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AN EFFECTIVE APPROACH OF FOUR-STEP METHOD FOR OPTIMAL SOLUTION OF TRANSPORTATION PROBLEM

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Abstract

Transportation problem (TP) in operation research is one of the most in use optimization technique to deal the problems that are related with transportation of goods from sources to destinations. Initial Basic Feasible Solution (IBFS) plays a vital role in TP which offers a way to obtain the optimal solution. The objective is to prevail the total transportation cost equivalent or nearer to optimal solution. In this paper, an effective approach of Four Step Method (FSM) for optimal solution of TP has been brought up in order to get optimal solution of TPs. In this method we construct the Maximum Column Table (MCT) and Maximum Row Table (MRT). Several problems has been solved using this method to get the optimal solution. The outcomes of proposed method are contrasted with results of North West Corner Method (NWCM), Least Cost Method (LCM) and Vogal's Approximation Method (VAM). It is observed that the proposed method is not only achieving better results but also overcoming the limitation of VAM.

Key Words: Transportation problem, Initial Basic Feasible Solution, Optimal solution

Linear programming problem

1. Introduction

Operation Research (OR) is a subject that uses application of progressed expository strategies to assist make superior decisions. The subject Operation Research was introduced during second World War and utilized for military techniques. During second World War a group of scientists from mathematics , physical and social sciences were grant to consider about of different army operations. This team was exceptionally successful in reducing cost of transporting the military equipment in different areas of war.

The transportation problem is the particular foreshadow of Linear Programming Problem (LPP) that was first presented by Hitchcock [1] in order to distribute the production of different numerous localities. After few years T.C. Koopmans [2] gave a method called 'Optimum Utilization of the Transportation System'. These two contributions are known as basement for the progress of transportation problem. The main ambition of Transportation problem was to minimize the cost of transportation of goods from *n* origin to *m* destinations.

Later, this type of problem was initially worked by Dantzig in 1951 and flourished by Charnes and Copper in 1953.Many scholars have worked on such type of problems. In 2014, M.Tanvir [3] and U.k. Das [4] proposed different methods which give almost optimal solution. In 2015, S.Akpan [5] modified the VAM by using the idea of Standard Deviation of rows and columns on balanced TP which showed better results. In 2016, M.S.Uddin [6] made modification on LCM and M.M. Ahmed [7] proposed a method called Incessant Allocation Method for solving transportation problem, this method was then checked on both profit maximizing and cost minimizing problems and concluded that it is giving much better results. In 2017, M.J.uddin [8] developed an algorithm named as Weighted Cost Opportunity which showed better results as compared to regular LCM.In the same year, M.R.S. Shaikh [9] proposed an algorithm and M.B. Hossain [10] gave a new method by considering average penalty of rows and average penalty of columns. The results obtained from this method is more acceptable as compared to the VAM. In 2018, R.Kumar [11] proposed a new approach called Direct Sum Method (DSM) which gave better IBFS as compared to well-known methods. In 2019, many methods proposed but among them BCE [12] and PGM [13] were considered to give best results. In 2020, A.M.Khoso [14] made modification on LCM and Hussain [15] made modification on VAM. Both modifications are considered to give better results in less time.

There are three well known methods that are being used to get the Initial Basic feasible Solution (IBFS) named as North West Corner Method (NWCM) , Least corner Method (LCM) and Vogal's Approximation Method (VAM). Among these three methods , VAM is considered to give best IBFS.

The three fundamental and well known methods to solve transportation problems are:

1.1 North West Corner Method (NWCM)

This is the basic and primary method that was used to minimize the total transportation cost. In this method we first allocate the northwest cell the required supply or demand and cross out that respective row or column. This process continues till the total cost is minimized

1.2 Least Cost Method (LCM)

This method is more effective than NWCM. In this method we allocate the lowest cost in transportation table the required supply or demand and cross out that respective row or column. This process continues till the total cost is minimized.

1.3. Vogal's Approximation Method (VAM)

This method is considered to give the best results among above methods. In this method we calculate penalty by subtracting the two consecutive lowest costs of each row and column. We allocate largest penalty the required supply or demand and cross out that respective satisfied row or column. This process continues till the total cost is minimized.

