A New Modified Algorithm Based Upon Exponential and Polynomial Functions for Solving Initial Value Problems

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Abstract
The main focus of this paper is to propose a fourth-order convergent modified numerical algorithm by using an exponential and polynomial type function for getting the approximate solution of initial value problems (IVPs) in ordinary differential equations (ODEs). The proposed algorithm has been tested and analyzed for the convergence, stability, consistency, of several numerical experiments that proved the superiority of the proposed algorithm against algorithms including the fourth-order arithmetic mean (AM), the fourth-order harmonic mean (HM), and the fourth-order Contra harmonic mean(C_oM).
Keywords: Modified numerical algorithm, Ordinary differential equation, Order of convergence, Initial value problems, Stability, and Consistency.

1. Introduction

Many researchers are working to solve many real-world physical problems. They are the ones that came up with the laws and theories that are related to natural physical problems. Most of the time, these laws take the form of differential equations. In the domains of engineering and physics, an ODE is crucial. It is the most significant tool for solving the physical and natural problems that belong to the field of science. The mostly real-world applications and pure mathematical research are interconnected to the ordinary differential equations. The most mathematical formulation of physical laws are defined in terms of ODEs and have applications in the field of fluid dynamics, chemistry, Biology, Medicine, Population dynamics, Economy, Social sciences, and many more. It is used to define the motion of projectile, rocket, charge, satellite planet, current in a circuit, conduction of heat, force on bodies, or bodies in motion depending on ODEs. There are many methods for solving initial value problems of the form given below:

$$y' = f(x,y), y(x_0) = y_0 \tag{1}$$

At present time, we have many exact and numerical techniques for solving the equation of the type (1) that gives approximate and analytical solutions, but some ODEs have no exact solutions. It is a known fact that several mathematical models arising from the real and physical life situations cannot be solved explicitly in most of the cases such as nonlinear lotka Volterra competition model and logistic equation in population dynamics, Lorenz system in meteorology, pendulum, and duffing equations in mechanical engineering, VanderPol equation in electrical engineering, Newton's law of cooling in thermodynamics, geodesic equation in geology, radioactive decay in nuclear physics, the motion of a charged particle, Fermi–Ulam–Pasta Oscillator and many more. In such situations, numerical methods of different characteristics and orders are needed mainly due to nonlinear terms involved in the practical problems.

These are all ODEs in which we need to use numerical methods to get the solutions. Some methods are explicit, and some are implicit methods. In continuation of this effort, many researchers have derived new numerical integration methods of explicit and implicit nature in an attempt to obtain better approximate results than various of the available ones in the present literature such as [1,2,5,6,10,12], just to mention a few. In [10] author developed a new numerical technique for the solution of initial value problems of first-order ODEs whereas in the present study, we have proposed a new modified algorithm based upon the exponential and polynomial function for the solution of initial value problems of first-order ODEs, the paper is organized as follows: the Section 2 starts with the derivation of the polynomial and exponential type function for a proposed new modified numerical algorithm, the section 3 to proves that proposed new modified numerical algorithm is fourth-order convergent and discuss the local truncation error, consistency and larger stability region of the proposed algorithm structured within its subsections. The Section 4 takes into consideration four numerical problems of varying nature to illustrate the performance of the algorithm based on the computation of errors. The last Section 5 summarizes the above discussion with highlights of major findings in the present research study.

2.1 Derivation of New modified algorithm

Let us presume that the theoretical solution of y(x) to the initial value problem (1) can be locally represented in the interval $[x_i, x_{i+1}], i \geq 0$ by the exponential and polynomial type function;

$$\emptyset(x) = (b_1 + b_2)e^{-3x} + b_3x^3 + b_4x^2 + b_5x + b_6 \tag{2}$$

where b_1, b_2, b_3, b_4, b_5 are real unknown coefficients and b_6 is a constant. We shall imagine y_n is a numerical approximate to the theoretical outcome y(x) and. $\emptyset_n = \emptyset(x_i, y_i)$.
We interpret mesh point as follows:
$x_{i+1} = x_i + h, \quad i = 0,1,2,...$ (3)
Imposing the following constraints on the exponential and polynomial type function (2) in order to get the undetermined coefficients.

