Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.12, No.1, 2022

Effects of inertia on Newtonian fluid in squeezed film by recursive approach

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Abstract

The thesis accomplishes the theoretical study of the effect of inertia on Newtonian fluid in squeezed. This research undertaking to get in ingenious knowledge for the procedure of the axisymmetric viscous fluid flow in between parallel plates steadily approaching to each other, as well the inertia effect is under consideration. Thereby, the crucial part of this thesis is to theoretically investigate rather than experimentally. Be sure that as it may, the expectation from this study is that it could be experimentally performed, so will get practical benefits, in the form of the improvement in the process of flow of oil in bearing and governs with capacity of load – bearing and improving the results of oil in bearing. The primary focused object of this work is to develop a mathematical model, thereby, to calculate the velocity profile likewise radial and axial velocity and pressure. In this research work, the squeezed film of Newtonian fluid between two disks is taken to obtain an analytical solution (PDE – Partial Differential Equations) subject to the favorable boundary conditions, as well as the inertia effect is under consideration. In this thesis the recursive approach is utilized to get an analytical solution and the obtained solution is examined with perturbation method. The examined solution has been found as the main objective of this study. It is analyzed that the recursive approach is easy to use and appears more effective.

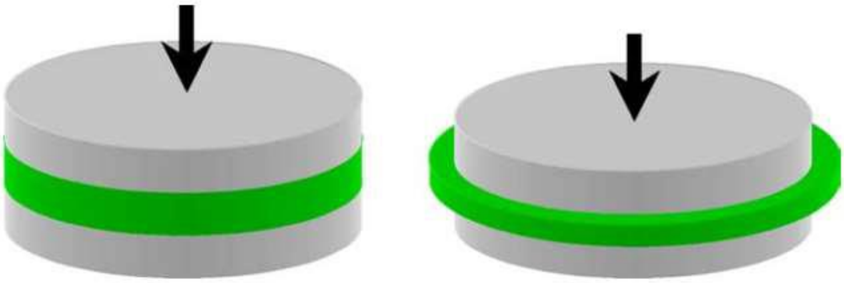
Keywords: Newtonian fluid, Non-Newtonian fluids. Ideal fluids Squeezed fluid, Inertia effect, Velocity Profile, Pressure.

DOI: 10.7176/MTM/12-1-04

Publication date: February 28th 2022

1. Introduction

Computational fluid dynamics (CFD) is a discipline of mathematics that studies the mechanics of fluids, such as liquids, plasmas, and gases, as well as the forces that operate over them. CFD is based on the Navier-Stoke equations, that explain the relationship involving pressure, velocity, density, and temperature in a fluid motion. It solves and examines problems that involve fluid motion using numerical and analytical techniques, computational analysis, and productivity tools, and it models and simulates liquid and gas interactions with surfaces as defined by boundary conditions using the latest in computing devices and elegant programming techniques. The fluid is a material which continuously collapses with the applied shear stress. In this part, the research has been investigated with behavior of Newtonian fluid, As the fluids which obey Newton's law of viscosity, it is known as a Newtonian fluid. Whereas, it covered various applications in the field of engineering and applied science. The number of fields defined as Atmospheric physics, material processing, environmental engineering, mining engineering, industrial systems and a lot of others. In the industries and chemical process, the behavior of fluid is justified for pumping and mixing. Usually usage of fluid is mentioned as, toothpaste, honey, diesel, engine oil, custard, nail paint and much more. The fluid could be used between two circular disks either would be Newtonian or Non – Newtonian. In the techniques of squeezing flows involve sample squeeze of molten material, the sample is between axisymmetric and disc-shaped. (Osvaldo H. companella & Mich peleg 2002). Squeeze flows are those flows in which the substance is compressed in between two circular disks and it radially squeezed out within the space or out of space. as the density of fluid changes with the application of external force, it is known as compressible fluid.



