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SQUARE ROOT OF NON–SQUARE NUMBERS

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ABSTRACT- In this paper an algorithm for finding the square root of non-square numbers with higher precision is developed. This high precision algorithm is designed to help in processing applications and implementation of concrete design of various processors/circuits involving non square numbers.

Keywords: Square roots; Non-square numbers; Algorithm.

INTRODUCTION

Square root of non-square numbers is an important arithmetic operation which poses a lot of problem both to students, designers and programmers in different fields of study. The popularity of square root of numbers and the recent publications in this area has created the need to develop the algorithm for solving square root of non-square numbers with higher precision which is believed would solve the problem of designers of circuits and programmers a like. A comprehensive list of references on various square root algorithms for solving square root of circuits has been presented in this paper.

There are several different methods for determining the square root of a number, some giving a rough approximation, others giving an exact value. Some of this methods include using Prime Factorization, Long division method, Newton method , Digit by digit method, Babilonian method, Rough Estimation method, Restoring method, and Non-restoring method (Jena and Panda,2016), Literature on square root of non-square numbers include the following: Arpiia et. al.(2016), Panda et al (2016), Ainbedkar and Becli (2011), Kaur et al (2014), O'Eeary et al.(1994), Ghalle et al. (2014), Higham (1986), Olga (2009), Siba abd Sahu (2015), Taek-Jen et al. (2007), Sutikno (2011), Tatar et al. (2010). In this work, an algorithm have been developed for solving any problem involving square root of non-square numbers.

1. NON-SQUARE NUMBERS

Any whole number that can be expressed as a square of an integer is referred to as a square number. Example of square numbers include - $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$ and so on. It is quite easy to find the square root of a square number without using calculator. For example $\sqrt{4} = 2$ *since* $2 \times 2 = 4$; $\sqrt{9} = 3$ *since* $3 \times 3 = 9$

A non-square number is the whole number that cannot be expressed as a square of an integer. Examples of non-square numbers are 2, 3, 5, 6 and so on. It is not so easy to find the square root of a non- square number without using calculator.

2. TECHNIQUECS FOR FINDING THE SQUARE ROOT OF NON-SQUARE NUMBER

However, some mathematicians have developed techniques on how to estimate the square root of such numbers. The commonly used method for estimating square root of non-square numbers is

$$\sqrt{Z} \simeq x + \frac{Z - x^2}{2x} \qquad (1)$$

Where x^2 is the next square number less than Z and $x = \sqrt{x^2}$

For example, using equation (1), Let us find $\sqrt{5}$

Solution

The next square numberless than 5 is 4, therefore

$$\sqrt{5} = Z + \frac{5-4}{2 \times 2} = 2 + 0.25 = 2.25$$

Check

Multiplying $2.25 \times 2.25 = 5.0625 > 5$ meaning that the formula in (1) over estimates the square root of 5.

Let us also find the square root of 15 using the same formula. Here the next square number less than 15 is 9 meaning that $Z = 15$, $X^2 = 9$ and $X = \sqrt{X^2} = \sqrt{9} = 3$,

Substituting into (1) above gives

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$\sqrt{15} = 3 + \frac{15-9}{3\times2} = 3 + 1 = 4$ therefore $\sqrt{15}$ is estimated to be 4.

Check

Multiplying 4 by itself gives 16 which is greater than 15. This means that the formula over estimates the square root of 15. Generally, the above formula provides an upper bound for the square root Z.

ALTHERNATIVE METHOD

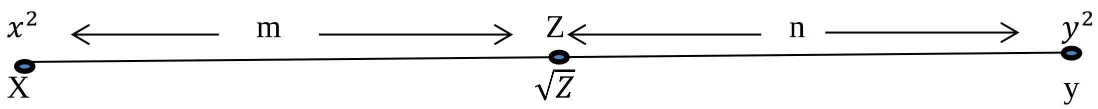
Suppose we wish to find the square root of a non-square number Z. The alternative procedure is as follows

Step 1

- Find the next square number $x^2 < Z$
- Find also the next square number $y^2 > Z$

Step II

- Find the distance $m = Z - X^2$ and the distance $n = y^2 - Z$. Let us demonstrate this on a number line



