of Currency in Circulation in Ghana using Seasonal Autoregressive Integrated Moving Average model.

2. Material and Methods

This study was carried out in Ghana using secondary data on monthly Currency in Circulation, from January, 2000 to December, 2011. The data was obtained from the Bank of Ghana database.

2.1 Regression with Periodic Dummies

To test the existence of monthly seasonality in the data, the following regression was run for the period 2000 to 2011.

$$\mathrm{CIC_t} = \sum_{i=1}^{12} \beta_i M_i + \varepsilon_t \tag{1}$$

where M_i is a dummy variable taking a value of one for month *i* and zero otherwise (where i= 1, 2, …, 12), β_i are parameters to be estimated, and ε_t is the error term. The hypothesis tested is $H_0: \beta_1 = \beta_2 = \cdots = \beta_{12} = 0$ against the alternative that not allβ_i are equal to zero. If the null hypothesis is rejected, then the Currency in Circulation exhibit month-of-the-year seasonality.

2.2 Seasonal Autoregressive Integrated Moving Average Model

The data on the Currency in Circulation was modelled using the Seasonal Autoregressive Integrated Moving Average (SARIMA) model. An ARIMA (*p, d, q*) model is a mixture of Autoregressive (AR) which indicates that there is a relationship between present and past values, a random value and a Moving Average (MA) which shows that the present value has relationship with the past shocks. If the data has a seasonal component, then ARIMA (*p, d, q*) is extended to included the seasonal component. Thus, the SARIMA model is expressed as ARIMA $(p, d, q)(P, D, Q)_S$. The orders *p* and *q* represent the non-seasonal AR and MA components respectively. The orders of the seasonal AR and MA components are *P* and *Q* respectively. Also, the orders of differencing for the seasonal and non-seasonal are *D* and *d* respectively. The SARIMA model denoted by ARIMA $(p, d, q)(P, D, Q)_S$ can be expressed using the lag operator as;

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D = \theta(L)\Theta(L^s)\varepsilon_t \tag{2}$$

where

$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p$

$\Phi(L^s) = 1 - \Phi_1 L^s - \Phi_2 L^{2s} - \ldots - \Phi_P L^{Ps}$

$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_q L^q$

$\Theta(L^s) = 1 - \Theta_1 L^s - \Theta_2 L^{2s} - \ldots - \Theta_Q L^{Qs}$

L represent the lag operator

ε_t represent white noise error at period *t*

ϕ_i represent the parameters of the non-seasonal autoregressive component

Φ_i represent the parameters of the seasonal autoregressive component

θ_j represent the parameters of the non-seasonal moving average component

Θ_j represent the parameters of the moving average component

The estimation of the model involves three steps, namely: identification, estimation of parameters and diagnostics. The identification step involves the use of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) to identify the tentative orders of both the non-seasonal and seasonal components of the model. The second step involves estimation of the parameters of the tentative models that have been selected. In this study, the model with the minimum values of Akaike Information Criterion (AIC), modified Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) is adjudged the best model. The last stage which is the diagnostic stage involves checking whether the selected model adequately represents the Currency in Circulation. An overall check of the model adequacy was made at this stage using the Ljung-Box test and ARCH-LM test. These tests were performed to check for higher order autocorrelation and homoscedasticity respectively.

2.3 Unit Root Test

This test was performed to check whether the data on Currency in Circulation was stationary. In view of this, the Augmented Dickey-Fuller (ADF) test was used for the test. The test is based on the assumption that a time series data Y_t follows a random walk:

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{3}$$

where $\rho = 1$, thus Y_{t-1} is subtracted from both sides. $\Delta Y_t = \beta Y_{t-1} + \varepsilon_t$ and $\beta = \rho - 1$. The null hypothesis is $H_0\!: \beta = 0$ and therefore $\rho = 1.0$ against the alternative that $H_1\!: \beta < 0$ and $\rho < 1$. If the p-value of the associated coefficient β is less than the 0.05 significance level, then the data is stationary.

3. Results and Discussion

Table 1 displays the descriptive statistics of the Currency in Circulation. The maximum and minimum values for the Currency in Circulation for the entire period under consideration were 4222.27 and 154.47 billion Ghana cedi respectively. The Currency in Circulation was positively skewed and leptokurtic in nature with the average and coefficient of variation been 1105.30 billion Ghana cedi and 84.57% respectively.

The time series plot (Figure 1) of the Currency in Circulation depicts an increasing trend with an unstable mean, as the mean keeps increasing and decreasing at certain points.