The properties of fluid flow between the parallel plates is a crucial point for researchers to interpret the squeezed flow problems. The contemporary account of this study has a great concern. whereas, the shade of this study reflects all over the endeavor of life sciences. Thereby arms of this flow spreads around the fields life sciences: likewise, Physical Sciences, Bio – Sciences, and Engineering, etc., such as, it is utilized in foodstuff processing, under the teeth chewing or eating any substance resembling the compression in between the (irregular) plates, comprising the food in between the palate and tongue expressed as squeezed fluid (Jackson, J. D. 1963). flow inside nasogastric tubes and syringes, injections, compression, polymer processing and hydraulic lifts, etc. In such a context fluid flow applications processes have a great concern to analyze the momentum equation. The fateful point is that the pioneers work under the assumption of lubrication, the formulation is presented by (Stefan, J. 1875). It makes it intense to obligate researchers and scientists towards squeezed fluid flow. The type of Newtonian squeezed fluid, in between the plates the rigid space is a system of quality type hydrodynamics. The main trigger is that the attraction of researchers towards this problem taking place. Its significance in theoretical and practical lubricants have great concern to this problem. In view of fact is that it often utilized for flow of oil in bearing and governs with capacity of load – bearing (Tayler, A. B. 1986). In the view of other side (Jackson, J. D. 1963), provide a theoretical concept of analyzing the squeezed flow problem. Since, there is lack of study on creeping steady axisymmetric squeeze flow of viscous fluid between two circular disks, hence the study is being carried out. The governing equations of the squeeze flow of viscous fluid film are partial differential equations on axisymmetric form with non- homogenous boundary conditions. Generally, this type of equation is solved either numerically or analytically. The recursive approach given by Langlois has been used to linearize these equations and analytical solutions have been obtained on definite conditions. Also terminologies for the physical parameters such as velocity profile and pressure gradient to be found.

2. Fluid and its importance

The universe is surrounded with different kinds of materials. Such kind of materials having not any affection with shearing force is named as fluid (liquid and gasses). Furthermore, in a particular way it can be classified as the substance which subsequently distorts (stream) in an attaches shear forces on a single form. The stream found in all fields of sciences. As for living things concerns it needs to be liquid stream, furthermore, it is found in nature, likewise, environments dependent on the liquid stream, such as lakes stream, rivers, oceans and more ever barometrical streams forms affecting climate change in form of the atmospheric change in the whole region. Likewise, wind converted in clouds on basis of precipitation which is replying to the topographical conditions. Therefore, the universe is going toward great changes due natural changes in fluids such as changes in particles of sea water, changes in fluids inside the earth which have great effects on environmental materials likewise changes in gasses stream and liquids stream. On the other hands it is perceive that stream is founded in all the areas connecting with life sciences, unless without liquid fluid stream life sciences would be unthinkable. Naturally, the environment depends on the liquid fluid stream, that steam is found in lakes, oceans, rivers at all. The changes in gasses like in wind, which directly affects life sciences, on this basis environmental changes create a lot of problems for the life of living things. Likewise, changes in wind stream making odd atmosphere which declining agriculture and non-agriculture areas in dark situation s. In this way, in an atmosphere wind stream plays a crucial role in agricultural growth. The fluid flow is essential for growth of living things such as, (plants, animal and human) in this way few of flows depending on the transport process which makes oxygen accessible for the living things particularly for human body which is necessary for long life, as like in human body through veins necessary supplements are transferred to the cells with mass flow. The fluid is further divided into two groups based on how they react to shear and normal stresses, which affect fluid properties. At the moment where some fluid parameters modify their volume, density in response to applied force or normal force (temperature or pressure), fluid parameters are identified as incompressible fluid flow (Durst, 2008).

Furthermore, the fluids are distinguished by their shear (thickness) behavior, ideal fluids, Newtonian fluids, and non-Newtonian fluids.

3. Ideal fluids or inviscid fluids

Whenever there is a negligibly small frictional force between fluid particles, such as $\eta = 0$. This type of fluid is known as inviscid or ideal fluid since it has no shear stress or viscosity.

4. Newtonian Fluids

In literature, Newtonian fluids are fluid materials in which the rate of deformation is determined by a linearly linked shear stress. These fluid forms also follow Newton's Second Law of Motion, sometimes known as like.

$$\mathrm{T} \propto \dot{\gamma} \qquad 4.1$$

T denotes the fluid's stress tensor, whereas denotes the fluid's strain rate. The presented Newtonian fluid equation with relation to Ericksen tensor A_1 can be defined in a specific form like that,

$$\mathrm{T} = \eta A_1 \qquad 4.2$$

As denotes the viscosity coefficient whereas, A1 is defined in specific form by mean of,

$$A_1 = \nabla \mathrm{V} + \nabla \mathrm{V}^{\mathrm{T}} \qquad 4.3$$

Therefore, ∇ signifies the gradient, although the superscript T denotes the tensor's transposition, and V indicates the vector's velocity. Whereas, milk, water, mineral oil, sugar solution, most fluid solutions, corn syrup, and glycerin are the maximum of listed examples for Newtonian fluids.

