

An Improved Algorithm for Optimal Solution of Unbalanced Transportation Problems

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Abstract
Unbalanced transportation problems are particular kind of transportation problems, but an optimal solution is hard to find for unbalanced transportation problems. Still there is a need for minimizing the transportation cost. Unbalanced–TP deals with two different cases, (i) Excess of accessibility $\sum_i \mathbf{A_i} > \sum_j \mathbf{B_j}$, (ii) Deficiency in accessibility $\sum_i \mathbf{A_i} < \sum_j \mathbf{B_j}$, here in this paper both the cases for getting better optimal solution are discussed. Proposed algorithm is based on dummy rows and dummy columns, by taking the absolute differences (penalty) of Initial & Last cost cells of each row/column in transportation cost-matrix, where the objective function is to find an optimal solution. This method is easy to understand and apply than the other existing methods using Initial Basic Feasible Solution–IBFS. Therefore, the proposed method is very helpful to get optimal solution for unbalanced transportation problems.
Keywords: Initial Basic Feasible Solution–IBFS, Unbalanced Transportation Problems, Dummy Rows & Dummy Columns, Optimal Solution.
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1. INTRODUCTION

In Operation Research–OR, Transportation Problem–TP is specific kind of sub-division of linear programming–LP problems. It is commonly significant in the scheme of decision making [1]. Wherein purpose is to lessen the cost of transportation for businesses with number of destination, while sustaining supply limit and demand prerequisite [2]. To get this aim of cost, we have the quantity and locality of already accessible supplies and the amount demanded on top of the participation united accompanied by each 'sending'. The term 'Transportation Model' is some time deceive because it can be used for plant location, machine assignment, product mix problems and many more as well. Whereas the model is really not limited to transportation/distribution only [5].
Linear programming is a mathematical technique which is widely used in fields of social sciences and business domain. For example: Transportation Problems, Allocation Problems, Diet Problems, Agriculture Problems, Network Flow Problems, Marketing Engineering and several others. L.F. Hitchcock (1941) initially established basic transportation problems. The motive of transportation problem controls optimal distribution arrangements between origins/sources and destinations, which are basically accumulated at many origins to diverse destinations in such a way that the cost of total transportation is minimized [4, 7, 10]. There are two forms of transportation problems.

1.1 Balanced and Unbalanced Transportation Problem

A transportation problem is known as balanced when the summation of all supply bases are equal to the summation of all demand purposes, i.e. $\sum_{i=1}^{m} \mathbf{A_i} = \sum_{j=1}^{n} \mathbf{B_j}$ otherwise it is known as unbalanced transportation problem, i.e. $\sum_{i=1}^{m} \mathbf{A_i} > \sum_{j=1}^{n} \mathbf{B_j}$ or $\sum_{i=1}^{m} \mathbf{A_i} < \sum_{j=1}^{n} \mathbf{B_j}$, to satisfy both the conditions of unbalanced transportation problems, they are presented with a dummy row or dummy column. If the sum of sources is greater than the sum of requirements, then the dummy destinations are presented with requirements, to solve the difference between sum of sources and sum of requirements cost, the dummy rows and dummy columns are set equal to zero transportation cost [6, 8].

In previous, studies many researchers have discussed different methods to solve the transportation problems of Initial Basic Feasible Solution–IBFS and optimal methods. The most advantageous and useful ways for initial basic feasible solution are North-West Corner Method–NWCM, Least-Cost Method–LCM, Vogel's Approximation Method–VAM and several other. Other methods like Modified Distribution–MODI Method and Stepping Stone Method–SSM are used for an optimal solution of transportation problems. Some important related works in last few years has dealt with. Z.A.M.S. Juman & N.G.S.A. Nawarathne [8] Considered only row penalties using Juman and Hoque (2015)'s Method–JHM, for solving the I.B.F.S of conservative transportation problem. A. Rashid [9] He proposed a different approach which can help the decision makers in handling unbalanced transportation assignment problems to minimize the costs of total transportation and maximize the total profits. A. Sridhar & N. Girmay, et al [11, 12] suggests a Heuristic Approach to get improved results of unbalanced transportation problems, and improved the existing VAM. This study has minimized the transportation costs in an easy and effective manner. A. Quddoos, et al [13] has proposed method named ASM–Method to find optimal solution of transportation problems, directly. This method requires easy arithmetical and logical calculation with less time period, and results are comparable to existing MODI–Method and Stepping Stone Method. Rajendra B. Patel [14] the presented method is a little modification of ASM–Method. M. Palanivel, et al [15] analyzed an innovative procedure for solving problems of transportation by using Harmonic Mean Approach. Ravi Kumar R, et al [16] they proposed a new technique named Direct Sum Method using VAM, to discover the I.B.F.S of transportation problems. A. Seethalakshmy & N. Srinivasan [17] proposed a different technique to resolve of unbalanced transportation assignment problems, based on finding the position of '1' by using systematic procedure. This article gives same result as that of the Hungarian Method with less consumption of time. M. A. Metlo, et al [18] Modified North-West Corner Method–MNWCM is developed to discover I.B.F.S based on NWCM. The main objective of this study is to reduce the size of iterations and is used to achieve an optimal solution with minimum time estimation, also the results are compared with existing NWCM.