2.2 The Exponential and Polynomial type Function.
a. The exponential and polynomial function must correspond with the theoretical solution at
$$x = x_i \ and \ x = x_{i+1}.$$
Hence we require that
$\emptyset(x_i) = (b_1 + b_2)e^{-3x_i} + b_3 x_i^3 + b_4 x_i^2 + b_5 x_i + b_6$. (4)

$\emptyset(x_{i+1}) = (b_1 + b_2)e^{-3x_{i+1}} + b_3 x_{i+1}^3 + b_4 x_{i+1}^2 + b_5 x_{i+1} + b_6$. (5)
b. Secondly, the derivatives of the polynomial and exponential type function are required to correspond with the differential equation, furthermore its first, second, third and fourth derivatives with respect to x at $x = x_i$ we denote nth total derivatives of $\emptyset(x, y)$ with respect to x with $\emptyset^{(n)}$ such that
$\emptyset^1(x_i) = \Theta_n$. (6)

$\emptyset^2(x_i) = \Theta_n^{(1)}$. (7)

$\emptyset^3(x_i) = \Theta_n^{(2)}$. (8)

$\emptyset^4(x_i) = \Theta_n^{(3)}$. (9)

$\Theta_n = -3(b_1 + b_2)e^{-3x_i} + 3b_3 x_i^2 + 2b_4 x_i + b_5$. (10)

$\Theta_n^{(1)} = 9(b_1 + b_2)e^{-3x_i} + 6b_3 x_i + 2b_4$. (11)

$\Theta_n^{(2)} = -27(b_1 + b_2)e^{-3x_i} + 6b_3$. (12)

$\Theta_n^{(3)} = (b_1 + b_2)e^{-3x_i}$. (13)

Solving $b_1 + b_2$ from equation (13),

$(b_1 + b_2) = \frac{1}{81}\Theta_n^{(3)}e^{3x_i}$. (14)
Substitute (14) into (12) we have,

$b_3 = \frac{1}{6}\left(\Theta_n^{(2)} + \frac{1}{3}\Theta_n^{(3)}\right)$. (15)

Substitute (14),(15) into (11), we have,

$b_4 = \frac{1}{2}\left\{\Theta_n^{(1)} - \frac{1}{9}\Theta_n^{(3)} - \left(\Theta_n^{(2)} + \frac{1}{3}\Theta_n^{(3)}\right)x_i\right\}$. (16)

Substitute (14),(15),(16) into (10) we have,

$b_5 = \frac{1}{27}\Theta_n^{(3)} - \left(\Theta_n^{(1)} - \frac{1}{9}\Theta_n^{(3)}\right)x_i + \frac{1}{2}\left(\Theta_n^{(2)} + \frac{1}{3}\Theta_n^{(3)}\right)x_i^2$. (17)

Replace
$\emptyset(x_i) = y_n$. (18)

$\emptyset(x_{i+1}) = y_{n+1}$, (19)

implies that

$$\emptyset(x_{i+1}) - \emptyset(x_i) = y_{n+1} - y_n. \tag{20}$$

$$y_{n+1} - y_n = (b_1 + b_2)[e^{-3x_{i+1}} - e^{-3x_i}] + b_3[x_{i+1}^3 - x_i^3] + b_4[x_{i+1}^2 - x_i^2] + b_5[x_{i+1} - x_i]. \tag{21}$$
Recall that

$$x_{i+1} = x_i + h, \qquad \text{with} \qquad i = 0,1,2\ldots \tag{22}$$

Substitute (14), (15), (16), (17) into (21), we have and neglecting the terms in x_i, x_i^2 ....

$$y_{n+1} = y_n + \frac{1}{81}\theta_n^{(3)}(e^{-3h} - 1) + \frac{1}{6}\left(\theta_n^{(2)} + \frac{1}{3}\theta_n^{(3)}\right)h^3 + \frac{1}{2}\left(\theta_n^{(1)} - \frac{1}{9}\theta_n^{(3)}\right)h^2 + \left(\theta_n + \frac{1}{27}\theta_n^{(3)}\right)h. \tag{23}$$
Hence (23) is the new modified algorithm for solution of the first order differential equation.

