

Exponentiated Inverse Power Pranav distribution: Properties and Application

Kingsley C. Nwankwo[1], Onyeagu Sidney I[2]., Nwankwo C. H[3], George A. Osuji[4], Chrisogonus K. Onyekwere[5].

[1]Department of Statistics, Nnamdi Azikiwe University Awka, Nigeria
2 Department of Statistics, Nnamdi Azikiwe University Awka, Nigeria
[3]Department of Statistics, Nnamdi Azikiwe University Awka, Nigeria
[4]Department of Statistics, Nnamdi Azikiwe University Awka, Nigeria
[5]Department of Statistics, Nnamdi Azikiwe University Awka, Nigeria

Corresponding author: chrisogonusjohnson@gmail.com

ABSTRACT

In this article, we proposed a new distribution known as the Exponentiated Inverse Power Pranav distribution for modeling lifetime data sets with monotone and non-monotone shapes in their hazard rates. Along with some of the basic properties, we however, studied the maximum likelihood estimation of the parameters of the proposed distribution. The model was subjected to life application with a dataset and compared to other sub-models. The new distribution was found to have a best fit more than the competing sub-models.

Keywords: Pranav distribution, Inverse Power Pranav distribution, Exponentiated distributions, Maximum Likelihood estimation, Exponentiated Inverse Power Pranav distribution

1 Introduction

In statistics, specifically survival analysis, many parametric models have been developed for modeling survival data. Notable among these models are the famous and widely employed Exponential, Weibull, Lindley and Gamma distributions. These models have their area of applications ranging from biological sciences, engineering, insurance, finance, epidemiology, demography amongst others. The inevitable and inherent instability and failure of systems and the need to measure the expected duration of time until events occur as well as system reliability gave birth to formulation of models. Data from these fields are regarded as lifetime data and can take bathtub or upside down bathtub shapes for their hazard rates. Since events naturally can have monotone or non-monotone shape, it is pertinent to formulate a flexible model that can assist to describe and predict such events. Numerous literatures exist on this, but none has been able to get exactly a model flexible enough to characterize the behavior of these events, specifically, events with non-monotone behavior in hazard rates. Few among the numerous literatures are the articles recently written by Dimitrakopoulou *et al* (2007), Vikas *et al* (2014), Vikas *et al* (2015), Eliwa *et al* (2018), Rameesa *et al* (2018), Amal *et al* (2019), Onyekwere *et al* (2020), Enogwe *et al* (2020) amongst others. Shukla (2018) followed the arguments of some literatures as stated in Lindley (1958), Shanker (2015) and Ghitany *et al* (2008), hence claim that he could develop a new life time distribution which may be better than Lindley, Exponential, Ishita, Shanker, Sujatha and Akash distribution. To buttress his claim, he proposed a new lifetime distribution known as Pranav distribution that can be apply to biological data.

The random variable Y is said to have Pranav distribution (PD) with scale and shape parameter $(\alpha, 4)$ if its probability density function (pdf) and cumulative density function (cdf) are defined by

$$f(y,\alpha)=\frac{\alpha^4}{\alpha^4+6}\left(\alpha+y^3\right)e^{-\alpha y}; y>0, \alpha>0 \tag{1}$$

$$F(y,\alpha)=1-\left[1+\frac{\alpha y\left(\alpha^2 y^2+3\alpha y+6\right)}{\alpha^4+6}\right]e^{-\alpha y}, y>0, \alpha>0 \tag{2}$$

As stated earlier, in practical situation, classical distributions do not always offer sufficient fits to lifetime datasets, thus giving rise to the need for more supple models. One of the limitations of Pranav distribution is its inability to model effectively datasets that have monotone or non-monotone shape in their hazard rate. In order to take care of this problem, the foremost goal of the author in this article is to propose a new distribution flexible enough to model adequately datasets with such characteristics. Authors have written several articles on Pranav distribution. See Uwaeme *et al* (2018), Umeh *et al* (2019), Odom *et al* (2019), Wani *et al* (2020) and so on. These extensions of Pranav distribution were developed to model specific situations thus may not fit well a dataset that has bathtub curve or upside down bathtub curve.

The rest of the paper is organized as follows: section 2 introduces the Inverse Power Pranav distribution, section 3 introduce also the Exponentiated Inverse Power Pranav distribution, section 4 presents the mathematical and statistical characteristics of Exponentiated Inverse Power pranav distribution, section 5 presents the maximum likelihood estimation, section 6 contains the applications to datasets and in section 7, we conclude the article.

2 Inverse Power Pranav distributions

Definition 2.1: A random variable X is said to have an Inverse Power Pranav distribution if the p.d.f. and c.d.f. are respectively given as

$$f_{IP}\left(x,\alpha,\beta\right)=\frac{\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}e^{-\alpha x^{-\beta}};x>0,\alpha,\beta>0 \tag{3}$$

$$F_{IP}\left(x,\alpha,\beta\right)=\left\{1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right\}e^{-\alpha x^{-\beta}};x>0,\alpha,\beta>0 \tag{4}$$

Proof: Given the random variable Y from a one parameter Pranav distribution defined in (1). Assume that another random variable X is related to Y by an inverse power function $X=g\left(Y\right)=Y^{-\frac{1}{\beta}}$. Suppose the observed value of X is denoted by $x=y^{-\frac{1}{\beta}}$, then $x^{\beta}=y^{-1}$, then $y=x^{-\beta}$ and $\frac{dy}{dx}=-\beta x^{-(\beta+1)}$. Putting $y=x^{-\beta}$ into (1) we obtain

$$f\left(g^{-1}\left(x\right)\right)=\frac{\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)e^{-\alpha x^{-\beta}};x>0,\alpha,\beta>0 \tag{5}$$

As specified in Hogg *et al.* (2019), the probability density function of a continuous random variable X could be obtained using the relation

$$f\left(x\right)=f\left(g^{-1}\left(x\right)\right)\left|\frac{dy}{dx}\right| \tag{6}$$

Thus, the probability density function $f\left(x\right)$ of the Inverse Power Pranav distribution can be obtained by substituting (5) and $\frac{dy}{dx}=-\beta x^{-(\beta+1)}$ into (6). Thus, we obtain

$$f\left(x,\alpha,\beta\right)=\frac{\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}e^{-\alpha x^{-\beta}};\ x>0,\alpha,\beta>0$$

Similarly, the cumulative density function (cdf) of the inverse power Pranav distribution (IPP) is derived as follows

$$F\left(x,\alpha,\beta\right)=P\left[X\le x\right]=\int_0^x f\left(x,\alpha,\beta\right)dx \tag{7}$$

$$=\frac{\beta\alpha^4}{\alpha^4+6}\int_0^x\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}e^{-\alpha x^{-\beta}}dx$$

$$=\frac{\beta\alpha^4}{\alpha^4+6}\left\{\alpha\int_0^x x^{-(\beta+1)}e^{-\alpha x^{-\beta}}dx+\int_0^x x^{-(4\beta+1)}e^{-\alpha x^{-\beta}}dx\right\} \tag{8}$$