This paper is a motivation for finding better optimal solution for unbalanced transportation problems. This leads to a different approach than the previous methods to minimize the total cost and to maximize the profit in transporting commodities from sources to destinations. The proposed method is introduced below:

2. METHODOLOGY:

2.1 Mathematical Formulation of Unbalanced Transportation Model:

Mathematically, the general transportation model is given as follows:

Minimize: (Total transportation cost) $\mathbf{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}U_{ij}$ (1)

Subject to: (Supply from sources) $\sum_{i=1}^{m} U_{ij} \leq A_i;\quad i = 1, 2, \ldots, m$ (2)

(Demand from destinations) $\sum_{j=1}^{n} U_{ij} \geq B_j;\quad j = 1, 2, \ldots, n$ (3)

Where $U_{ij} \geq 0;\quad \forall\ i\ \&\ j$ (4)

(For an unbalanced transportation problem) $\sum_{i=1}^{m} A_i \neq \sum_{j=1}^{n} B_j$ (5)

The unbalanced transportation problems are of two cases **(i)** $\sum_i A_i > \sum_j B_j$ and **(ii)** $\sum_i A_i < \sum_j B_j$. There are two feasible cases:

i. (Excess accessibility) $\sum_{i=1}^{m} A_i > \sum_{j=1}^{n} B_j$ (6)

(A dummy purpose) D_{n+1} (7)

$B_{n+1} = \sum_{i=1}^{m} A_i - \sum_{j=1}^{n} B_j;\quad U_{i,\,n+1} = 0;\quad i = 1, 2, \ldots, m$ (8)

ii. (Deficiency in accessibility) $\sum_{i=1}^{m} A_i < \sum_{j=1}^{n} B_j$ (9)

(A dummy basis) S_{m+1} (10)

$A_{m+1} = \sum_{j=1}^{n} B_j - \sum_{i=1}^{m} A_i;\quad U_{m+1,\,j} = 0;\quad j = 1, 2, \ldots, n$ (11)

Necessary condition for presence of a feasible solution of unbalanced transportation problems are $\sum_{i=1}^{m} A_i = \sum_{j=1}^{n+1} B_j$ or $\sum_{i=1}^{m+1} A_i = \sum_{j=1}^{n} B_j$. i.e., the summation of total supply must equal to the summation of total demand [3].

2.1.1 PROPOSED ALGORITHM:

Step#01:- Case (i) whenever $\sum_i A_i > \sum_j B_j$, add $C_{ij+1} = 0\ \forall$ dummy column based on the demand (B_{j+1}), and calculate the absolute differences (penalty) between Initial & Last cost cells of each column in transportation cost-matrix, (ignoring differences (penalty) of Initial & Last cost cells of dummy column and all rows), then identify maximum penalty (P_j) for allocation.

Case (ii) whenever $\sum_i A_i < \sum_j B_j$, add $C_{ij+1} = 0\ \forall$ dummy row based on the supply (A_{i+1}), and calculate the absolute differences (penalty) between Initial & Last cost cells of each row in transportation cost-matrix, (ignoring differences (penalty) of Initial & Last cost cells of dummy row and all columns), then identify maximum penalty (P_i) for allocation.

Mathematically,

$\mathbf{C_{ij+1} = 0}\ \forall$ dummy rows & dummy columns.

$\mathbf{P_i = |\,C_{1n} - C_{mn}\,|}\ \forall$ rows penalty & $\mathbf{P_j = |\,C_{m1} - C_{mn}\,|}\ \forall$ columns penalty.