3.1. Local Truncation Error Analysis
We decide the order of convergence of any numerical algorithm by analysis of local truncation error. In order to check the order of the algorithm, we subtract the numerical algorithm (23) from the well-known Taylor's series expansion for y(x) in powers of h which has been described below:

We know that Taylor's series expansion is
$$y(x_n) = y(x_n) + h\theta + \frac{1}{2!}h^2\theta^{(1)} + \frac{1}{3!}h^3\theta^{(2)} + \frac{1}{4!}h^4\theta^{(3)} + \frac{1}{5!}h^5\theta^{(4)} + O(h^6). \tag{24}$$
Local truncation error is defined as
$$LTE = Y(x_{n+1}) - y_{n+1}. \tag{25}$$

$$LTE = y(x_n) + h\theta + \frac{1}{2!}h^2\theta^{(1)} + \frac{1}{3!}h^3\theta^{(2)} + \frac{1}{4!}h^4\theta^{(3)} + \frac{1}{5!}h^5\theta^{(4)} - \left[y_n + \frac{1}{81}\theta_n^{(3)}(e^{-3h} - 1) + \frac{1}{6}\left(\theta_n^{(2)} + \frac{1}{3}\theta_n^{(3)}\right)h^3 + \frac{1}{2}\left(\theta_n^{(1)} - \frac{1}{9}\theta_n^{(3)}\right)h^2 + \left(\theta_n + \frac{1}{27}\theta_n^{(3)}\right)h\right] + O(h^6). \tag{26}$$

Replacing the terms e^{-3h} by its Maclaurin's series, one obtains.

$$LTE = y(x_n) + h\theta + \frac{1}{2!}h^2\theta^{(1)} + \frac{1}{3!}h^3\theta^{(2)} + \frac{1}{4!}h^4\theta^{(3)} + \frac{1}{5!}h^5\theta^{(4)} - \left[y_n + \frac{1}{81}\theta_n^{(3)}\left(1 - 3h + \frac{(-3h)^2}{2!} + \frac{(-3h)^3}{3!} + \frac{(-3h)^4}{4!} + \frac{(-3h)^5}{5!} + \cdots - 1\right) + \frac{1}{6}\left(\theta_n^{(2)} + \frac{1}{3}\theta_n^{(3)}\right)h^3 + \frac{1}{2}\left(\theta_n^{(1)} - \frac{1}{9}\theta_n^{(3)}\right)h^2 + \left(\theta_n + \frac{1}{27}\theta_n^{(3)}\right)h\right] + O(h^6). \tag{27}$$

$$LTE = y(x_n) + h\theta + \frac{1}{2!}h^2\theta^{(1)} + \frac{1}{3!}h^3\theta^{(2)} + \frac{1}{4!}h^4\theta^{(3)} + \frac{1}{5!}h^5\theta^{(4)} - \left[y_n + \frac{1}{81}\theta_n^{(3)}(-3h + \frac{9h^2}{2} + \frac{27h^3}{6} + \frac{81h^4}{24} + \frac{243h^5}{120}) + \frac{1}{6}\left(\theta_n^{(2)} + \frac{1}{3}\theta_n^{(3)}\right)h^3 + \frac{1}{2}\left(\theta_n^{(1)} - \frac{1}{9}\theta_n^{(3)}\right)h^2 + \left(\theta_n + \frac{1}{27}\theta_n^{(3)}\right)h\right] + O(h^6). \tag{28}$$
Under local assumption the terms up to h^4 have been cancelled from (28) and we are left with the following expression:
$$LTE = \frac{1}{5!}h^5\left(\theta_n^{(4)} - 3\theta_n^{(3)}\right) + O(h^6). \tag{29}$$
Thus, the leading term of the local truncation error involves h^5 which confirms that the proposed new numerical modified algorithm based upon polynomial and exponential function given by (29) has fourth order of convergence.