By transformation technique, we let $y=x^{-\beta}$. When $x=0, y=\infty$ and when $x=x, y=x^{-\beta}$

Applying this in (8), we obtain

$$F(x,\alpha,\beta)=\frac{\alpha^4}{\alpha^4+6}\left\{\alpha\int_{x^{-\beta}}^{\infty}e^{-\alpha y}dy+\int_{x^{-\beta}}^{\infty}y^3e^{-\alpha y}dy\right\} \tag{9}$$

Using the integration by parts method, (9) can be written as

$$F(x,\alpha,\beta)=\frac{\alpha^4}{\alpha^4+6}\left\{e^{-\alpha x^{-\beta}}+\frac{x^{-3\beta}e^{-\alpha x^{-\beta}}}{\alpha}+\frac{3x^{-2\beta}e^{-\alpha x^{-\beta}}}{\alpha^2}+\frac{6x^{-\beta}e^{-\alpha x^{-\beta}}}{\alpha^3}+\frac{6e^{-\alpha x^{-\beta}}}{\alpha^4}\right\} \tag{10}$$

Consequently, the cumulative distribution function of the inverse power Pranav distribution is

$$F(x,\alpha,\beta)=\left\{1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right\}e^{-\alpha x^{-\beta}};x>0,\alpha,\beta>0$$

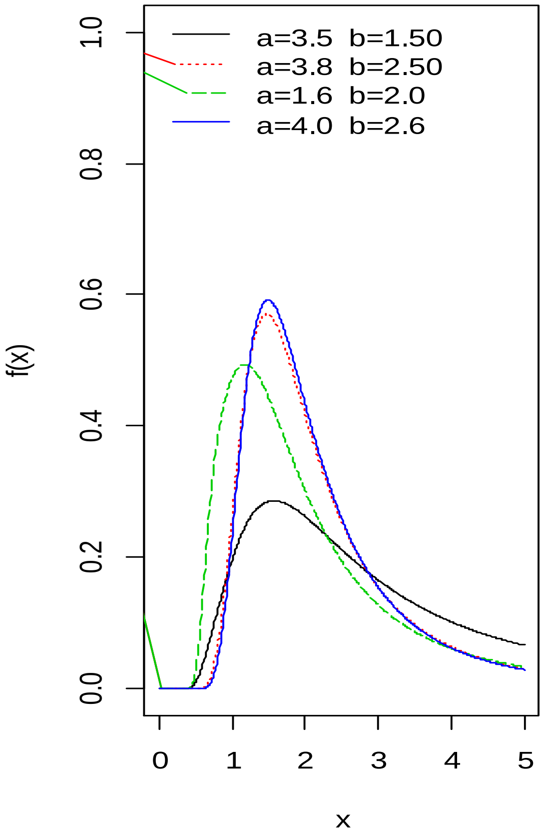

Fig 1a:pdf plot of IPP

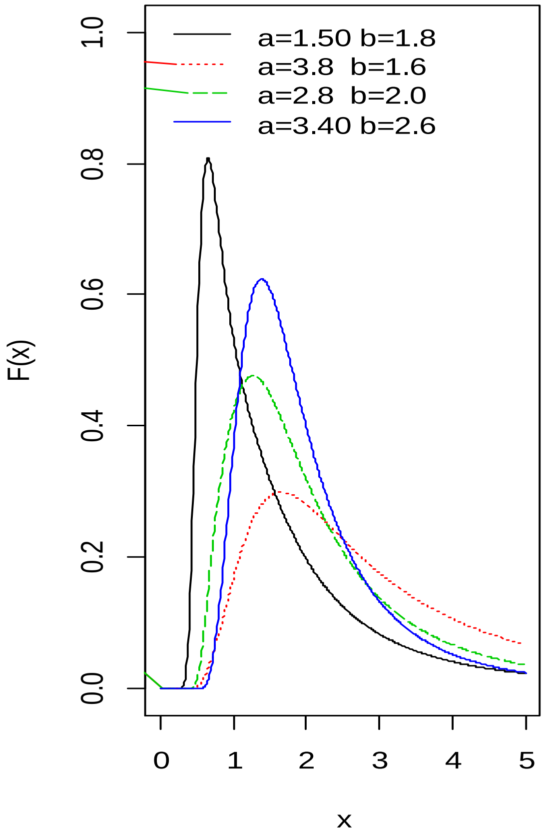

Fig 1b:cdf plot of EIPP

Figures 1a and1b show the plot of the pdf and cdf of the Inverse power Pranav distribution for varying values of the parameters α and β .

3 Exponentiated Inverse Power Pranav Distribution

Definition 3.1: Let X be a ranom variable with pdf and cdf defined in (3) and (4) above, then, the cdf and pdf of the Exponentiated Inverse Power Pranav distribution are respectively given by

$$G_p(x,\lambda)=\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda};x>0,\alpha,\beta,\lambda>0 \tag{11}$$

$$g_p(x,\lambda)=\frac{\lambda\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}e^{-\alpha x^{-\beta}};\ x,\alpha,\beta,\lambda>0 \tag{12}$$

Proof: A random variable X is said to follow an Exponentiated distribution if the cumulative density function cdf and probability density function pdf are given by

$$G_p(x,\lambda)=\left[F(x,\lambda)\right]^{\lambda};\square\in\Re^{|},\lambda\geq 0 \tag{13}$$

$$g_p(x,\lambda)=\lambda\left[F(x,\lambda)\right]^{\lambda-1}f(x,\lambda),\square\in\Re^{|},\lambda\geq 0 \tag{14}$$

Consequently, the cdf and the pdf of the new Exponentiated Inverse Power Pranav distribution (EIPP) are obtained using equations (13) and (14) above. Thus

$$G_p(x,\lambda)=\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda};x>0,\alpha,\beta,\lambda>0$$

$$g_p(x,\lambda)=\frac{\lambda\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}e^{-\alpha x^{-\beta}};\ x,\alpha,\beta,\lambda>0$$

Figures 2a, 2b, 2c and 2d show the plot of the pdf and cdf of the Exponentiated Inverse power Pranav distribution for varying values of the parameters α,β and λ .