Also, the existence of seasonal factors was investigated by regressing the Currency in Circulation on full set of periodic dummies. The regression results revealed that the series was characterised by monthly seasonal factors. The F-statistic of the regression was 17.30 with a p-value of 0.00. The slow decay of the ACF plot (Figure 2) and a very dominant spike at lag one of the PACF plot of the series indicates that the series is not stationary. The non-stationarity of the series was affirmed using the ADF test. The results of the ADF test in Table 2 confirm the existence of unit root in the series. The Currency in Circulation was thus transformed using logarithmic transformation in order to stabilise the variance of the series. Due to evidence of seasonality the transformed series was seasonally differenced and tested for stationarity. The ADF test of the seasonally differenced series revealed that the series was not stationary as shown in Table 3.The transformed seasonal differenced series was again non-seasonal differenced and tested for stationarity. The ADF test in Table 4, affirms that the transformed seasonal and non-seasonal differenced Currency in Circulation is stationary.

The stationarity of the series was also affirmed by the time series plot of the transformed seasonal and non-seasonal differenced series. As shown in Figure 3, the series fluctuates about the zero line confirming stationarity in mean and variance. After obtaining the order of integration of the series, the orders of the Autoregressive and Moving Average for both seasonal and non-seasonal components were obtained from the ACF and PACF plots (Figure 4) based on the Box-Jenkins (1976) approach. From Figure 4, the ACF plot have significant spike at the non-seasonal lag 1 and seasonal lag 12, with significant spike at other non-seasonal lags. The PACF plot also has significant spike at the non-seasonal lags 1 and 2 and seasonal lags 12 and 24. The PACF plot also has spike at other non-seasonal lags. Using the lower significant lags of both the ACF and PACF and their respective significant seasonal lags, tentative models for the Currency in Circulation were identified. As shown in Table 5, ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ was chosen as the appropriate model that fit the data well because it has the minimum values of AIC, AICc, and BIC compared to other models. Using the method of maximum likelihood, the estimated parameters of the derived model are shown in Table 6. The ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ model can be expressed in terms of the lag operator as;

$$(1-L)(1-L^{12})\ln \mathrm{CiC} = (1-0.3809L)(1-0.7109L^{12})\varepsilon_t$$

Observing the *p*-values of the parameters of the model, it can be seen that both the non-seasonal and seasonal Moving Average components are highly significant at the 5% level. Thus, the model appears to be the best model among the proposed models.

To ensure the adequacy of the estimated model, the ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ was diagnosed. As shown in Figure 4, the standardised residuals revealed that almost all the residuals have zero mean and constant variance. Also, the ACF of the residuals depict that the autocorrelation of the residuals are all zero that is they are uncorrelated. Finally, in the third panel, the Ljung-Box statistic indicates that there is no significant departure from white noise for the residuals as the *p*-values of the test statistic clearly exceeds the 5% significant level for all lag orders. To buttress the information depicted in Figure 4, the ARCH-LM test and t-test were employed to test for constant variance and zero mean assumption respectively. The ARCH-LM test result shown in Table 7, failed to reject the null hypothesis of no ARCH effect in the residuals of the selected model. Also, the t-test gave a test statistic of -1.3281 and a p-value of 0.1865 which is greater than the 5% significance level. Thus, we fail to reject the null hypothesis that the mean of the residuals is approximately equal to zero. Hence, the selected model satisfies all the assumptions and it can be concluded that ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ model provides adequate representation of the Currency in Circulation.

4. Conclusion

In this study, the monthly volume of Currency in Circulation in Ghana was modelled using the Seasonal Autoregressive Integrated Moving Average model. The best model identified for the Currency in Circulation was ARIMA (0, 1, 1)(0, 1, 1)$_{12}$. Diagnostic checks on the model residuals revealed that the model is adequate for representing the Currency in Circulation in Ghana. Thus, we proposed ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ model for predicting the Currency in Circulation in Ghana. However, continues monitoring of the forecasting performance of this model is necessary to make the use of this model more realistic.

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Table 1: Descriptive statistics for Currency in Circulation

Variable	Mean	Minimum	Maximum	CV (%)	Skewness	Kurtosis
Currency in Circulation	1105.30	154.47	4222.27	84.50	1.29	0.95

Table 2: ADF test of Currency in Circulation in level form

Test	Constant		Constant+ Trend	
	Test Statistic	*P*-value	Test Statistic	*P*-value
ADF	5.4972	1.0000	5.2189	1.0000

Table 3: ADF test of seasonal differenced Currency in Circulation

Test	Constant		Constant+ Trend	
	Test Statistic	*P*-value	Test Statistic	*P*-value
ADF	-2.4924	0.1173	-2.4762	0.3401