1.4. Proposed Four-Step Method

In this proposed method we calculate the Maximum Row Table (MRT) and Maximum Column table (MCT) by subtracting the largest cost of each row (Column) from respective costs of that row (Column). After that we add MRT and MCT and name this table as revised table. We allocate the largest cost of revised table and cross out that respective row or column. This process continues till all supplies and demands are satisfied. Then we make same positioned allotment in original table as revised table

Let $x_{ij} \geq 0$ be the amount transported from the source *i* to the destination *j*, The mathematical formulation of the problem is stated as:

Minimize	$\mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} Cij\ xij$	(Total transportation cost)
Subject to	$\sum_{j=1}^{n} xij = ai$	(Supply from sources)
	$\sum_{i=1}^{m} xij = bj$	(Demand from destinations)
	$x_{ij} \geq 0$,	for all i and j

Where

Z	:	Total transportation cost to be minimized.
C_{ij}	:	transportation cost of the goods from each sources i to destination j.
x_{ij}	:	Amount of goods driven away from source i to destination j.
a_i	:	Amount of supply at each source i.
b_j	:	Amount of demand at each destination j.

Optimal solution of Transportation Model (TM) has two pace: initially we obtain an initial basic feasible solution (IBFS) using m + n -1 destinations, where n and m are origins and destinations respectively.

2. Methodology

In this section we will explain step by step process of proposed method.

Step 1: Calculate the Maximum Row (MRT) and Column Table (MCT) by subtracting the largest cost of each row (Column) from respective costs of that row (Column). i.e. if C_{11} is largest cost in first row (Column) then

For MRT,	For MCT
$C_{11}^{+} = C_{11} - C_{11}$	$C_{11}^{-} = C_{11} - C_{11}$
$C_{12}^{+} = C_{11} - C_{12}$	$C_{21}^{-} = C_{11} - C_{21}$
$C_{13}^{+} = C_{11} - C_{13}$	$C_{31}^{-} = C_{11} - C_{31}$

And so on

Table 2.1: Maximum Row Table (MRT)

Destination / Sources	D_1	D_2	D_3	…	D_n	Supply
S_1	$C_{11}^+ = C_{12} - C_{11}$	$C_{12}^+ = C_{12} - C_{12}$	$C_{13}^+ = C_{12} - C_{13}$	…	$C_{1n}^+ = C_{12} - C_{1n}$	S_1
S_2	$C_{21}^+ = C_{22} - C_{21}$	$C_{22}^+ = C_{22} - C_{22}$	$C_{23}^+ = C_{22} - C_{23}$	…	$C_{2n}^+ = C_{22} - C_{2n}$	S_2
S_3	$C_{31}^+ = C_{33} - C_{31}$	$C_{32}^+ = C_{33} - C_{32}$	$C_{33}^+ = C_{33} - C_{33}$	…	$C_{3n}^+ = C_{33} - C_{3n}$	S_3
	…	…	…	…	…	…
S_m	$C_{m1}^+ = C_{m2} - C_{m1}$	$C_{m2}^+ = C_{m2} - C_{m2}$	$C_{m3}^+ = C_{m2} - C_{m3}$	…	$C_{mn}^+ = C_{m2} - C_{mn}$	S_m
Demand	q_1	q_2	q_3	…	q_n	

Table 2.2: Maximum Column Table (MCT)

Destination / Sources	D_1	D_2	D_3	…	D_n	Supply
S_1	$C_{11}^- = C_{31} - C_{11}$	$C_{12}^- = C_{42} - C_{11}$	$C_{13}^- = C_{13} - C_{13}$	…	$C_{1n}^- = C_{2n} - C_{1n}$	S_1
S_2	$C_{21}^- = C_{31} - C_{21}$	$C_{22}^- = C_{42} - C_{22}$	$C_{23}^- = C_{13} - C_{23}$	…	$C_{2n}^- = C_{2n} - C_{2n}$	S_2
S_3	$C_{31}^- = C_{31} - C_{31}$	$C_{32}^- = C_{42} - C_{32}$	$C_{33}^- = C_{13} - C_{33}$	…	$C_{3n}^- = C_{2n} - C_{3n}$	S_3
	…	…	…	…	…	…
S_m	$C_{m1}^- = C_{31} - C_{m1}$	$C_{m2}^- = C_{42} - C_{m2}$	$C_{m3}^- = C_{13} - C_{m3}$	…	$C_{mn}^- = C_{2n} - C_{mn}$	S_m
Demand	q_1	q_2	q_3	…	q_n	

Step 2: Add MRT and MCT and name the new table as revised table.