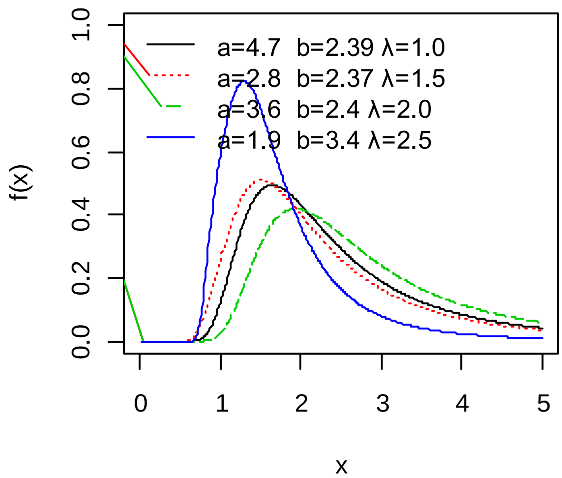

Fig 2a:pdf plot of EIPP

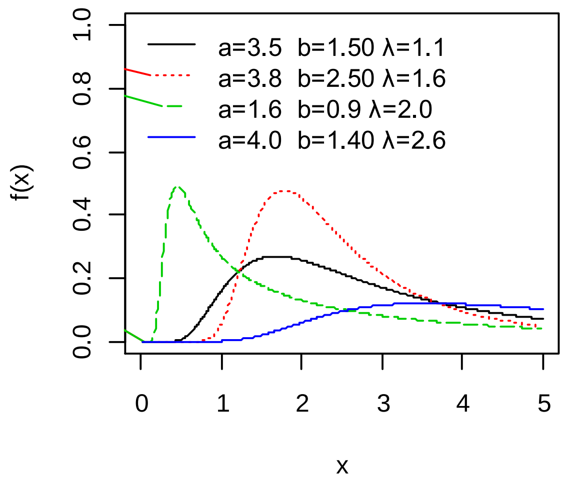

Fig 2b:pdf plot of EIPP

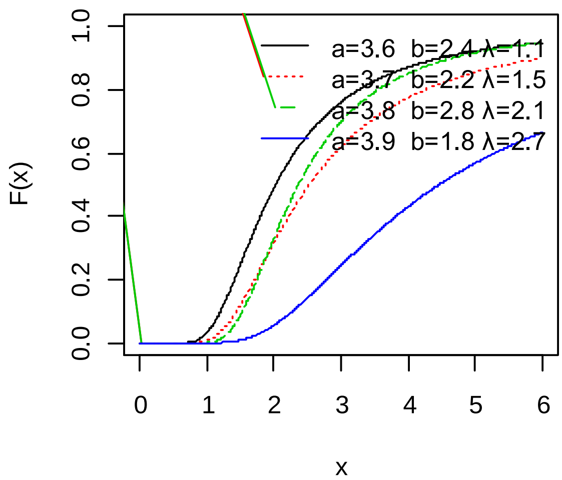

Fig 2c:cdf plot of EIPP

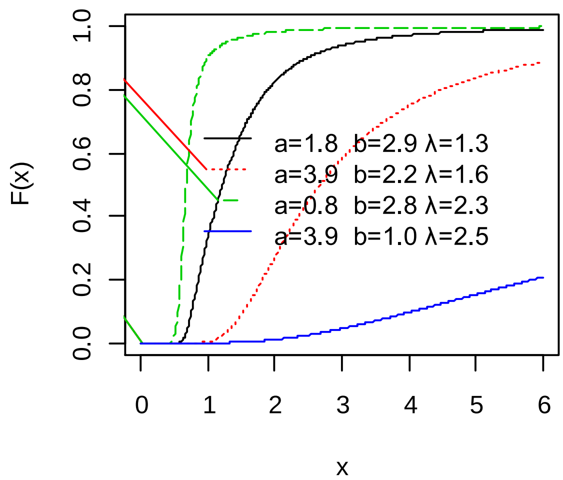

Fig 2d:cdf plot of EIPP

4 Mathematical and Statistical Characteristics of Exponentiated Inverse Power Pranav distribution

4.1 Moments

Definition 4.1: Suppose X is a random variable which has the cdf and pdf defined in equations (11) and (12) above, then the rth moment about the origin, $E\left(X^r\right)$ is given by

$$E\left(X^r\right)=\xi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+1\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+1)}}+\psi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+4\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+4)}} \tag{15}$$

Where

$$\xi_{i,j,k}=\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+5}3^j2^k}{\left(\alpha^4+6\right)^{i+1}}$$

$$\psi_{i,j,k}=\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+4}3^{j}2^{k}}{\left(\alpha^{4}+6\right)^{i+1}}$$

Proof: The rth moment of a random variable X is given by

$$E\left(X^{r}\right)=\int_{0}^{\infty}x^{r}g_{p}\left(x,\alpha,\theta,\lambda\right)dx \tag{16}$$

$$E\left(X^{r}\right)=\frac{\alpha^{4}\lambda\beta}{\alpha^{4}+6}\int_{0}^{\infty}x^{r}\left(\alpha x^{-(\beta+1)}+x^{-(4\beta+1)}\right)e^{-\alpha x^{-\beta}}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}$$

$$=\left\{\begin{array}{l}\dfrac{\alpha^{5}\lambda\beta}{\alpha^{4}+6}\displaystyle\int_{0}^{\infty}x^{-\beta+r-1}e^{-\alpha x^{-\beta}}\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}dx\\ +\dfrac{\alpha^{5}\lambda\beta}{\alpha^{4}+6}\displaystyle\int_{0}^{\infty}x^{-4\beta+r-1}e^{-\alpha x^{-\beta}}\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}dx\end{array}\right\} \tag{17}$$

By employing the series expansions

$$\left(1+z\right)^{n}=\sum_{j=0}^{\infty}\binom{n}{j}z^{j},$$

(17) becomes

$$E\left(X^{r}\right)=\left\{\begin{array}{l}\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\dfrac{\lambda\beta\alpha^{3i-j-k+5}3^{j}2^{k}}{\left(\alpha^{4}+6\right)^{i+1}}\displaystyle\int_{0}^{\infty}x^{r+j\beta+\beta k-\beta-3i\beta-1}e^{-\alpha(1+i)x^{-\beta}}dx\\ +\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\dfrac{\lambda\beta\alpha^{3i-j-k+4}3^{j}2^{k}}{\left(\alpha^{4}+6\right)^{i+1}}\displaystyle\int_{0}^{\infty}x^{r+j\beta+\beta k-4\beta-3i\beta-1}e^{-\alpha(1+i)x^{-\beta}}dx\end{array}\right\} \tag{18}$$

If we let $y=x^{-\beta}$. By transformation techniques, we obtain the following

$$E\left(X^{r}\right)=\left\{\begin{array}{l}\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\dfrac{\lambda\alpha^{3i-j-k+5}3^{j}2^{k}}{\left(\alpha^{4}+6\right)^{i+1}}\displaystyle\int_{0}^{\infty}y^{3i-j-k-r/\beta}e^{-\alpha(1+i)y}dy\\ +\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\dfrac{\lambda\alpha^{3i-j-k+4}3^{j}2^{k}}{\left(\alpha^{4}+6\right)^{i+1}}\displaystyle\int_{0}^{\infty}y^{3i-j-k-r/\beta+3}e^{-\alpha(1+i)y}dy\end{array}\right\} \tag{19}$$

Recall that

$$\int_{0}^{\infty}x^{n}e^{-ax}dx=\frac{\Gamma\left(n+1\right)}{a^{n+1}}$$

thus, we have

$$E\left(X^r\right)=\left\{\begin{array}{l}\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+5}3^j2^k}{\left(\alpha^4+6\right)^{i+1}}\frac{\Gamma\left(3i-j-k-r/\beta+1\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+1)}}\\+\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+4}3^j2^k}{\left(\alpha^4+6\right)^{i+1}}\frac{\Gamma\left(3i-j-k-r/\beta+4\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+4)}}\end{array}\right\}$$