Table 4: ADF test of transformed seasonal and non-seasonal differenced series

Test	Constant		Constant+ Trend	
	Test Statistic	*P*-value	Test Statistic	*P*-value
ADF	-5.0165	0.0000	-4.9081	0.0001

Table 5: Tentative Seasonal Autoregressive Integrated Moving Average Models

Model	AIC	AICc	BIC
ARIMA $(1, 1, 1)(1, 1, 1)_{12}$	-368.92	-368.44	-354.54
ARIMA $(1, 1, 1)(2, 1, 1)_{12}$	-367.05	-366.37	-349.80
ARIMA $(2, 1, 1)(1, 1, 1)_{12}$	-366.92	-366.25	-349.67
ARIMA $(1, 1, 0)(1, 1, 0)_{12}$	-353.80	-353.62	-345.18
ARIMA $(0, 1, 1)(0, 1, 1)_{12}$	-372.16^{*}	-371.97^{*}	-363.53^{*}

*: Means best based on the selection criteria

Table 6: Estimates of parameters for ARIMA $(0, 1, 1)(0, 1, 1)_{12}$

variable	Coefficient	Standard error	*z*-statistic	*P*-value
θ_1	0.3809	0.0786	4.8400	0.0000
Θ_1	0.7109	0.0849	8.3570	0.0000

Table 7: ARCH-LM test of Residuals of ARIMA (0, 1, 1)(0, 1, 1)$_{12}$ Model

Lag	Test statistic	df	*P*-value
12	2.8814	12	0.9963
24	4.7132	24	1.0000
36	6.3775	36	1.0000