Then the distance from $z - x^2 = m \quad while \quad y^2 - z = n$

Step III

Estimate the square root of Z using the interpolation formula given below;

$$\sqrt{Z} \simeq \frac{m \times y + n \times x}{m+n} \tag{2}$$

Or

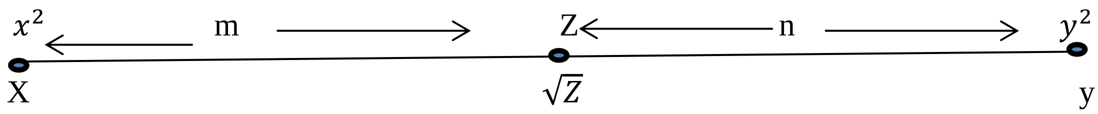
$$\sqrt{Z} \simeq x + \frac{m}{m+n} \tag{3}$$

This is called lower interpolation formula because x is the square root of the next square number less than Z

Or

$$\sqrt{Z} \simeq y - \frac{n}{m+n} \qquad (4)$$

This is called the upper interpolation formula since y is the square roots of the next square number greater than Z



The above formulas provides a lower bound for square root of Z

Example I

Find $\sqrt{10}$

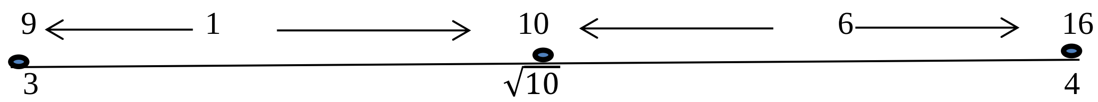
Solution

The next square number less than 10 is 9 and the next square number greater 10 is 16 and

$x^2 = 9,\ y^2 = 16, \qquad Z = 10$

Then the distance m is m = 10 – 9 = 1 and the distance n is n = 16-10 = 6

Showing this on a number line gives



So the total distance from 9 to 16 is $m+n=1+6=7$ so we can therefore estimate the $\sqrt{10}$ to be

$$\sqrt{10} = \frac{1\times4+6\times3}{7} = \frac{22}{7} = 3.143$$

Or

Using the lower interpolation formula

$$\sqrt{Z} = x + \frac{n}{m+n}$$

We can also find the $\sqrt{10}$ as follows

$$\sqrt{10} = 3 + \frac{1}{7} = 3 + 0.143 = 3.143$$

Or

Using the upper interpolation formula

$$\sqrt{Z} = y - \frac{n}{m+n}$$

$$\sqrt{10} = 4 - \frac{6}{7} = 4 – 0.857 = 3.143$$

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Check

Multiply 3.143 by itself gives 3.143x 3.143 = 9.878 <10

This implies that the above formulas under estimate the square root of 10

Proposition

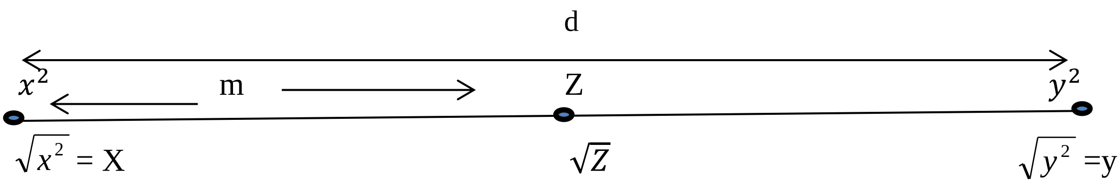
The formula

$$\sqrt{Z} \simeq x + \frac{2m}{2d-1} \tag{5}$$

Where

m = Z –x^2 and x^2 is the next square number less than Z, d = y^2 – x^2 and y^2 is the next square number greater than Z, provides a better estimate than the proceeding one.

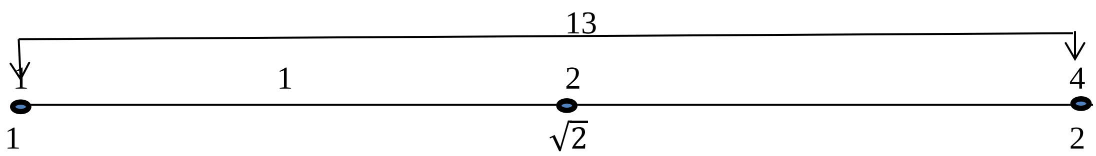
Illustrating this on a number line gives



d is the distance between x^2 and y^2 .For example using our proposed formula to find square root of some non-square numbers

$$\sqrt{Z} = x + \frac{2m}{2d-m}$$

$$\sqrt{2} = 1 + \frac{2x1}{2x3-1} = 1+0.4 = 1.4$$



Check

1.4 x1.4 = 1.96 < 2, meaning that our proposed formula also under estimate the $\sqrt{2}$. Also using the formula to find $\sqrt{5}$ gives

$$\sqrt{5} = 2+ \frac{2x1}{2x5-1} = 2+ 0.\ 222 = 2.222$$

Check

2.222 x 2.222 = 4.9372

Meaning that the formula also under estimate the square root of 5.