Let $\xi_{i,j,k}=\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+5}3^j2^k}{\left(\alpha^4+6\right)^{i+1}}$

and $\psi_{i,j,k}=\sum_{i=0}^{\infty}\binom{\lambda-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{j}{k}\frac{\lambda\alpha^{3i-j-k+4}3^j2^k}{\left(\alpha^4+6\right)^{i+1}}$

Consequently, the rth moment of Exponentiated Inverse Power Pranav distribution becomes

$$E\left(X^r\right)=\xi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+1\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+1)}}+\psi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+4\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+4)}}$$

4.2 Moment generating function of EIPP distribution

Definition 4.2: Given a random variable X, such that $X\sim EIPPD\left(\alpha,\beta,\lambda\right)$, the moment generating function is given by

$$M_x\left(t\right)=\sum_{l=0}^{\infty}\frac{t^l}{l!}\left\{\xi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+1\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+1)}}+\psi_{i,j,k}\frac{\Gamma\left(3i-j-k-r/\beta+4\right)}{\left(\alpha\left(i+1\right)\right)^{(3i-j-k-r/\beta+4)}}\right\} \quad (20)$$

Proof: The moment generating function of a random variable X , is given by

$$M_x\left(t\right)=E\left(e^{tX}\right)=\int_0^{\infty}e^{tx}f_p\left(x,\alpha,\beta,\lambda\right)dx \quad (21)$$

Using Taylor's series, we obtain the following

$$M_x\left(t\right)=\int_0^{\infty}\left(1+tx+\frac{(tx)^2}{2!}+\cdots\right)f_p\left(x,\alpha,\beta,\lambda\right)dx$$

$$=\int_0^{\infty}\sum_{l=0}^{\infty}\frac{t^l}{l!}x^l f\left(x,\alpha,\beta,\lambda\right)dx$$

$$=\sum_{l=0}^{\infty}\frac{t^l}{l!}\int_0^{\infty}x^l f\left(x,\alpha,\beta,\lambda\right)dx=\sum_{l=0}^{\infty}\frac{t^l}{l!}E\left(X^l\right)$$

Where $E\left(X^l\right)=E\left(X^r\right)$. Thus, the mgf of EIPP distribution becomes

$$M_x(t)=\sum_{l=0}^{\infty}\frac{t^l}{l!}\left\{\xi_{i,j,k}\frac{\Gamma(3i-j-k-r/\beta+1)}{(\alpha(i+1))^{(3i-j-k-r/\beta+1)}}+\psi_{i,j,k}\frac{\Gamma(3i-j-k-r/\beta+4)}{(\alpha(i+1))^{(3i-j-k-r/\beta+4)}}\right\}$$

4.3 Distribution of order statistics

Definition 4.3: Suppose $x_1,x_2,\cdots,x_n$ are a random sample of size n drawn from exponentiated inverse power Pranav distribution. Also, if we let $x_{(1)}<x_{(2)}<\cdots<x_{(n)}$ denote, the corresponding order statistics. The pdf and the cdf of the pth order statistics, say $Y=X_{(p)}$ are given by

$$f_X(x)=\left\{\begin{array}{l}\dfrac{\lambda\beta n!\left(\alpha+x^{-3\beta}\right)e^{-\alpha(1+j)x^{-\beta}}}{\left(\alpha^4+6\right)^{j+1}(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i\sum_{j=0}^{\infty}\binom{\lambda(p+i)-1}{j}\\ \times\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\dfrac{\alpha^{3j-k-l+4}3^k2^l}{\left(\alpha^4+6\right)^j}x^{-\beta(3j-k-l)}\end{array}\right\} \tag{22}$$

$$F_X(x)=\left\{\begin{array}{l}\sum_{j=p}^{n}\binom{n}{j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^k\sum_{l=0}^{\infty}\binom{\lambda(j+k)}{l}\sum_{m=0}^{l}\binom{l}{m}\\ \times\sum_{\tau=0}^{m}\binom{m}{\tau}\dfrac{\alpha^{3l-m-\tau}3^m2^\tau}{\left(\alpha^4+6\right)^l}x^{-\beta(3l-m-\tau)}e^{-\alpha lx^{-\beta}}\end{array}\right\} \tag{23}$$

Proof: The probability density function (pdf) of the kth order statistic are given by

$$f_{X_{(p)}}(x)=\frac{n!}{(p-1)!(n-p)!}\left[F(x)\right]^{p-1}\left[1-F(x)\right]^{n-p}f(x)$$

$$=\frac{n!}{(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^iF^{p+i-1}(x)f(x) \tag{24}$$

$$=\left\{\begin{array}{l}\dfrac{\lambda\beta\alpha^4n!\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}e^{-\alpha x^{-\beta}}}{(p-1)!(n-p)!\left(\alpha^4+6\right)}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^i\\ \times\left[\left(1+\dfrac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right)e^{-\alpha x^{-\beta}}\right]^{\lambda(p+i)-1}\end{array}\right\} \tag{25}$$

Using series expansion,

$$\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda(p+i)-1}$$

$$=\sum_{j=0}^{\infty}\binom{\lambda(p+i)-1}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{\alpha^{3j-k-l}3^{k}2^{l}}{\left(\alpha^{4}+6\right)^{j}}x^{-\beta(3j-k-l)}e^{-\alpha jx^{-\beta}} \tag{26}$$

Substituting (26) into (25), we obtain the pdf of the pth order statistics for EIPP distribution. Thus,

$$f_X(x)=\left\{\begin{array}{l}\dfrac{\lambda\beta n!\left(\alpha+x^{-3\beta}\right)e^{-\alpha(1+j)x^{-\beta}}}{\left(\alpha^{4}+6\right)^{j+1}(p-1)!(n-p)!}\sum_{i=0}^{n-p}\binom{n-p}{i}(-1)^{i}\sum_{j=0}^{\infty}\binom{\lambda(p+i)-1}{j}\\ \times\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\dfrac{\alpha^{3j-k-l+4}3^{k}2^{l}}{\left(\alpha^{4}+6\right)^{j}}x^{-\beta(3j-k-l)}\end{array}\right\}$$

Similarly, the cdf of the pth order statistics is given by

$$F_{X_{(p)}}(x)=\sum_{j=p}^{n}\binom{n}{j}F^{j}(x)\{1-f(x)\}^{n-j} \tag{27}$$

$$=\sum_{j=p}^{n}\binom{n}{j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^{k}F^{j+k}(x) \tag{28}$$

$$=\sum_{j=p}^{n}\binom{n}{j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^{k}\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\lambda(j+k)} \tag{29}$$

Where

$$\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\lambda(j+k)}$$

$$=\sum_{l=0}^{\infty}\binom{\lambda(j+k)}{l}\sum_{m=0}^{l}\binom{l}{m}\sum_{\tau=0}^{m}\binom{m}{\tau}\frac{\alpha^{3l-m-\tau}3^{m}2^{\tau}}{\left(\alpha^{4}+6\right)^{l}}x^{-\beta(3l-m-\tau)}e^{-\alpha lx^{-\beta}} \tag{30}$$