3.2 Consistency Analysis
The numerical algorithm needs to be consistent that the truncation errors to be zero when the step size h gets smaller and ultimately extend to zero. Among many, the consistency analysis of a numerical algorithm is one of the ways to check that whether
$$\lim_{h\to 0}\frac{LTE}{h} = 0. \tag{30}$$
From the above criterion, it is easy to observe that the proposed modified numerical algorithm has consistency characteristics. Further, it is also known that any algorithm considered to be consistent has an order of accuracy greater than 0.

Based upon this fact, consistency of the proposed algorithm can safely be claimed since the fourth-order accuracy of the proposed algorithm has already been proved in the previous section.

3.3 Stability
The algorithm is said to be numerically stable if it is efficient of damping out the small change execute in the input data. The idea of stability may be taken in different conditions: it may be correlated with the specific numerical algorithm used, or the step size h used in numerical computations or with the particular problem being solved, one

of the popular way to apply the test problem for stability analysis of the proposed numerical algorithm is $\left(\frac{dy}{dx}\right) y(x) = \emptyset y(x); y(0) = 1$ which has known as exact solution $y(x) = \exp(-\emptyset x)$ where $\emptyset > 0$ is, in general, a complex constant. For an integration interval $[x_i, x_{i+1}]$, where $h = x_i - x_{i+1}$; the exact solution at the point $x = x_{i+1}$ is:

$$y(x_{i+1}) = e^{\emptyset x_{i+1}} = e^{\emptyset(x_i+h)} = e^{\emptyset x_i} e^{\emptyset h} = y(x_i) e^{\emptyset h}, \emptyset < 0 \tag{31}$$

When applied the proposed numerical algorithm on this test problem; it yields

$$y_{i+1} = \psi y_i \quad \text{where} \quad \psi = 1 + \emptyset h + \frac{(\emptyset h)^2}{2} + \frac{(\emptyset h)^3}{6} + \frac{(\emptyset h)^4}{24} \tag{32}$$

Comparing the Equations (31) and (32), it is clearly seen that the Equation (32) is a five term approximation for the function $e^{\emptyset h}$ in the exact solution. The error growth factor given by (32) can be controlled by $|\psi| < 1$ so that the errors may not amplify. Thus, the stability function of the proposed numerical algorithm (23) requires that

$$\left| 1 + \emptyset h + \frac{(\emptyset h)^2}{2} + \frac{(\emptyset h)^3}{6} + \frac{(\emptyset h)^4}{24} \right| < 1$$

Setting z=$\emptyset h$, then (32) yields $\left| 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} \right| < 1$

The region of absolute stability for the proposed numerical algorithm (23) is defined by the region in the complex plane such that $\left| 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} \right| < 1$.

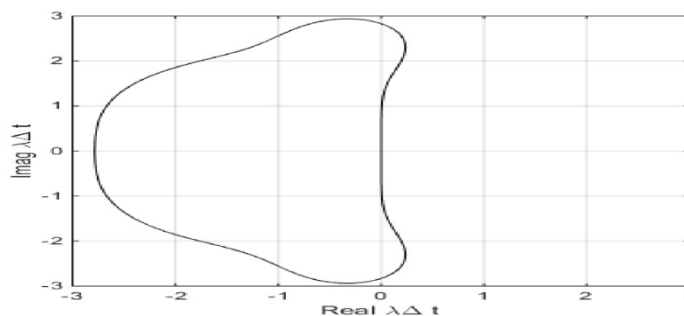The stability region is plotted in the figure 1 given below.

Figure 1. The Stability Region of the Fourth-Order Proposed Numerical Algorithm

4. NUMERICAL PROBLEMS

Examples 1-4 in this part discuss numerical values of y(x) for initial value problems (IVPs) and absolute inaccuracy of the proposed algorithm and a few known algorithms. The results of the proposed new modified algorithm based on exponential and polynomial functions, AM (The fourth-order arithmetic mean), (The fourth-order harmonic mean), (The fourth-order Contra harmonic mean method), and their absolute percentage errors and exact solution at various x-values are shown in Tables 1-4. These findings were validated using the MATLAB software (version 5.0). (R-2014-b).