5. Non-newtonian fluid

The fluids which do not obey Newton's law of viscosity are defined as non-Newtonian fluids. Number of fluids with long chain molecules and various complex molecular structures included in this category such as numerous rubber-like fluids does not obey the Newtonian hypothesis law of viscosity. This is the component of a fluid whose shear stress is non-linear in terms of shear rate, or whose fluid flow does not flow through the origin in a curve form. Flow of fluid somehow doesn't depend on conditions of fluid velocity such as in the geometry of the problem, like shear rate, and even on the kinematic of the fluid element are all in under certain consideration. Peanut butter, toothpaste, drilling mud, egg whites, multi-grade engine oils, slurries, molten polymers, and other non-Newtonian fluids are being considered. These fluids are divided into three categories: time independent (Visco-Inelastic), time dependent (Visco-Dependent), and viscoelastic fluids.

5.1 Time-Independent Fluids

These fluids are viscous inelastic or generalized Newtonian fluids in which the rate of deformation at each point can be determined purely by the value of the shear stress at that point. At rest, these fluids are homogenous and isotropic. A generic constitutive relation of the form can be used to define the flow behavior of such fluids.

$$S = F(\dot{\gamma}) \qquad 5.1$$

Eq. 5.1 indicates that the strain rate at each location inside the sheared fluid is solely governed by the shear stress at that point, or vice versa. The fluids in this category are further divided into three types: pseudo plastic (shear thinning), dilatant (shear thickening), and visco – plastic fluids.

5.2 Time-Dependent Fluids

Time-dependent fluids are those whose stress is determined by both the rate of deformation and the time of deformation. The apparent viscosity is determined by the rate of deformation as well as the length of time it takes to distort. Because applied shear alters the fluid's whole structure, viscosity of time-dependent fluids increases or decreases with the passage of time in the presence of a constant rate of deformation. Rheopectic fluid, also known as shear thickening over time, is a fluid whose viscosity increases with time, Thixotropic fluid, or time thinning fluid, is a fluid whose viscosity decreases with time. Shear-dependent rheopexy and thixotropic are both time-dependent phenomena. Yogurt and paint are examples of thixotropic materials, while gypsum pastes and printer inks are examples of rheopectic materials.

5.3 Viscoelastic Fluid

Viscoelastic fluids are non-Newtonian fluids having elasticity and memory that are time-dependent. Although viscoelastic fluids contain a certain amount of energy due to flow, when applied stresses are eliminated, the fluid strives to revert to its prior condition totally or partially. The natural state of viscoelastic fluids changes continuously throughout flow; they seek to attain an instantaneously deformed state but only partially recover; this property of the fluids is known as elasticity and is also known as fluid memory: Mucus, liquid soap, pudding, toothpaste, and clay, which contain both fluids, are examples of viscoelastic fluids.

5.4 Deborah Number

Its named after Deborah, an Old Testament prophetess who said, "The mountains flowed before the Lord," inspired the name. The Deborah number (De) is a mathematical term to describe the ratio of stress relaxation time to process stress time (t). As defined by mathematics,

$$D_e = \frac{\lambda}{t} \tag{5.2}$$

Deborah's number is proportional to the fluid's relaxation time. It's an elasticity number, and in viscoelastic fluids, it's an essential dimensionless parameter. Deborah's number refers to characteristic velocity, which is defined as U, and character length, which is defined as R,

$$D_e = \frac{\lambda \mathrm{U}}{t} \tag{5.3}$$

This number is also known as the Weiss-Enberg number. As the smallest size of Deborah's number is under consideration then the material acts like a Newtonian fluid. As the relaxation period increases, fluidity converts to solidity, and the Deborah number rises, the material's behavior changes to non-Newtonian. Viscoelastic materials act like solids at very high Deborah numbers.

5.5 Reynolds Number

The Reynolds number is a dimensionless number that defines whether a flow pattern in a pipe is laminar or turbulent. The Reynolds number is determined by the ratio of inertial forces to viscous forces.

$$R_e = \frac{\rho \mathrm{vH}}{\mu} \tag{5.4}$$

The Reynolds number is defined as: R_e, as fluid density is represented by ρ, flow velocity is represented by v, and H is the requisite geometrical distance/diameter, while fluid viscosity is expressed by μ.

