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On The Mechanism And Behavior Of Plasma

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Abstract

The charged particles' action of E and B fields have three diverse levels of modeling, Starting with the simplest one to the most complicated. In this paper we consider the generalization of Newtonian force law in geometrical term is to describe charged particles' (plasma) trajectories on electromagnetic fields in the kinetic or microscopic model.

Keyword: Gravitational field, Time–dependent Flow, Integral Curves, Exterior Differential Systems, Kinetic Energy.

1 Introduction

Universe contain plasma which a collection of particles with electric charges moving and giving rise to EM fields, The motion of a charged particle in an electromagnetic fields can be thought of it as the superposition of fast circular motion around a Guiding center system (GCS) and slow drift of this system's points[5]. Knowing of Plasma's behavior for observer required motion's trajectories (or say flow lines), which a solution to the equation of motion in gravitational field.

Recall that The gravitational field X is a generalization of the vector form that describes the gravita-tional force which would be applied on an object in any given state in space, note that work done by gravity is path-independent because gravitational fields are also conservative [4]. So the charged particles trajectories that are near neighbors cannot suddenly be separated, since points cannot diverge faster than exponentially in time if the derivative of X is uniformly bounded [4].

Therefor we see that plasma components behave in complex way which produce complicated motion , hence we need specified description for individual particles because a large part of phenomena understood by the single particle motion , and later we give in this work a geometrical a generalization which allow us to solve equations of motion analytically in a general case.

2 Autonomous and Flow

Consider the dynamical system of a particles moving under the influence of the central force field, and suppose a particle have the state (positions and momenta), (x_i, p_i) in a set $U \subseteq \mathbb{R}^3 \times \mathbb{R}^3$. When the state $\gamma_0 \in U$ at time t_0 changes to γ at time t , we have the evolution operator given by

$$F_{t,t_0}(\gamma_0) = \gamma$$

So if F_{t,t_0} depends only on $(t - t_0)$ then the Chapman–Kolmogorov law or (autonomous)

$$F_{\tau,t} \circ F_{t,t_0} = F_{\tau,t_0} \quad (F_{t,t} \equiv \text{Identity}). \qquad (2)$$

has the group property:

$$F_t \circ F_{t_0} = F_{t+t_0} \quad (F_0 \equiv \text{Identity}). \qquad (3)$$

Here such F_t called a flow and an evolution operator F_{t,t_0} a time–dependent flow, (Note that if the dynamical system is irreversible, that is, defined only for $t \ge t_0$, then we speak of a semi–flow)

Since usually, the laws of motion are given in the ODEs form, instead of evolution operator F_{t,t_0}, so we must solve to find the flow. These equations of motion have the form:

$$\dot{\gamma} = \gamma(X), \qquad \gamma(0) = \gamma_0 ,$$

where X is a (possibly time–dependent) vector–field on U .

Also above particle motion can be described by Newtonian equation of motion:

$$m\ddot{x}_i = F_i(x) \qquad i = 1,2,3$$

then we can splits it into two Hamiltonian equations:

$$\dot{x}_i = p^i / m, \qquad \dot{p}^i = F_i(x),$$

just by introducing momenta $p^i = m\dot{x}_i$.

But in general case of a Riemannian manifold $p_i = mg_i\dot{x}^j$,so above equations takes form

$$\dot{x}_i = \frac{g_{ij}p^j}{m}, \qquad \dot{p}_i = F_i(x). \qquad (4)$$

The RHS. of equations (4) Define a Hamiltonian vector–field on this 6D- manifold by

$$X(x,p) = [(x_{i,}p^i),(p^i/m, F_i(x))]$$

Solution of eq. (4) produces trajectories, which comprise the flow F_t of the vector–field $X(x,p)$

3 Integral Curves and Time–dependent Flow

So if we can describe these trajectories well we can get more information of plasma behavior in universe, for this we go on with the Integral Curves:

Recall that integral curve is a parameterized curve $\gamma : I \to M$ from an open interval $I \subset \mathbb{R}$ into n -manifold M [2], which satisfying

$$\dot{\gamma}(t) = X(\gamma(t)) , \quad \text{for all } I \subset \mathbb{R} . \qquad (5)$$

Here X is a smooth tangent vector–field on the smooth $n-$ manifold M ,

And the unique γ satisfying conditions

(i) $\gamma(0) = m$, $m \in M$

(ii) If $\beta : \tilde{I} \to M$ is any other parameterized curve in M satisfying (i) and (5), then $\tilde{I} \subset I$ and $\beta(t) = y(t)$ for all $t \in \tilde{I}$.

Is called maximal integral curve

Note that if the tangent vector determined by $\gamma = X$ at every point $m \in M$ then γ represents an integral curve or *flow line* of a vector–field X

Recall that the velocity $\dot{\gamma}$ of the γ is a vector–field along γ given by

$$\dot{\gamma}(t) = [\gamma(t), \dot{x}_1(t), ..\dot{x}_n(t)]$$

And the rate of change of the vector parts of $X(t)$ a long γ measured by $\dot{X}(t)$ .thus, acceleration $\ddot{\gamma}(t)$ of γ is the vector–field along γ getting by differentiation of the velocity field $\dot{\gamma}$. Also a parameterized curve $\gamma : I \to M$ is said to be a geodesic if its acceleration $\ddot{\gamma}$ everywhere orthogonal to M So $\gamma : I \to M$ is a geodesic if it satisfies

$$\ddot{\gamma}(t) + \dot{N}(\gamma(t)) \cdot N(\gamma(t)) = 0, \qquad N(\gamma(t)) \text{ is orientation} \qquad (6)$$

This represents the systems of second–order component ODEs.

$$\ddot{x}_i + N_i(x+1,...,x_n)\frac{\partial N_j}{\partial x_k}(x+1,...,x_n)\dot{x}_j\dot{x}_k = 0$$

Here we can reduce these systems to the first–order differential system just by substitute $\dot{x}_i = u_i$, where

$$\dot{u}_i = -N_i(x+1,...,x_n)\frac{\partial N_j}{\partial x_k}(x+1,...,x_n)\dot{x}_j\dot{x}_k$$

This form is just the differential equation for the integral curves of X in$\mathbb{R} \times U$., U is open chart on manifold M [2].