Putting equation (30) into (29), we obtain the cdf of the EIPP distribution. Thus,

$$F_X(x)=\left\{\begin{array}{l}\sum_{j=p}^{n}\binom{n}{j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^{k}\sum_{l=0}^{\infty}\binom{\lambda(j+k)}{l}\sum_{m=0}^{l}\binom{l}{m}\\ \times\sum_{\tau=0}^{m}\binom{m}{\tau}\dfrac{\alpha^{3l-m-\tau}3^{m}2^{\tau}}{\left(\alpha^{4}+6\right)^{l}}x^{-\beta(3l-m-\tau)}e^{-\alpha lx^{-\beta}}\end{array}\right\}$$

4.4 Quantile function of EIPP distribution

Definition 4.4: Let X be a random variable with cdf defined in (11). Then, the pth quantile function of the random variable X, denoted by $x_p=Q(p)=F^{-1}(p)$, can be obtained by inverting (11). Thus,

$$Q(p)=\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda} \tag{31}$$

$$\frac{Q_{(p)}^{1/\gamma}}{e^{-\alpha x^{-\beta}}}=1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}$$

$$Q_{(p)}^{1/\gamma}e^{\alpha x^{-\beta}}=1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}$$

$$e^{\alpha x^{-\beta}}=\frac{1}{Q_{(p)}^{1/\gamma}}+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{Q_{(p)}^{1/\gamma}\left(\alpha^4+6\right)}$$

$$\alpha x^{-\beta}=ln\left[\frac{1}{Q_{(p)}^{1/\gamma}}+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{Q_{(p)}^{1/\gamma}\left(\alpha^4+6\right)}\right]$$

$$x=\left\{\frac{1}{\alpha}ln\left[\frac{1}{Q_{(p)}^{1/\gamma}}+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{Q_{(p)}^{1/\gamma}\left(\alpha^4+6\right)}\right]\right\}^{-\frac{1}{\beta}} \tag{32}$$

The quantile function derived in (32) is beneficial for generating random numbers from the EIPP distribution.

4.5 Rényi entropy of EIPP distribution

Entropy is used to measure the unpredictability of systems and it is extensively employed in fields like physics, molecular imaging of tumours, sparse kernel density estimation, high-resolution scalar quantization, estimation of the number of components of a multi-component non-stationary signal , identification of cardiac autonomic neuropathy in diabetes and signal segmentation in time-frequency plane . A large value of entropy implies that there is greater uncertainty in the data. The Rényi, Shannon and Tsallis entropy, among others, are some different forms of entropy.

Definition 4.5: Given that X is a random that follows Exponentiated Inverse Power Pranav distribution defined in (12), the Rényi entropy is given by

$$R_e(\eta)=\frac{1}{1-\eta}\log\left\{\phi_{i,j,k,l}\frac{\Gamma\left(3j+4\eta-k-l+\frac{\eta}{\beta}-\frac{1}{\beta}\right)}{\left(\alpha(\eta+j)\right)^{\left(3j+4\eta-k-l+\frac{\eta}{\beta}-\frac{1}{\beta}\right)}}\right\} \tag{33}$$

Where

$$\phi_{i,j,k,l}=\sum_{i=0}^{\infty}\binom{\eta}{i}\sum_{j=0}^{\infty}\binom{\eta(\lambda-1)}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{\lambda^{\eta}\beta^{\eta-1}\alpha^{4\eta+3j-k-l}3^k2^l}{\left(\alpha^4+6\right)^{\eta+j}}$$

Proof: The Rényi entropy is defined for a continuous random variable X as

$$R_e(\eta)=\frac{1}{1-\eta}\log\left\{\int_x f^{\eta}(x)\,dx\right\},\eta>0\ \text{and}\ \eta\neq 0 \tag{34}$$

$$=\frac{1}{1-\eta}\log\left\{\int_0^{\infty}\left[\frac{\lambda\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}e^{-\alpha x^{-\beta}}\right]^{\eta}dx\right\}$$

$$=\frac{1}{1-\eta}\log\left\{\frac{\lambda^{\eta}\beta^{\eta}\alpha^{4\eta}}{\left(\alpha^4+6\right)^{\eta}}\int_0^{\infty}\left(\alpha+x^{-3\beta}\right)^{\eta}x^{-\eta(\beta+1)}\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\eta(\lambda-1)}e^{-\eta\alpha x^{-\beta}}dx\right\}$$

$$=\frac{1}{1-\eta}\log\left\{\begin{array}{l}\frac{\lambda^{\eta}\beta^{\eta}\alpha^{4\eta}}{\left(\alpha^4+6\right)^{\eta}}\int_0^{\infty}\alpha^{\eta}\left(1+\alpha^{-1}x^{-3\beta}\right)^{\eta}x^{-\eta(\beta+1)}\\ \times\left\{\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right)e^{-\alpha x^{-\beta}}\right\}^{\eta(\lambda-1)}e^{-\eta\alpha x^{-\beta}}dx\end{array}\right\} \tag{35}$$

Using series expansion as indicated in section of this article, (35) becomes

$$R_e(\eta)=\frac{1}{1-\eta}\log\left\{\begin{array}{l}\sum_{i=0}^{\infty}\binom{\eta}{i}\sum_{j=0}^{\infty}\binom{\eta(\lambda-1)}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{\lambda^{\eta}\beta^{\eta}\alpha^{4\eta+3j-k-l}3^k2^l}{\left(\alpha^4+6\right)^{\eta+j}}\\ \times\int_0^{\infty}x^{\beta k+\beta l-3\beta j-4\eta\beta-\eta}e^{-\alpha(\eta+j)x^{-\beta}}dx\end{array}\right\} \tag{36}$$

Letting $y=x^{-\beta}$ and applying appropriate transformations, equation (37) becomes

$$R_e(\eta)=\frac{1}{1-\eta}\log\left\{\begin{array}{l}\sum_{i=0}^{\infty}\binom{\eta}{i}\sum_{j=0}^{\infty}\binom{\eta(\lambda-1)}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{\lambda^{\eta}\beta^{\eta-1}\alpha^{4\eta+3j-k-l}3^k2^l}{\left(\alpha^4+6\right)^{\eta+j}}\\ \times\int_0^{\infty}y^{3j+4\eta-k-l+\frac{\eta}{\beta}-\frac{1}{\beta}-1}e^{-\alpha(\eta+j)y}dy\end{array}\right\}$$

Recall that

$$\int_0^{\infty}x^ne^{-ax}dx=\frac{\Gamma(n+1)}{a^{n+1}}$$

Consequently, we obtain the Rényi entropy of EIPP distribution as

$$R_e(\eta)=\frac{1}{1-\eta}\log\left\{\phi_{i,j,k,l}\frac{\Gamma\left(3j+4\eta-k-l+\frac{\eta}{\beta}-\frac{1}{\beta}\right)}{\left(\alpha(\eta+j)\right)^{\left(3j+4\eta-k-l+\frac{\eta}{\beta}-\frac{1}{\beta}\right)}}\right\} \tag{37}$$

Where

$$\phi_{i,j,k,l}=\sum_{i=0}^{\infty}\binom{\eta}{i}\sum_{j=0}^{\infty}\binom{\eta(\lambda-1)}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\frac{\lambda^{\eta}\beta^{\eta-1}\alpha^{4\eta+3j-k-l}3^{k}2^{l}}{\left(\alpha^{4}+6\right)^{\eta+j}}$$

4.6 Reliability analysis of EIPP distribution

In this section, we present the survival function, hazard rate and odds functions of the EIPP distribution, which are very significant in reliability analysis.