6. Methodology.

The theme of this study is to obtain analytical solutions by recursive approach, for the following parameters, velocity profile and pressure distribution, for this sake following equations for velocity profile, pressure distribution, are sought in the form of perturbation of state of rest.

$$u(r,z) = \sum_{i=1}^{2} \varepsilon^{(i)} \; u^{(i)}(r,z) \tag{6.1}$$

$$w(r,z) = \sum_{i=1}^{2} \varepsilon^{(i)} \; w^{(i)}(r,z) \tag{6.2}$$

$$p(r,z) = constant + \sum_{i=1}^{2} \varepsilon^{(i)}\ p^{(i)}(r,z) \qquad 6.3$$

The boundary conditions for $[u_i^{(1)},\ p^{(1)}], [u_i^{(2)},\ p^{(2)}]$ guide to dynamical equations system, whereas, $[u_i^{(1)},\ p^{(1)}]$ as defined by equations *(4.1) & (4.2)* leads to a solution of the motion equations for Rivlin – Ericksen' arbitrary constants. The negations of terms up to order two. At every computation, gets linear equations for dynamical system. The first order describe the $[u_i^{(1)},\ p^{(1)}]$ leads to exactly for the equations, known as governing Newtonian flow fluid.

The equations $[u_i^{(2)},\ p^{(2)}]$ looks likes similar to the previous equations, except involving nonhomogeneous terms $[u_i^{(1)}]$ with contribution of the inertial force term.

Equations (4.1 – 4.3) are written in component form and the mentioned assumptions are defined as under,

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad 6.4$$

r – components

$$\rho\left[u\ \frac{\partial u}{\partial r} + w\ \frac{\partial u}{\partial z}\right] = -\frac{\partial p}{\partial r} + \mu\ \left(\nabla^2 \mu - \frac{1}{r^2}\ \mu\right) \qquad 6.5$$

z – components

$$\rho\left[u\ \frac{\partial w}{\partial r} + w\ \frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\ (\nabla^2 w) \qquad 6.6$$

With the use of boundary conditions of the problem according to the assumptions let the velocity vector is given by,

$$v = \left[u^{(1)}(r,z)\ ,0\ ,w^{(1)}(r,z)\right] \qquad 6.7$$

Whereas $u^{(1)}$indicates the redial velocity and $w^{(1)}$ is the normal velocity. ρ defined as constant density, p is pressure, following assumptions are used as under.

Equation of motion governing flow of study incompressible Newtonian fluid. The following equations indicates continuity, momentum (Ghori, 2007).

$$O\left(\epsilon^{(1)}\right):\ \nabla \cdot v^{(1)} = 0 \qquad 6.8$$

$$-\ \nabla \cdot p^{(1)} +\ \mu\ \underline{A}_1^{(1)} = 0 \qquad 6.9$$

Whereas,

$$\underline{A}_1^{(1)} = \nabla \cdot \underline{v}^{(1)} + \nabla \cdot \left(\underline{v}^{(1)}\right)^T \qquad 6.10$$

Substituting (12-13) into equations (5-6) and collecting the coefficients of equal powers of ε, we get the following first and second boundary value problems,

First order $\boldsymbol{O(\varepsilon)}$ problem associated with non-homogenous boundary conditions

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad 6.11$$

$$\rho\left[u^{(1)}\ \frac{\partial u^{(1)}}{\partial r}+w^{(1)}\frac{\partial u^{(1)}}{\partial z}\right]=\ -\ \frac{\partial p^{(1)}}{\partial r}+\mu\ \left(\nabla^2 u^{(1)}-\ \frac{1}{r^2}\ u^{(1)}\right) \qquad 6.12$$

$$\rho\left[u^{(1)}\ \frac{\partial w^{(1)}}{\partial r}+w^{(1)}\ \frac{\partial w^{(1)}}{\partial z}\right]=\ -\ \frac{\partial p^{(1)}}{\partial z}+\ \mu\left(\nabla^2 w^{(1)}\right) \qquad 6.13$$