Table C: **Revised Table** = MRT + MCT

Destination / Sources	D_1	D_2	D_3	…	D_n	Supply
S_1	$C_{11}^* = C_{11}^+ + C_{11}^-$	$C_{12}^* = C_{12}^+ + C_{12}^-$	$C_{13}^* = C_{13}^+ + C_{13}^-$	…	$C_{1n}^* = C_{1n}^+ + C_{1n}^-$	S_1
S_2	$C_{21}^* = C_{21}^+ + C_{21}^-$	$C_{22}^* = C_{22}^+ + C_{22}^-$	$C_{23}^* = C_{23}^+ + C_{23}^-$	…	$C_{2n}^* = C_{2n}^+ + C_{2n}^-$	S_2
S_3	$C_{31}^* = C_{31}^+ + C_{31}^-$	$C_{32}^* = C_{32}^+ + C_{32}^-$	$C_{33}^* = C_{33}^+ + C_{33}^-$	…	$C_{3n}^* = C_{3n}^+ + C_{3n}^-$	S_3
	…	…	…	…	…	…
S_m	$C_{m1}^* = C_{m1}^+ + C_{m1}^-$	$C_{m2}^* = C_{m2}^+ + C_{m2}^-$	$C_{m3}^* = C_{m3}^+ + C_{m3}^-$	…	$C_{mn}^* = C_{mn}^+ + C_{mn}^-$	S_m
Demand	q_1	q_2	q_3	…	q_n	

Step 3: Allocate largest cost U_{ij} (as shown in eq (1))of revised table a maximum supply a_i (or demand b_j) and cross out the satisfied row of supply (or column of demand). Then in same table select second largest cost U_{kl} of revised table and repeat the process until all supply a_i and demand b_j are satisfied. If there are similar costs in the revised table, then select the cost **P** which gives us maximum sum of supply and demand of respective cost as shown in eq 2.

$U_{ij} = \max(c_{ij})$, i,j= 1,2,3 ... --------- (1) **P** $= \max(a_i + b_j)$ --------- (2)

Step 4: Make the same positioned allotment in the original table as revised table.

$$\text{Min } Z = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{21}x_{21} + \cdots + c_{ij}x_{ij}$$

3. Numerical Illustration

In this paper, we have examined a balanced transportation problems chosen from literature. The result of these problems are then compared with well-known methods named as NWCM , LCM and VAM. We solve example 1 step-by-step.

3.1. Example 1: Consider the following balanced transportation problem.

Table 3.1:Transportation Problem

	D1	D2	D3	Sources
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Destinations	6	10	15	31

Step 1: For MRT, subtract 8, 9, 6 from each elements of 1st, 2nd and 3rd row respectively.

Table 3.2: **Maximum Row Table** (MRT)

	D1	D2	D3	Sources
S1	8 – 6 = 2	8 – 8 = 0	8 – 4 = 4	14
S2	9 – 4 = 5	9 – 9 = 0	9 – 8 = 1	12
S3	6 – 1 = 5	6 – 2 = 4	6 – 6 = 0	5
Destinations	6	10	15	31

For MCT, subtract 6, 9, 8 from each elements of 1st, 2nd and 3rd column respectively.

Table 3.3: **Maximum Column Table** (MCT)

	D1	D2	D3	Sources
S1	6 – 6 = 0	9 – 8 = 1	8 – 4 = 4	14
S2	6 – 4 = 2	9 – 9 = 0	8 – 8 = 0	12
S3	6 – 1 = 5	9 – 2 = 7	8 – 6 = 2	5
Destinations	6	10	15	31

Step 2: Add maximum row table and maximum column table and name the new table as revised table.

Table 3.4: **Revised Table** = MRT + MCT

	D1	D2	D3	Sources
S1	2 + 0 = **2**	0 + 1 = **1**	4 + 4 = **8**	14
S2	5 + 2 = **7**	0 + 0 = **0**	1 + 0 = **1**	12
S3	5 + 5 = **10**	4 + 7 = **11**	0 + 2 = **2**	5
Destinations	6	10	15	31

Step 3 : Allocations in revised table are shown in table 3.5

Table 3.5: Allocations in Revised Table

	D1	D2	D3	Supply
S1	2	1	8 **14**	14
S2	7 **6**	0 **5**	1 **1**	12
S3	10	11 **5**	2	5
Demands	6	10	15	31

- First Allocation: In table 3.1.5, we see that 11 is the largest cost so we allocate 5 in cell (3, 2) of supply and cross-out third row. The demand of D2 is reduced to 5 = (10 – 5)

- Second Allocation: In the table next largest cost is 8 so we allocate 14 in cell (1, 3) of supply and cross-out first row. The demand of D3 is reduced to 1 = (15 – 14)
- By considering the process, third allocation in cell (2, 1) is 6, fourth allocation in cell (2, 3) is 1 and fifth allocation in cell (2, 2) is 5

Step 4 : Make the same positioned allotment in the original table as revised table.