Problem 4.1 consider initial value problem

$$\frac{dy}{dx} = xy^3 - y, y(0) = 1, \qquad 0 \le x \le 1, \qquad h = 0.1$$

The exact solution of problem 4.1 is

$$y(x) = \frac{2}{\sqrt{2+4x+2e^{2x}}}$$

Table 1. Shows the results of example 4.1

X-Value	Exact	Proposed	Error	AM	Error	HM	Error	C_0M	Error
0	1.0000	1.0000	0	1.0000	0.0002	1.0000	0	1.0000	0
0.1000	0.9088	0.9088	0.000	0.9090	0.0002	0.9087	0.0001	0.9088	0.0001
0.2000	0.8316	0.8316	0.000	0.8319	0.0003	0.8314	0.0002	0.8315	0.0001
0.3000	0.7645	0.7645	0.000	0.7648	0.0003	0.7643	0.0002	0.7643	0.0001
0.4000	0.7049	0.7049	0.000	0.7052	0.0003	0.7046	0.0002	0.7047	0.0001
0.5000	0.6511	0.6511	0.000	0.6514	0.0003	0.6508	0.0003	0.6509	0.0002
0.6000	0.6019	0.6019	0.000	0.6023	0.0003	0.6017	0.0003	0.6018	0.0002
0.7000	0.5566	0.5566	0.000	0.5570	0.0003	0.5564	0.0003	0.5565	0.0002
0.8000	0.5146	0.5146	0.000	0.5149	0.0003	0.5143	0.0003	0.5144	0.0002
0.9000	0.4754	0.4754	0.000	0.4757	0.0003	0.4751	0.0003	0.4752	0.0002
1.0000	0.4388	0.4388	0.000	0.4391	0.0003	0.4385	0.0002	0.4386	0.0002

Problem 4.2 consider initial value problem

$$\frac{dy}{dx} = x + y, y(0) = 1, \qquad 0 \le x \le 1, \qquad h = 0.1$$

The exact solution of problem 4.2 is

$$y(x) = 2e^x - x - 1$$

Table 2. Shows the results of example 4.2

X-Value	Exact	Proposed	Error	AM	Error	HM	Error	C_0M	Error
0	1.0000	1.0000	0	1.0000	0	1.000	0	1.0000	0
0.1000	1.1103	1.1103	0.0000	1.1100	0.0003	1.1105	0.0001	1.1104	0.0001
0.2000	1.2428	1.2428	0.0000	1.2421	0.0007	1.2431	0.0003	1.2430	0.0002
0.3000	1.3997	1.3997	0.0000	1.3986	0.0011	1.4001	0.0004	1.4000	0.0002
0.4000	1.5836	1.5836	0.0000	1.5821	0.0015	1.5842	0.0006	1.5840	0.0003
0.5000	1.7974	1.7974	0.0000	1.7955	0.0020	1.7982	0.0008	1.7979	0.0004
0.6000	2.0442	2.0442	0.0000	2.0417	0.0025	2.0452	0.0009	2.0447	0.0005
0.7000	2.3275	2.3275	0.0000	2.3244	0.0031	2.3286	0.0011	2.3281	0.0006
0.8000	2.6511	2.6511	0.0000	2.6473	0.0038	2.6524	0.0014	2.6518	0.0007
0.9000	3.0192	3.0192	0.0000	3.0146	0.0046	3.0208	0.0016	3.0201	0.0009
1.0000	3.4366	3.4365	0.0000	3.4311	0.0054	3.4384	0.0019	3.4376	0.0010