Subject to boundary conditions,

$$w^{(1)}=-\ V(t),\qquad u^{(1)}\ =\ 0,\qquad at,\qquad z=H(t)$$

$$w^{(1)}=V(t),\qquad w^{(1)}=\ 0,\qquad at,\qquad z=-\ H(t) \qquad 6.14$$

Second order $O(\varepsilon^2)$ problem with associated homogeneous boundary conditions

$$\frac{\partial u}{\partial r}+\ \frac{u}{r}+\ \frac{\partial w}{\partial z}=0 \qquad 6.15$$

$$\rho\left[u^{(1)}\ \frac{\partial u^{(1)}}{\partial r}+w^{(1)}\frac{\partial u^{(1)}}{\partial z}\right]=\ -\ \frac{\partial p^{(2)}}{\partial r}+\mu\ \left(\nabla^2 u^{(2)}-\ \frac{1}{r^2}\ u^{(2)}\right) \qquad 6.16$$

$$\rho\left[u^{(1)}\ \frac{\partial w^{(1)}}{\partial r}+w^{(1)}\ \frac{\partial w^{(1)}}{\partial z}\right]=\ -\ \frac{\partial p^{(2)}}{\partial z}+\ \mu\left(\nabla^2 w^{(2)}\right) \qquad 6.17$$

Subject to boundary conditions,

$$w^{(2)}=0,\qquad u^{(2)}\ =\ 0,\qquad at,\qquad z=H(t)$$

$$w^{(2)}=0,\qquad w^{(2)}=\ 0,\qquad at,\qquad z=-\ H(t) \qquad 6.18$$

Velocity field for $O(\varepsilon)$ first order problem

Rewriting the system of partial differential equations (1.8.14 – 1.8.16) with associated boundary conditions (1.8.17) in terms of stream function. Defining the stream function for first order problem as,

$$u^{(1)}=\ -\frac{1}{r}\left(\frac{\partial\psi^{(1)}}{\partial z}\right),\ \ w^{(1)}=\ \frac{1}{r}\left(\frac{\partial\psi^{(1)}}{\partial r}\right) \qquad 6.19$$

$$E^4\psi^{(1)}(r,z)=0 \qquad 6.20$$

$$\frac{1}{r}\left(\frac{\partial\psi^{(1)}}{\partial r}\right)=\ -V,\qquad \frac{\partial\psi^{(1)}}{\partial z}=0,\qquad at,\qquad z\ =\ H(t) \qquad 6.21$$

$$\frac{\partial\psi^{(1)}}{\partial r}\ =\ 0,\qquad \frac{\partial\psi^{(1)}}{\partial z}=0,\qquad at,\qquad z\ =\ 0 \qquad 6.22$$

To obtain the solution of equation (24) corresponding to boundary conditions (25-26), assuming the following solution of stream function are as given

$$\psi^{(1)}(r,z) = r^2 \phi_1 \qquad 6.23$$

After simplification of above given equation gets results as follows,

$$u^{(1)}(r,z) = \frac{3rV}{4H}\left(1 - \left(\frac{z}{h}\right)^2\right), \qquad 6.24$$

$$w^{(1)} = \frac{V}{4}\left(\left(\frac{z}{h}\right)^3 - \frac{3z}{h}\right) \qquad 6.25$$

$$\mathrm{p}^{(1)} = \frac{3\mu V}{H}\left[2\left(\frac{z}{H}\right)^2 - 2\left(\frac{z}{H}\right) - \left(\frac{r}{H}\right)^2\right] \qquad 6.26$$

Velocity field and Pressure for $\boldsymbol{O}(\boldsymbol{\varepsilon}^2)$ **s**econd order problem,

Defining the stream function for second order approximation as below,

$$u^{(2)} = -\frac{1}{r}\left(\frac{\partial\psi^{(2)}}{\partial z}\right), \qquad w^{(2)} = \frac{1}{r}\left(\frac{\partial\psi^{(2)}}{\partial r}\right) \qquad 6.27$$

Equation (15) is identically satisfied and second order approximation equation (16-17) after eliminating the pressure and boundary conditions (19) take the following form,

$$E^4\ \psi^{(2)}(r,z) = \frac{3r^2v^2\rho}{4H^3\mu}\left[\left(\frac{z}{H}\right)^3 - 3\left(\frac{z}{H}\right)\right] \qquad 6.28$$

Whereas,

$$\psi^{(2)}(r,z) = \frac{3r^2v^2\rho}{4H^3\mu}\ \phi^2(z) \qquad 6.29$$

Therefore,

$$\phi^2(z) = H^4\left(-\frac{19z}{840H} + \frac{13z^3}{280H^3} - \frac{1z^5}{40H^5} + \frac{1z^7}{840H^7}\right) \qquad 6.30$$