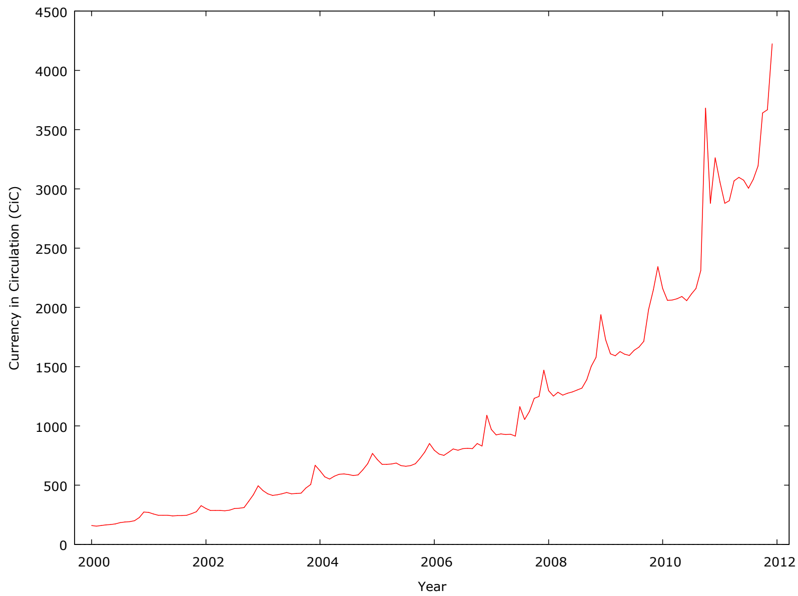

Figure 1: Time series plot of Currency in Circulation

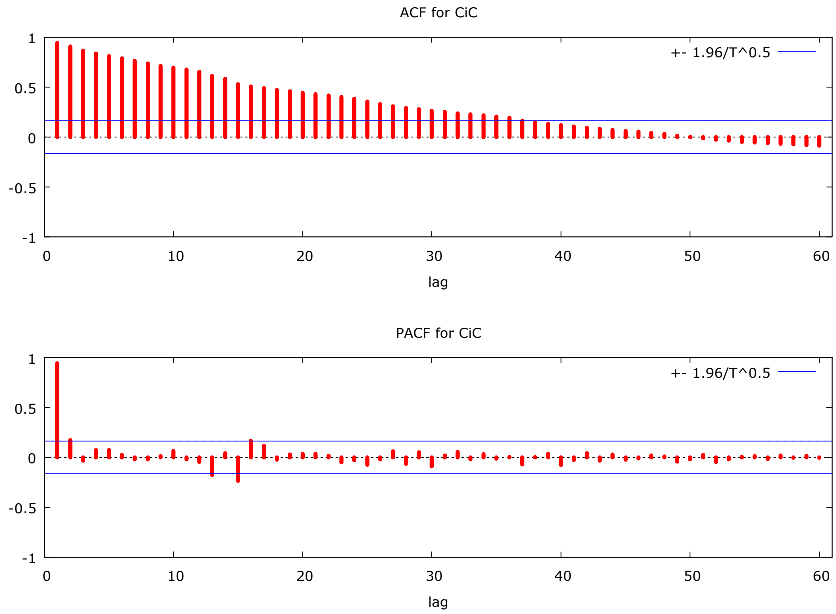

Figure 2: ACF and PACF plot of Currency in Circulation

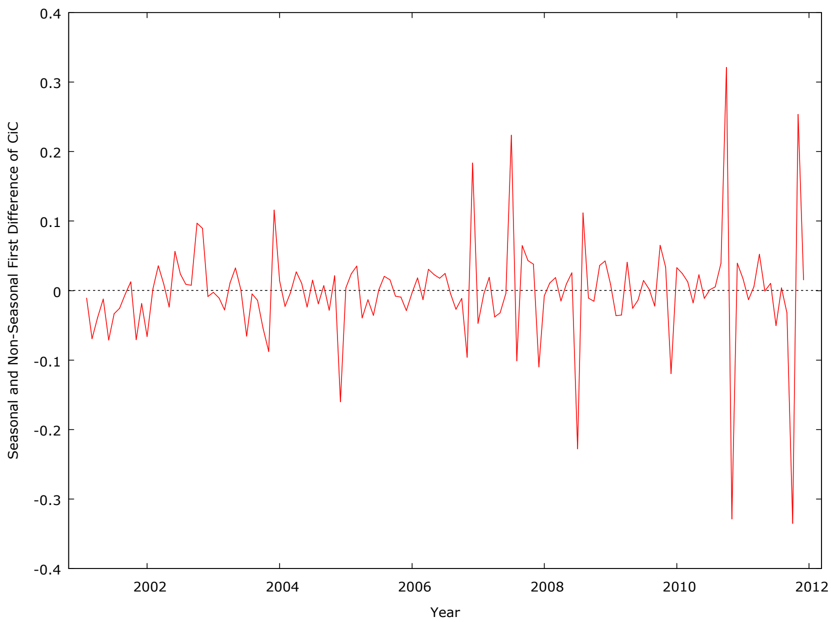

Figure 3: Time series plot of seasonal and non-seasonal first differenced series

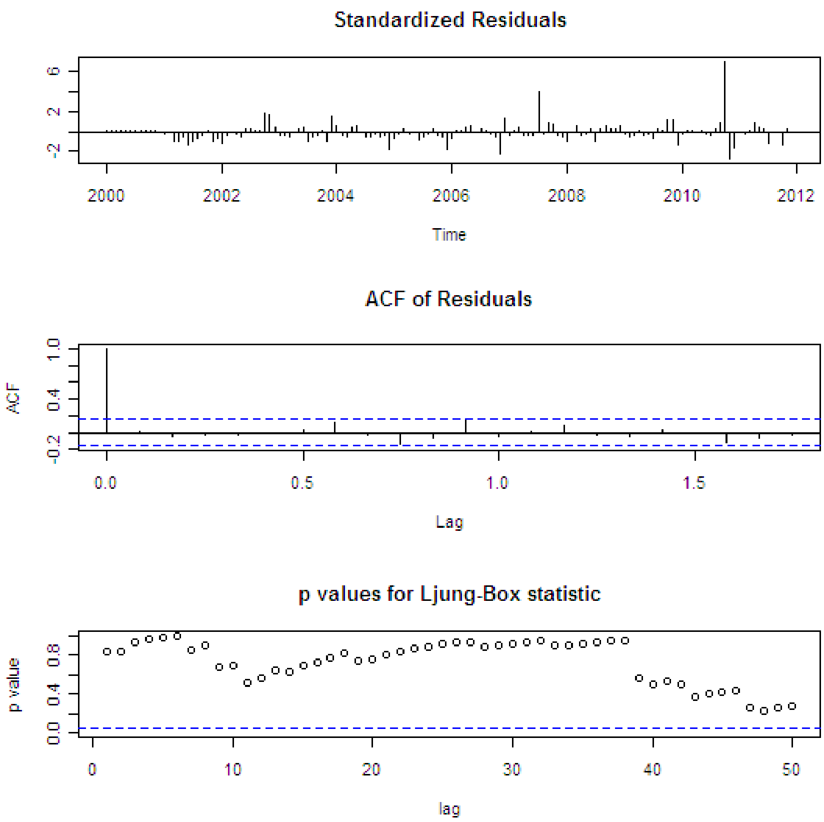

Figure 4: Diagnostic plot of ARIMA (0, 1, 1)(0, 1, 1)$_{12}$

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