Step#02:- Choose the smallest cost cell C_{ij} in row/column and allocate with selected maximum penalty (P_i/P_j).

Step#03:- "In case of ties", If two or more of the smallest cost cells C_{ij} in row/column are same then select the top entry of them to allocate. If two or more of maximum penalty (P_i/P_j) are same then select the smallest cost cell C_{ij} in row/column.

Step#04:- Repeat steps (1 to 3) for remaining transportation cost-matrix, continue this procedure until all the supply (A_i) and demand (B_j) are become zero.

Note: The dummy row/column should be allocated in last.

Step#05:- Finally, calculate minimizing the total cost of transportation.
i.e., **Tcost**, $\{Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} U_{ij}\}$.

3. NUMERICAL EXAMPLES

Consider the following different – sizes of both the cases of unbalanced transportation problems, selected from literature. We solve them using proposed algorithm and compare these results with the solution of NWCM, LCM & MODI–Method.

3.1 Example#1:- The Unbalanced–TP of case (i) Excess of accessibility $\sum_i A_i > \sum_j B_j$.

Sources\Destinations	D_1	D_2	D_3	Supply (A_i)
S_1	3	4	6	100
S_2	7	3	8	80
S_3	6	4	5	90
S_4	7	5	2	120
Demand (B_j)	110	110	60	∑ 280\390

Table: 3.1.1

Sources\Destinations	D_1	D_2	D_3	Dummy column	Supply (A_i)
S_1	3	4	6	0	100
S_2	7	3	8	0	80
S_3	6	4	5	0	90
S_4	7	5	2	0	120
Demand (B_j)	110	110	60	110	∑ 390\390

Table: 3.1.2

Sources\Destinations	D_1	D_2	D_3	Dummy column	Supply (A_i)
S_1	3	4	6	0	100
S_2	7	3	8	0	80
S_3	6	4	5	0	90
S_4	7	5	**2** **60**	0	120 60
Demand (B_j)	110	110	60 0	110	∑ 330\330
Column penalty (P_j)	(4)	(1)	(4)		

Table: 3.1.3

Sources\Destinations	D_1	D_2	Dummy column	Supply (A_i)
S_1	**3** **100**	4	0	100 0
S_2	7	3	0	80
S_3	6	4	0	90
S_4	7	5	0	60
Demand (B_j)	110 10	110	110	∑ 230\230
Column penalty (P_j)	(4)	(1)		

Table: 3.1.4

Sources\Destinations	D_1	D_2	Dummy column	Supply (A_i)
S_2	7	**3** **80**	0	80 0
S_3	6	4	0	90
S_4	7	5	0	60
Demand (B_j)	10	110 30	110	∑ 150\150
Column penalty (P_j)	(0)	(2)		

Table: 3.1.5

Sources\Destinations	D_1	D_2	Dummy column	Supply (A_i)
S_3	6	**4** **30**	0	90 60
S_4	7	5	0	60
Demand (B_j)	10	30 0	110	∑ 120\120
Column penalty (P_j)	(1)	(1)		

Table: 3.1.6

Sources\Destinations	D_1	Dummy column	Supply (A_i)
S_3	**6** **10**	0	60 50
S_4	7	0	60
Demand (B_j)	10 0	110	∑ 110\110
Column penalty (P_j)	(1)		

Table: 3.1.7

Sources\Destinations	Dummy column	Supply (A_i)
S_3	**0** **50**	50 0
S_4	**0** **60**	60 0
Demand (B_j)	110 60 0	∑ 0\0

Table: 3.1.8

$U_{43} = 60$, $U_{11} = 100$, $U_{22} = 80$, $U_{32} = 30$, $U_{31} = 10$, $U_{34} = 50$, & $U_{44} = 60$.

Tcost, $Z = (2*60) + (3*100) + (3*80) + (4*30) + (6*10) + (0*50) + (0*60) = 840/-$

Optimality Test

Formulate an optimality test by MODI–Method, to find whether the obtained feasible solution is optimal or not. Here, the number of allocations is equal to $(m + n – 1) = 4 + 4 – 1 = 7$, hence optimality test can be achieved. Find X's and Y's values using the formula $X_i + Y_j = C_{ij}$ ∀ allocated cells, and set $X_1 = 0$. Then, find $d_{ij} = (X_i + Y_j) – C_{ij} \leq 0$ ∀ unallocated cells.