4.6.1 Survival function of EIPP distribution

Survival function $S_p(x)$ is the probability that the survival time is greater than or equal to x .It is also known as reliability function and refers to the probability of surviving an age x or becoming older than x. We use survival function in reliability analysis to determine the survival time of items. Let X be a continuous random variable with CDF, $F(x)$, the survival function of X is

$$S_p(x)=1-G_p(x) \tag{38}$$

Therefore, the survival function of EIPP distribution is

$$S_p(x)=1-\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda} \tag{39}$$

4.6.2 Hazard function

Hazard function is the probability that an individual dies at time x given that the individual has lived to that time x. It is extensively used to show the hazard of an incident for instance, death happening at certain time t.

Given a random variable X from a continuous distribution, the hazard rate $h(x)$ is given by

$$h_p(x)=\frac{g_p(x)}{1-G_p(x)} \tag{40}$$

$$h_p(x)=\frac{\frac{\lambda\beta\alpha^{4}}{\alpha^{4}+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1}e^{-\alpha x^{-\beta}}}{1-\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^{4}+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda}} \tag{41}$$

The graphs in figures 3a, 3b, 3c and 3d show some possible shapes of the survival function and hazard rate for different values of α,β and λ

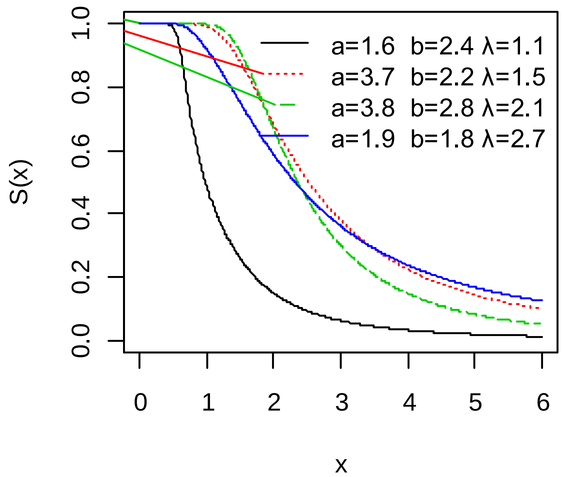

Fig 3a:survival plot of EIPP

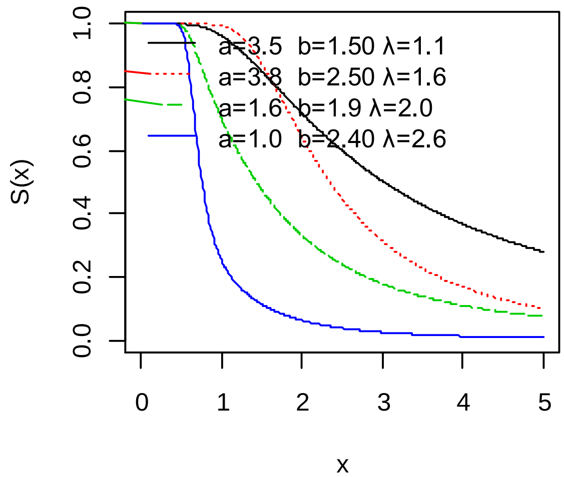

Fig 3b:survival plot of EIPP

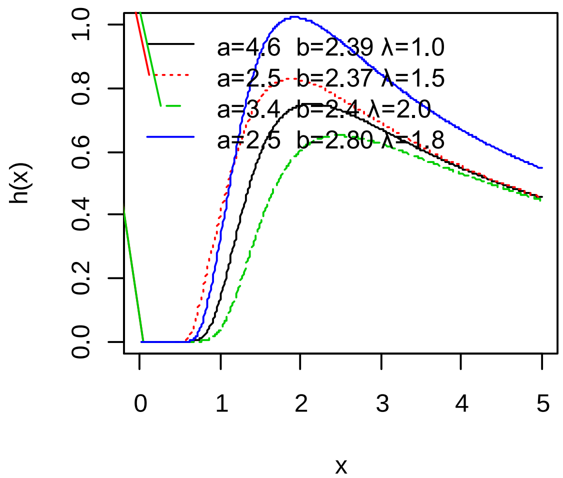

Fig 3c:hazard plot of EIPP

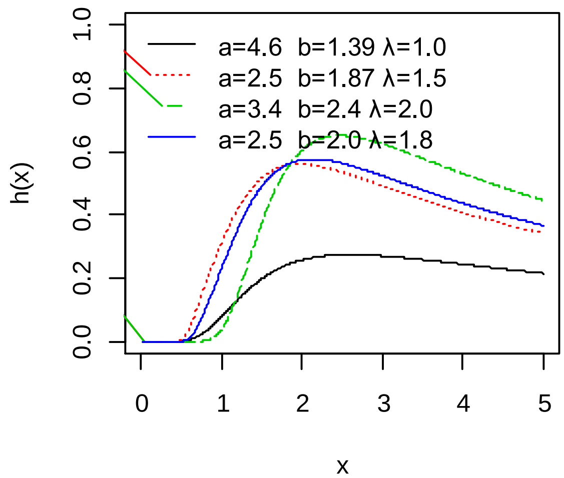

Fig 3d: hazard plot of EIPP

4.6.3 Odds function

The odds function of the EIPP distribution is given by

$$\pi_{O_{(p)}}(x)=\frac{F_p(x)}{S_p(x)} \tag{42}$$

$$\pi_{O_{(p)}}(x)=\frac{\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda}}{1-\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2 x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda}} \tag{43}$$

5 Maximum Likelihood Estimators

Let $X_1, X_2, ..., X_n$ denote a random sample of size n from the EIPP distribution having parameters α, β and λ. To estimate the parameters α, β and λ using the maximum likelihood method, we state the likelihood function of the random sample from the EIPP distribution as

$$L = L(\alpha,\beta,\lambda|x) = \prod_{i=1}^{n} \frac{\lambda\beta\alpha^4}{\alpha^4+6}\left(\alpha+x^{-3\beta}\right)x^{-(\beta+1)}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1} e^{-\alpha x^{-\beta}}$$

$$=\left(\frac{\lambda\beta\alpha^4}{\alpha^4+6}\right)^n \prod_{i=1}^{n}\left(\alpha+x^{-3\beta}\right)\sum_{i=1}^{n} x^{-(\beta+1)}\prod_{i=1}^{n}\left\{\left[1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\alpha^4+6}\right]e^{-\alpha x^{-\beta}}\right\}^{\lambda-1} e^{-\alpha\sum_{i=1}^{n} x^{-\beta}}$$