Now if the mechanical system follows an integral curve $\gamma(t)$, then γ represents trajectory. Thus, the motion of the mechanical system is fully described by

$$\dot{\gamma}_i(t) = X_i(\gamma(t)) \qquad \forall t \in I, I \subseteq \mathbb{R}$$

in case t represents the time.

Recall that the flow F_t of a C^k vector–field $X \in \chi^k(M)$ is the one-parameter group of diffeomor-phisms $F : M \to M$ such that $t \mapsto F_t(m)$ is the integral curve of X with initial condition m for all $m \in M$ and $t \in I$. So by induction the flow $F_t(m)$ is C^k on k . It is defined as

$$\frac{d(F_t(x))}{dt} = X(F_t(x))$$

Here the smoothness of F_t guaranteed by existence and uniqueness theorems for ODEs, so by which the property (3) getting

This property generalizes the situation from linear spaces ($M = V$) to the nonlinear case, thus we can think of F_t as a formal exponential

$$F_t(m) = \exp(tX) = \sum_{k=0}^{\infty} \frac{X^k t^k}{k!}$$

Therefore we can get the time–dependent flow property

$$F_{t,r} = F_{t,s} \circ F_{s,r},$$

by uniqueness of existence and uniqueness theorem again.

Where the flow is one–parameter group of diffeomorphisms $F_{t,s} : M \to M$ such that $t \mapsto F_{t,s}(m)$ is the integral curve $\gamma(t)$ with initial condition $\gamma(t) = m$ at $t = s$.
Note that If X happens to be time independent; the two notions of flows are related by $F_{t,s} = F_{t-s}$.

4 Motion Law Generalization

Since charged particles have some kinds of motions then study of motion trajectories require us to generalize our basic tools in term of exterior forms language to be valid for generic physicals.

Below we go to generalize the Newtonian Force law, $F = ma$ to thought of it as The Covariant Force Law by next process.

Note that the internal velocity vector–field is defined by the set of ODEs, and Analytically, vector–field is defined as a set of autonomous ODEs, then Its solution gives the flow, consisting of integral curves of the vector–field , but Geometrically, vector–field is defined as a cross–section of the tangent bundle TM ,
(velocity phase–space) which have a 1–form–field (represents a field of one–forms) as geometrical dual , these one – form are is defined as an exterior differential systems, which an algebraic dual to the autonomous set of ODEs, but Geometrically, it is defined as a cross–section of the cotangent bundle T^*M (momentum phase–space). Thus the scalar potential field is defined by the vector–field and its corresponding one –form–field together.

Therefore and since the internal acceleration vector–field $a_i = a_i(x_i, \dot{x}_i, t)$ defined by the set of ODEs, then we can define the internal force one –form field $F_i = F_i(x_i, \dot{x}_i, t)$ as a family of force one–forms,
(half is rotational and half translational) :

$$F_i = mg_{ij}(\ddot{x}_i + \Gamma^j_{ik}\dot{x}_i\dot{x}_k) \qquad (\Gamma^j_{ik} \text{ is Levi–Civita connections})$$

(Note: system's Riemannian kinetic energy form is $T = \frac{1}{2} mg_{ig} v_i v_j$),
Then the meaning of above expressing is:

Force 1–form–field = Mass distribution × Acceleration vector–field

So this covariant force law generalizes the fundamental Newtonian equation for the generic physical system. Thus this generalization allows us to give analytical solutions for equations of motion in a general case, where the fields depend on time and spatial coordinates.

Solution as microscopic model for charged particles under the influence of Lorentz force $F = q(E + v \times B)$ is a helical trajectory [3], (according to Larmor radius $\rho_L = m_q v_\perp / |q| B$), which increases with the energy of the particle and decreases with the strength of the magnetic field [3].

In inertial situation, the inertial trajectories of particles and radiation in the resulting geometry are then calculated by solving geodesic equation (6) or in general form:

$$\frac{d^2 x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = 0,$$

according to following theorem[4] :

Theorem:
On a smooth manifold M , an inertial trajectory of free particles is geodesics.

Instead, charged particles can show demeanor usually like waves, also particles can, originates superposition, this superposition property allows the particles to be in a quantum superposition state so we can describe the motion by wave equation, wave equations can be derived from Newton's laws [1], then the behavior of plasma can studied according to Schrödinger (wave–particle duality)[6], since particles' corresponding wavefunction satisfies the wave equation and The Schrödinger equation includes the wavefunction, this wavefunction summarizes the particles' quantum state in the system[7], so position and momentum at all times along trajectory are deterministic and can be jointly known.

Note that: when the Hamiltonian don't dependent on time then wavefunctions can form stationary states, (orbitals) , so this case can be described by time-independent Schrödinger equation.

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