Also let us use the formula to find the square root of 10

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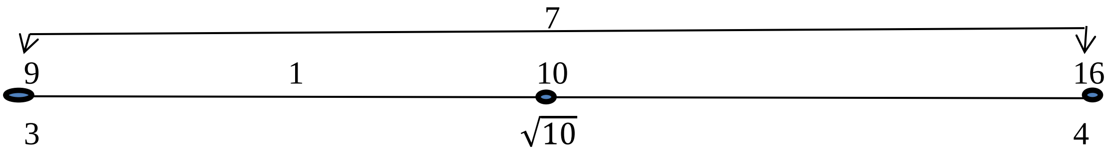
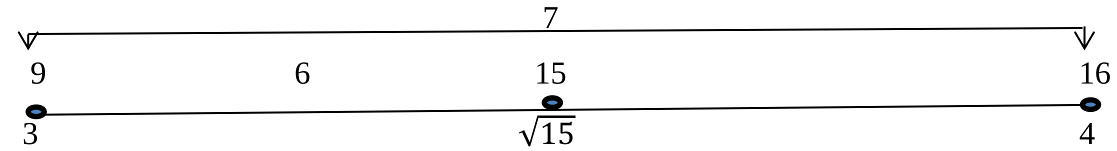


$\sqrt{10} = 3+ \frac{2x1}{2x7-1}$ 3 +0.154 = 3.154

Check

3.154 x 3.154 = 9.9477 < 10

Which implies that the formula also index estimates the square root of 10



$\sqrt{15} = 3+ \frac{2x6}{2x7-1}$ 3 +0.932 = 3.932

Check

3.932 x3.932 = 15.461 > 15

This implies that our formula over estimates the square root of 15

ALGORITHM FOR FINDING ACCURATE SQUARE ROOT OF NON-SQUARE NUMBERS

Step I

- Find the lower bound L of $\sqrt{Z}$
- And the upper bound U of $\sqrt{Z}$

Step II

- Find the average A of L and U obtained in step I
- i.e A $\simeq \frac{L+U}{2}$

Step III

- If $A^2 <$ Z, then set the lower bound L = A and repeat step II
- If $A^2 >$Z, then set U = A and repeat step II
- If $A^2 \simeq$ Z (to a given number of accuracy then stop and take the $\sqrt{Z} \simeq$ A.

Example

Let's find the square root of 10 so that its square is approximately equal to 10 to 3 decimal places

Solution

Using the formula

$$\sqrt{Z} \simeq x + \frac{m}{m+n}$$

L = 3 + $\frac{1}{7}$ = 3+ 0.143 = 3.143

Also using thee formula

$$\sqrt{Z} \simeq x + \frac{Z-x^2}{2x}$$

We obtain the happen bound U = 3 + $\frac{1}{6}$ = 3+ 0167 = 3.167

Now average the two given

A = $\frac{3.143+3.167}{2}$ = 3.155

Check

3.155 +3.155 = 9.954 < 10

This indicates that 3.155 should be our new lower bound so computing A with L = 3.155

our new A = $\frac{3.155+3.167}{2}$ = 3.161

Check

3.161 x 3.161 = 9.992 < 10, indicating that 3.161 is the new lower bound, so our new A is

A = $\frac{3.161+3.167}{2}$ = 3.164

Check

3.164 x 3.164 = 10.011 > 10, indicating that 3.164 is the new upper bound. So our A will now be A = $\frac{3.161+3.164}{2}$ = 3.163

Check

3.163 x 3.163 = 10.005 > 10

Indicating that 3.163 is a new upper bound, so continuing that way we can generate the table below;

S/N	LB	UB	A	A2
1	3.155	3.167	3.161	9.992
2.	3.161	3.167	3.164	10.011
3.	3.161	3.164	3.163	10.005
4.	3.161	3.163	3.162	9.998
5.	3.162	3.163	3.1625	10.001
6.	3.162	3.1625	3.16225	10.000

So stop and conclude that the square root of 10 is appropriately 3.16225

i.e $\sqrt{10} \simeq 16225$

Check

3.16225 x 3.16225 = 10.000

CONCLUSION

With the steps/algorithm stated in this work, different types of problems involving non-square numbers can be solved without stress by all students. This work is deem to helping researchers understand and implement solutions to solving problems with non-square numbers for designing various square root processors or circuits. A lot of limitations in the area of square root of non-square numbers will be solved by the hybrid algorithm developed in this work.

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