Taking the natural logarithm, the likelihood function is obtained. Thus,

$$\ln L = \left\{\begin{aligned} & n\ln\lambda + n\ln\beta + 4n\ln\alpha - n\ln\left(\alpha^4+6\right)+\sum_{i=1}^{n}\ln\left(\alpha+x^{-3\beta}\right)-(\beta+1)\ln\sum_{i=1}^{n}x \\ & +(\lambda-1)\sum_{i=1}^{n}\ln\left[\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\left(\alpha^4+6\right)}\right)e^{-\alpha x^{-\beta}}\right]-\alpha\sum_{i=1}^{n}x^{-\beta} \end{aligned}\right\}$$

$$\frac{\partial L}{\partial\alpha} = \left\{\begin{aligned} & \frac{4n}{\alpha}-\frac{2n\alpha}{\alpha^4+6}+\sum_{i=1}^{n}\left(\frac{1+x^{-3\beta}}{\alpha+x^{-3\beta}}\right)-(\lambda-1) \\ & \times\sum_{i=1}^{n}\left[\frac{\left(\alpha^8x^{-\beta}+30\alpha^4x^{-\beta}+\alpha^5x^{-4\beta}+4\alpha^6x^{-3\beta}+12\alpha^5x^{-2\beta}+6\alpha x^{-4\beta}\right)}{\left(\alpha^4+6\right)^2\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\left(\alpha^4+6\right)}\right)}\right]-\sum_{i=1}^{n}x^{-\beta} \end{aligned}\right\}=0 \tag{44}$$

$$\frac{\partial L}{\partial\beta} = \left\{\begin{aligned} & \frac{n}{\beta}+\sum_{i=1}^{n}\left(\frac{3x^{-3\beta}\ln x}{\alpha+x^{-3\beta}}\right)-\ln\sum_{i=1}^{n}x+\alpha\sum_{i=1}^{n}x^{-\beta}\ln x+(\lambda-1) \\ & \times\sum_{i=1}^{n}\left[\frac{\left(\begin{aligned} & \alpha^{10}x^{-\beta}\ln x+12\alpha^6x^{-\beta}\ln x+\alpha^9x^{-4\beta}\ln x \\ & +3\alpha^8x^{-3\beta}\ln x+6\alpha^7x^{-2\beta}\ln x+36\alpha^2x^{-\beta}\ln x \\ & +6\alpha^5x^{-4\beta}\ln x+18\alpha^4x^{-3\beta}\ln x+36\alpha^3x^{-2\beta}\ln x \\ & +3\alpha^7x^{-3\beta}\ln x-6\alpha^6x^{-2\beta}\ln x-6\alpha^5x^{-\beta}\ln x \\ & -18\alpha^3x^{-3\beta}\ln x-36\alpha^2x^{-2\beta}\ln x-36\alpha x^{-\beta}\ln x \end{aligned}\right)}{\left(\alpha^4+6\right)^2\left(1+\frac{\alpha x^{-\beta}\left(\alpha^2x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\left(\alpha^4+6\right)}\right)}\right] \end{aligned}\right\}=0 \tag{45}$$

$$\frac{\partial L}{\partial \lambda}=\frac{n}{\lambda}+\sum_{i=1}^{n}\ln\left[\left(1+\frac{\alpha x^{-\beta}\left(\alpha^{2}x^{-2\beta}+3\alpha x^{-\beta}+6\right)}{\left(\alpha^{4}+6\right)}\right)e^{-\alpha x^{-\beta}}\right]=0 \tag{46}$$

It is usually more convenient to use nonlinear optimization algorithms such as quasi-Newton algorithm to numerically maximize the log-likelihood function. The R package provides nonlinear optimization for solving such problems.

6 Numerical applications

To enable us demonstrate the applicability of Exponentiated Inverse Power Pranav distribution (EIPP), the data set given below represents the active repair times (hours) for an airborne communication transceiver. The data has been used extensively in many research works by numerous authors. Initially it was used by Jorgensen(1982) and reported by Rameesa *et al* (2018b).

0.50, 0.60, 0.60, 0.70, 0.70, 0.70, 0.80, 0.80,1.00, 1.00, 1.00, 1.00, 1.10, 1.30, 1.50, 1.50,1.50, 1.50, 2.00, 2.00, 2.20, 2.50, 2.70, 3.00,3.00, 3.30, 4.00, 4.00, 4.50, 4.70, 5.00, 5.40,5.40, 7.00, 7.50, 8.80, 9.00, 10.20, 22.00, 24.50

Some criteria such as the Akaike information criterion (AIC) and Bayesian information criterion (BIC) are used to discriminate the aforementioned distributions so as to ascertain which of them gives the best fit. The formulae for (AIC) and (BIC) are respectively given by

$$AIC=2k-2l \tag{47}$$

$$BIC=k\ln(n)-2l \tag{48}$$

where l denotes the log-likelihood function evaluated at the maximum likelihood estimates, k is the number of model parameters, n is the sample size. For calculation of the analytical measures, R software is used to produce the required solution from the nonlinear equations since one cannot easily obtain a close form solutions (44), (45) and (46). Also, the Kolmogorov-Smirnov goodness of fit test was carried out at 5% level of significance to find out if the data used in this study follows Exponentiated Inverse Power Pranav distribution. The test statistic is given by

$$K.S=\sup_{X}\left|F_{0}(x)-F_{c}(x)\right| \tag{49}$$

Where $F_0(x)$ denotes the cumulative distribution function of the hypothesized distribution and $F_c(x)$ denotes the empirical distribution function of the observed data. A distribution is said to provide the best fit to the data if among all the distributions under consideration, it corresponds to minimum values of AIC, BIC and the log-likelihood respectively.