Hence all $d_{ij} \leq 0$, the solution given below is optimal.

Sources\Destinations	D_1	D_2	D_3	Dummy column	Supply (A_i)	Values (X_i)
S_1	**3** **100**	4 1 -3	6 -1 -7	0 -3 -3	100	$X_1 = 0$
S_2	7 5 -2	**3** **80**	8 1 -7	0 -1 -1	80	$X_2 = 2$
S_3	**6** **10**	**4** **30**	5 2 -3	**0** **50**	90	$X_3 = 3$
S_4	7 6 -1	5 4 -1	**2** **60**	**0** **60**	120	$X_4 = 3$
Demand (B_j)	110	110	60	110	∑ 390\390	
Values (Y_j)	$Y_1 = 3$	$Y_2 = 1$	$Y_3 = -1$	$Y_4 = -3$		

Table: 3.1.9

3.2 Example#2:- The Unbalanced–TP of case (ii) Deficiency in accessibility $\sum_i \mathbf{A_i} < \sum_j \mathbf{B_j}$.

Sources\Destinations	D_1	D_2	D_3	D_4	Supply (A_i)
S_1	10	15	12	12	200
S_2	8	10	11	9	150
S_3	11	12	13	10	120
Demand (B_j)	140	120	80	220	∑ 560\470

Table: 3.2.1

Sources\Destinations	D_1	D_2	D_3	D_4	Supply (A_i)
S_1	10	15	12	12	200
S_2	8	10	11	9	150
S_3	11	12	13	10	120
Dummy row	0	0	0	0	90
Demand (B_j)	140	120	80	220	∑ 560\560

Table: 3.2.2

Sources\Destinations	D_1	D_2	D_3	D_4	Supply (A_i)	Row penalty (P_i)
S_1	**10** **140**	15	12	12	200 60	(2)
S_2	8	10	11	9	150	(1)
S_3	11	12	13	10	120	(1)
Dummy row	0	0	0	0	90	
Demand (B_j)	140 0	120	80	220	∑ 420\420	

Table: 3.2.3

Sources\Destinations	D_2	D_3	D_4	Supply (A_i)	Row penalty (P_i)
S_1	15	**12** **60**	12	60 0	(3)
S_2	10	11	9	150	(1)
S_3	12	13	10	120	(2)
Dummy row	0	0	0	90	
Demand (B_j)	120	80 20	220	∑ 360\360	

Table: 3.2.4

Sources\Destinations	D_2	D_3	D_4	Supply (A_i)	Row penalty (P_i)
S_2	10	11	9	150	(1)
S_3	12	13	**10** **120**	120 0	(2)
Dummy row	0	0	0	90	
Demand (B_j)	120	20	220 100	∑ 240\240	

Table: 3.2.5

Sources\Destinations	D_2	D_3	D_4	Supply (A_i)	Row penalty (P_i)
S_2	10	11	**9** **100**	150 50	(1)
Dummy row	0	0	0	90	
Demand (B_j)	120	20	100 0	∑ 140\140	

Table: 3.2.6

Sources\Destinations	D_2	D_3	Supply (A_i)	Row penalty (P_i)
S_2	**10** **50**	11	50 0	(1)
Dummy row	0	0	90	
Demand (B_j)	120 70	20	∑ 90\90	

Table: 3.2.7

Sources\Destinations	D_2	D_3	Supply (A_i)
Dummy row	**0** **70**	**0** **20**	90 0
Demand (B_j)	70 0	20 0	∑ 0\0

Table: 3.2.8

$U_{11} = 140$, $U_{13} = 60$, $U_{34} = 120$, $U_{24} = 100$, $U_{22} = 50$, $U_{42} = 70$, & $U_{43} = 20$.

Tcost, $Z = (10*140) + (12*60) + (10*120) + (9*100) + (10*50) + (0*70) + (0*20) = 4{,}720$/-

Optimality test, the number of allocations is equal to $(m + n - 1) = 4 + 4 - 1 = 7$. Hence all $d_{ij} \leq 0$ the solution given below is optimal.