Problem 4.3 consider initial value problem

$$\frac{dy}{dx} = xy^2 - y, y(0) = 1, \qquad 0 \le x \le 1, \qquad h = 0.1$$

The exact solution of problem 4.3 is

$$y(x) = \frac{1}{x+1}$$

Table 3. Shows the results of example 4.3

X-Value	Exact	Proposed	Error	AM	Error	HM	Error	C_0M	Error
0	1.0000	1.0000	0	1.0000	0	1.0000	0	1.0000	0.0001
0.1000	0.9091	0.9091	0.0000	0.9093	0.0002	0.9090	0.0001	0.9090	0.0001
0.2000	0.8333	0.8333	0.0000	0.8337	0.0004	0.8331	0.0002	0.8332	0.0002
0.3000	0.7692	0.7692	0.0000	0.7697	0.0004	0.7689	0.0003	0.7691	0.0002
0.4000	0.7143	0.7143	0.0000	0.7148	0.0005	0.7139	0.0003	0.7141	0.0002
0.5000	0.6667	0.6667	0.0000	0.6672	0.0006	0.6663	0.0004	0.6665	0.0002
0.6000	0.6250	0.6250	0.0000	0.6256	0.0006	0.6246	0.0004	0.6248	0.0002
0.7000	0.5882	0.5882	0.0000	0.5889	0.0006	0.5878	0.0004	0.5880	0.0002
0.8000	0.5556	0.5556	0.0000	0.5562	0.0007	0.5551	0.0004	0.5553	0.0003
0.9000	0.5263	0.5263	0.0000	0.5270	0.0007	0.5258	0.0005	0.5260	0.0003
1.0000	0.5000	0.5000	0.0000	0.5007	0.0007	0.4995	0.0005	0.4997	0.0004

Problem 4.4 consider initial value problem

$$\frac{dy}{dx} = \frac{y^3 \cos x}{2}, y(0) = 1, \qquad 0 \le x \le 1, \qquad h = 0.1$$

The exact solution of problem 4.4 is

$$y(x) = \frac{1}{\sqrt{1 - \sin x}}$$

Table 4. Shows the results of example 4.4

X-Value	Exact	Composed	Error	AM	Error	HM	Error	C_OM	Error
0	1.0000	1.0000	0	1.0000	0	1.0000	0	1.0000	0
0.1000	1.0540	1.0540	0.0000	1.0539	0.0001	1.0540	0.0000	1.0540	0.0000
0.2000	1.1171	1.1172	0.0001	1.1170	0.0001	1.1171	0.0000	1.1171	0.0000
0.3000	1.1914	1.1915	0.0001	1.1912	0.0002	1.1914	0.0000	1.1914	0.0000
0.4000	1.2798	1.2799	0.0002	1.2794	0.0003	1.2797	0.0000	1.2797	0.0000
0.5000	1.3860	1.3862	0.0002	1.3855	0.0005	1.3859	0.0001	1.3859	0.0001
0.6000	1.5156	1.5159	0.0003	1.5148	0.008	1.5154	0.0002	1.5154	0.0002
0.7000	1.6765	1.6770	0.0005	1.6753	0.0012	1.6762	0.0003	1.6762	0.0003
0.8000	1.8810	1.8816	0.0006	1.8790	0.0020	1.8804	0.0005	1.8803	0.0006
0.9000	2.1483	2.1490	0.0007	2.1449	0.0034	2.1475	0.0008	2.1471	0.00012
1.0000	2.5116	2.5119	0.0003	2.5054	0.0062	2.5105	0.0011	2.5091	0.00025

We notice that in Tables 1-4, the error incurred in the fourth-order arithmetic mean (AM), The fourth-order harmonic mean (HM) and The fourth-order Contra harmonic mean (C_OM) is greater than that of the proposed numerical algorithm and the same time get larger as *n* increases. Hence proposed numerical algorithm is more accurate than the fourth-order arithmetic mean (AM), The fourth-order harmonic mean (HM), and The fourth-order Contra harmonic mean (C_OM) as we can see from Tables 1-4.

5. CONCLUSION

In this research work, a new modified numerical algorithm has been proposed for solving the IVPs in ODEs (ordinary differential equations). The proposed algorithm has been created from exponential and polynomial type functions. The outcomes of the proposed algorithm have been compared with the fourth-order arithmetic means (AM), the fourth-order harmonic mean (HM), and the fourth-order Contra harmonic mean (C_OM), and found that the proposed algorithm has been fourth-order and performing greater accuracy as compared to all mentioned algorithms and others well known second-order algorithms and third-order algorithms. The proposed numerical algorithm has also been analyzed for stability, consistency, and the local truncation error; and found that the proposed algorithm is more stable, consistent, and fourth-order accurate.

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SOLUTION OF INITIAL VALUE PROBLEMS

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