Table 1: MLE's LL, AIC, BIC, K.S and P values of the fitted distributions of the data set

Model	Parameters	S.E	LL	AIC	BIC	KS	p
	α = 1.1720	0.4464					
EIPP	β = 1.2729	0.1982	-87.9937	181.9873	187.054	0.08499	0.9112
	λ = 6.9144	9.5748					
IPP	α = 2.3341	0.1934	-89.0177	182.0353	185.4131	0.098065	0.8009
	β = 1.3472	0.1592					
PD	α= 0.9435	0.0646	-118.76	239.5201	241.209	0.17435	0.1558
	α =1.1717	0.2088					
EIPL	β =3.7378	18.219	-89.4736	184.9472	190.0138	0.095991	0.8207
	θ= 0.4950	2.7785					

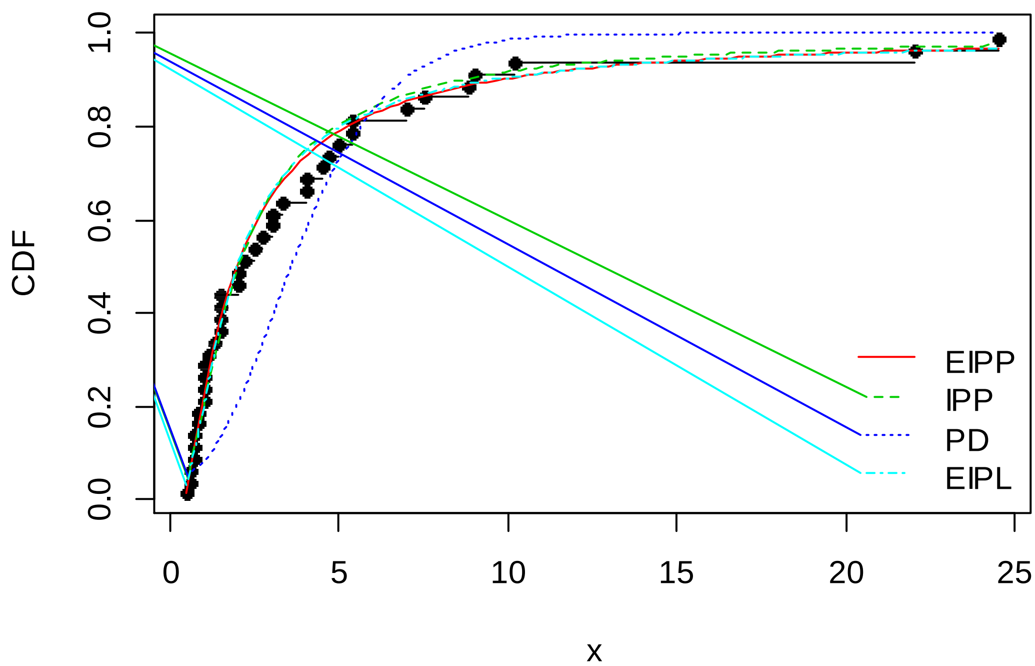

Figure 4: Graphs of the estimated cdfs based on the research data

The maximum likelihood estimates with the standard error of the fitted models, the corresponding AIC, BIC, K.S and the corresponding p values for the data set are presented in Tables 1 above. Based on the results presented, it is evident that the EIPP distribution has the least AIC, BIC and largest negative log-likelihood values among all competing models, and so it could be considered as the best model among all the distributions which have been fitted to the a real data set.

7 Conclusions

In distribution theory, concerted efforts have always been made to generalize distributions. Numerous methods exist that can help one to achieve this. Interest has always been centred on the quality of empirical results which lies on how well the proposed distribution fits the data sets under consideration. In this article, a new distribution has been introduced and the properties have been studied. Here, we derived moments and moment generating function, the quantile function, an expression for distribution of order statistics and Rényi entropy were derived.

Reliability measures such as survival function, hazard function and odds function were also derived. The distribution were subjected to life data to display its applicability, it was compared to other sub-models. Based on the selection criteria, the proposed model, that is, Exponentiated Inverse Power Pranav distribution was found to perform better that the other sub-models.

Author Contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of interest: The authors declare no conflict of interest.

References

Dimitrakopoulou, T., Adamidis, K. and Loukas, S. (2007) A Lifetime Distribution with an Upside-Down Bathtub-Shaped Hazard Function.IEEE Transactions on Reliability, Vol. 56, No. 2

Vikas K.S, Sanjay K. S, Umesh S, and Varun A.(2014) The Inverse Lindley distribution – A stressstrength reliability model with applications to head and neck cancer data, *Journal of industrial and Production Engineering*.32(3):162-173.DOI:10.1080/21681015.2015.1025901

Vikas K. S, Sanjay K. S and, Umesh U.(2015) A new upside-down bathtub shaped hazard rate model for survival data analysis, Applied Mathematics and Computation; 239:242253.DOI:10.1016/j.amc.2014.04.048

Eliwa M.S, El-Morshedy M., Mohamed Ibrahim (2018), Inverse Gompertz Distribution: Properties and Different Estimation Methods with Application to Complete and Censored Data. *Annals of Data Science*, https://doi.org/10.1007/s40745-018-0173-0

Rameesa J., Jan T.R, Peer B.B (2018), ExponentiatedInversePowerLindleyDistributionandits Applications, arXiv:1808.07410v1 (math.ST).

Amal S.H., Rokaya E.M. (2019) Weibull Inverse Lomax Distribution, *Pak.j.stat.oper.res.* Vol.XV No. III, pp587-603

Onyekwere C. K., George A. Osuji G. A and Enogwe S. U (2020),Inverted Power Rama Distribution with Applications to Life Time Data, *AJPAS*, 9(4): 1-21; Article no.AJPAS.62214

Enogwe S. U., Obiora-Ilouno H. O., Onyekwere C. K. (2021), Inverse Power Akash Probability Distribution with Applications, *Earthline J. Math. Sci.* Vol. 6 No. 1, 1-32

Shukla KK.(2018) Pranav distribution with properties and its applications. *Biom Biostat Int J*. 2018;7(3):244–254. DOI: 10.15406/bbij.2018.07.00215

Lindley DV (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society*, Series B. 1958;20(1):102–107.

Shanker R. (2015) Akash distribution and Its Applications. *International Journal of Probability and Statistics*. 2015;4(3):65–75.

Ghitany M E, Atieh B, Nadarajah S.(2008) Lindley distribution and its Applications. *Mathematics Computing and Simulation*. 2008;78:493– 506.

Uwaeme O.R, Akpan N. P., Orumie U. C. (2018) An Extended Pranav Distribution; *AJPAS*, 2(4); 1-15, Article no.AJPAS,46556

Umeh E. and Ibenegbu A. (2019) A Two-Parameter Pranav Distribution with Properties and its Application, *J Biostat Epidemiol*. 2019;5(1): 74-90

Odom C. C, Nduka E. C and Ijomah M. A. (2019) A Modification of Pranav Distribution using Quadratic Rank Transmutation Map Approach; *Int. J. Sci. Res. in Mathematical and Statistical Sciences*, Vol. 6(2), ISSN: 2348-4519

Wani S.A., Anwar H., Shaista S. and Sumeera S. (2020) Pranav Quasi Gamma Distribution: Properties and Applications; *Journal of statistical Theory and Applications*; DOI: https://doi.org/10.2991/jsta.d.201202.001; ISSN 1538-7887.

Hogg, R.V., J.W. McKean and A.T. Craig (2019). Introduction to Mathematical Statistics, 8edn, Pearson Education, Inc.

Jorgensen, B. (1982). Statistical properties of the generalized inverse Gaussian distribution. New York: *Springer-Verlag*.

Mahmoud, M.R. and R.M. Mandouh (2013). On the transmuted Frechet distribution. J. of Appl. Sci. Res., 9, 5553-5561

Mudholkar, G.S. and Srivastava, D.K. (1993). Exponentiated Weibull family for analyzing bathtub failure-rate data, *IEEE Transact. Reliab*, 42(2), 299 - 302.