Sources\Destinations	D_1	D_2	D_3	D_4	Supply (A_i)	Values (X_i)
S_1	**10** **140**	15 12 -3	**12** **60**	12 11 -1	200	$X_1 = 0$
S_2	8 8 0	**10** **50**	11 10 -1	**9** **100**	150	$X_2 = -2$
S_3	11 9 -2	12 11 -1	13 11 -2	**10** **120**	120	$X_3 = -1$
Dummy row	0 -2 -2	**0** **70**	**0** **20**	0 -1 -1	90	$X_4 = -12$
Demand (B_j)	140	120	80	220	∑ 560\560	
Values (Y_j)	$Y_1 = 10$	$Y_2 = 12$	$Y_3 = 12$	$Y_4 = 11$		

Table: 3.2.9

4. COMPARISON OF RESULTS TABLE & DISCUSSION:

Consider the **Table 4.1.1**, we have solved the different real life problems by the proposed method. The following examples of case (i) 1, 3, 5 and case (ii) 2, 4, 6 are solved and several others, selected from literature. Also tested the performance of proposed method in comparison to NWC–Method, LC–Method & MODI–Method, and it can be clearly seen that the proposed method is same as the optimal results.

Numerical Examples	Rows	Columns	NWC–Method	LC–Method	Proposed–Method	Optimal Solution–MODI Method
Ex:1 [Ref: 8]	4	4	1,010	990	**840**	**840**
Ex:2 [Ref: 3]	4	4	5,070	5,260	**4,720**	**4,720**
Ex:3 [Ref: 6]	4	6	19,700	12,100	**11,500**	**11,500**
Ex:4 [Ref: 9]	4	4	3,550	3,225	**3,100**	**3,100**
Ex:5 [Ref: 1]	2	4	31,875	20,875	**17,875**	**17,875**
Ex:6 [Ref: 4]	5	3	14,140	12,550	**11,720**	**11,720**

Table: 4.1.1

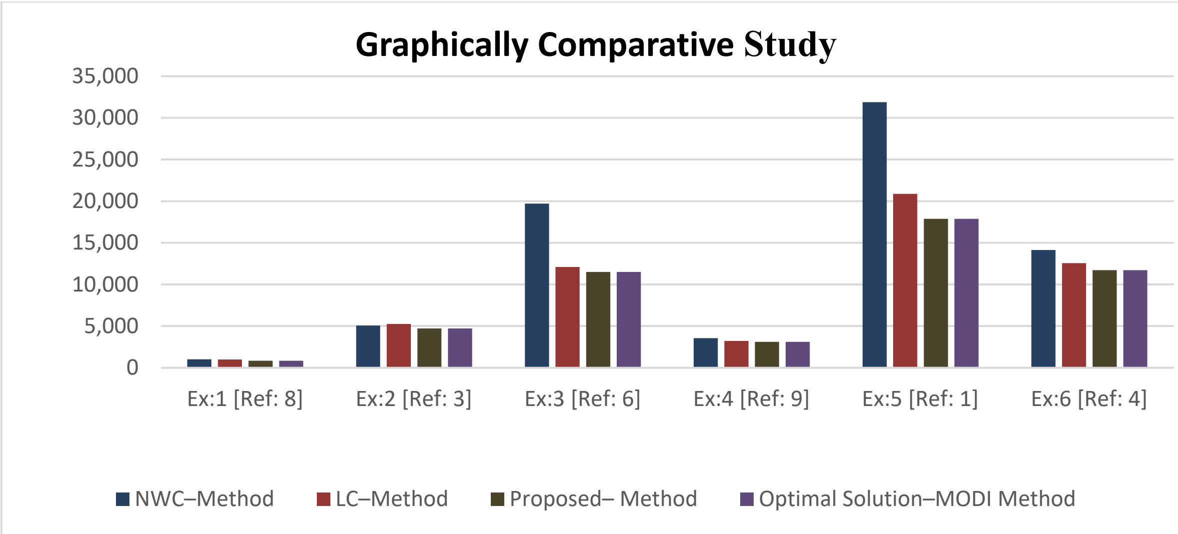

Fig: 1 Results are summarized in Table 4.1.1.

5. CONCLUSION

In this paper, we have promoted an improved algorithm for obtaining better optimal solution of unbalanced transportation problems. According to work computation, the proposed method is easy and less arithmetical calculation required to get optimal solution compared to existing MODI–Method. Also a comparison of proposed algorithm is made with NWC–Method, LC–Method and MODI–Method. Various numerical examples of different – sizes of both the cases are tested and critically observed that the proposed algorithm provide optimal solution directly.

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