Table 3.8: Allocations in original table.

	D1	D2	D3	Sources
S1	6	8	4 **14**	14
S2	4 **6**	9 **5**	8 **1**	12
S3	1	2 **5**	6	5
Destinations	6	10	15	31

$$\mathbf{Z = 4\times6 + 9\times5 + 2\times5 + 4\times14 + 8\times1 = 143}$$

4. Optimality test for example using Modi method

1st Iteration

Step: 1 Using table 3.8. Initial Basic Feasible Solution

Intend the values of dual variables U_i and V_j , $U_i + V_j = C_{ij}$

The initially we take $U_1 = 0$

$C_{13}=U_1 + V_3 = 4$ => $0 + V_3 = 4$ => $V_3 = 4$

$C_{21}=U_2 + V_1 = 4$ => $4 + V_1 = 4$ => $V_1 = 0$

$C_{22}=U_2 + V_2 = 9$ => $4+ V_2 = 9$ => $V_2 = 5$

$C_{23}=U_2 + V_3 = 8$ => $U_2 + 4 = 8$ => $U_2 = 4$

$C_{32}=U_3 + V_2 = 2$ => $U_3 + 5 = 2$ => $U_3 = -3$

Table 4.1 : Example-1: Step 1 of 1st iteration

	D_1	D_2	D_3	Supply	U_i
S_1	6	8	4	14	$U_1=0$
S_2	4	9	8	12	$U_2=4$
S_3	1	2	6	5	$U_3=-3$
Demand	6	10	15	$\sum$ 31	
V_j	$V_1=0$	$V_2=5$	$V_3=4$		

Count the occasion test using Modi method

$T_{11}= C_{11} – (U_1 + V_1)$ => $6 – (0 + 0)$ => $T_{11} = 6$

$T_{12}= C_{12} – (U_1 + V_2)$ => $8 – (0 + 5)$ => $T_{12} = 3$

$T_{31}= C_{31} – (U_3 + V_1)$ => $1 – (-3 + 0)$ => $T_{31} = 4$

$T_{33}= C_{32} – (U_3 + V_3)$ => $6 – (-3 + 4)$ => $T_{33} = 5$

Test the all placed cell sign

Table 4.2 : Example-1: Step 2 of 1st iteration

	D_1	D_2	D_3	Supply	U_i
S_1	6 **6**	8 **3**	4 **<u>14</u>**	14	$U_1=0$
S_2	4 **<u>6</u>**	9 **<u>5</u>**	8 **<u>1</u>**	12	$U_2=4$
S_3	1 **4**	2 **<u>5</u>**	6 **5**	5	$U_3=-3$
Demand	6	10	15	$\sum$ 31	
V_j	$V_1=0$	$V_2=5$	$V_3=4$		

All unoccupied cell sign are positive or equal to zero $T\geq0$

Table 4.3: Example-1: Step 3 of 1st iteration

	D1	D2	D3	Sources
S1	6	8	4 **14**	14
S2	4 **6**	9 **5**	8 **1**	12
S3	1	2 **5**	6	5
Destinations	6	10	15	∑31

$$Z = 4 \times 14 + 4 \times 6 + 9 \times 5 + 8 \times 1 + 2 \times 5 = 143$$

5. Results and Discussion

We have investigated the performance of proposed method by comparing its results with NWCM, LCM and VAM by testing six transportation problems, the results obtained using proposed in examples 2, 3, 5 and 6 are same as the optimal outcome. While in example 1 and 4 it is close to the optimal solution. It could be clearly seen that the proposed method gave reliable results as compared to the well-known existing methods like NWCM, LCM and VAM.

No. of Example	Type of problem	Result of NWCM	Result of LCM	Result of VAM	Result of proposed method	Optimal Solution
1	3 x 4	116	116	101	101	100
2	3 x 3	39	37	33	33	33
3	3 x 3	730	555	555	555	555
4	3 x 4	3680	3680	3520	3510	3460
5	3 x 3	163	228	143	143	143
6	3 x 3	6600	6540	5920	5920	5920

6. Conclusion

In this paper, for achieving better initial basic feasible solution of transportation problems, an effective approach of Four-Step Method for optimal solution of transportation problem has been developed. The proposed method has been checked on various transportation problems and also looked over for optimality. A compression of proposed method has been made with LCM, NWCM and VAM by examining six examples, it is resulted that the proposed method is achieving better outcomes as compared to the well-known existing methods mentioned above.

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