After simplifications of above equations gets the following results as follows,

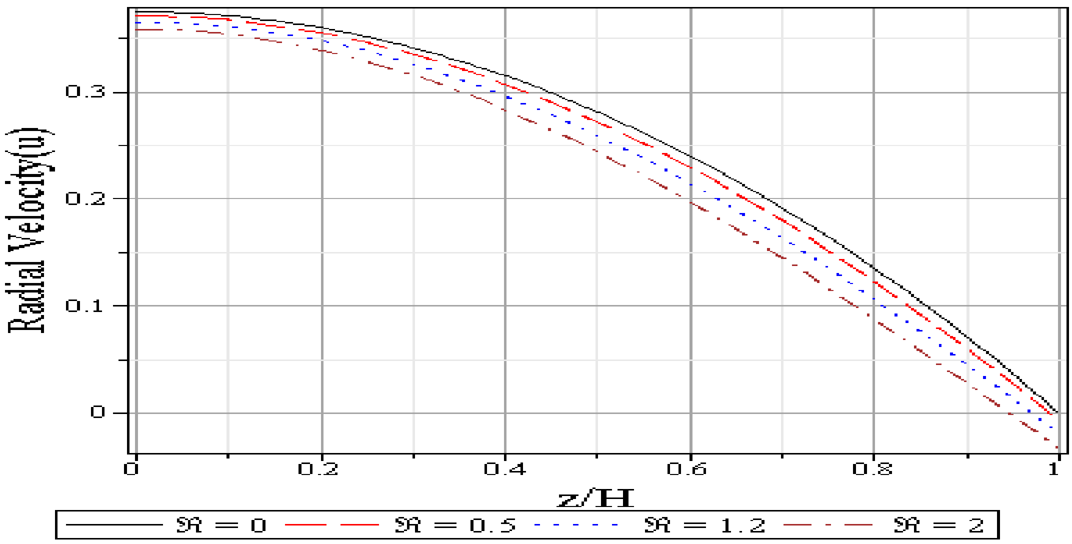
$$u^{(2)} = \frac{3rv^2\rho}{4\mu}\left(\frac{19}{840} + \frac{39z^2}{280H^2} - \frac{1z^4}{8H^4} + \frac{1z^6}{120H^6}\right) \qquad 6.31$$

$$w^{(2)} = \frac{3Hv^2\rho}{2\mu}\left(-\frac{19z}{840H} + \frac{13z^3}{280H^3} - \frac{1z^5}{40H^5} + \frac{1z^7}{840H^7}\right) \qquad 6.32$$

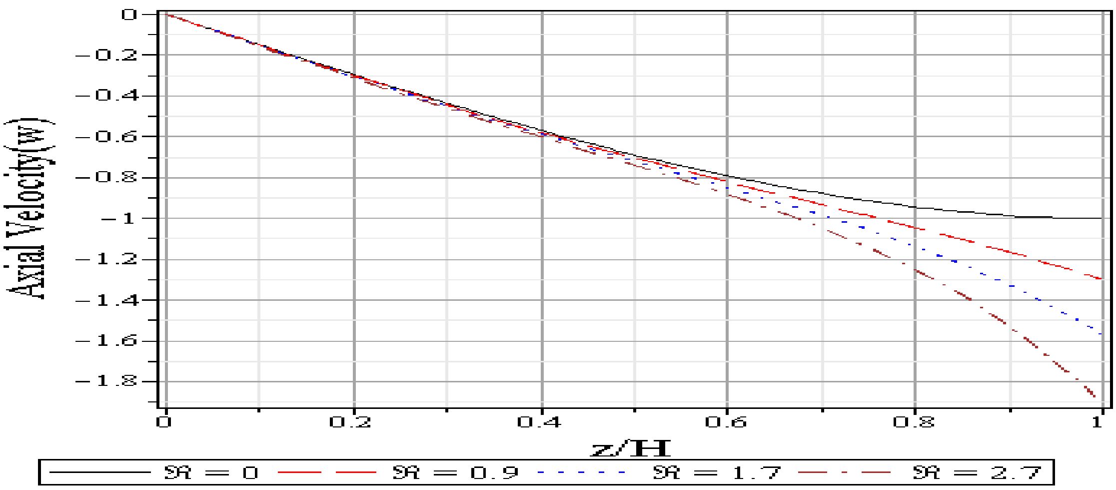
$$p^{(2)} = -\frac{27v\mu}{70H^3}Re\ (r^2 - R^2) - \frac{27v\mu}{70H^3}Re\left[57\left(\frac{z}{H}\right)^2 - 35\left(\frac{z}{H}\right)^4 + 7\left(\frac{z}{H}\right)^6 + 29\right] \qquad 6.33$$

5. Results and discussions

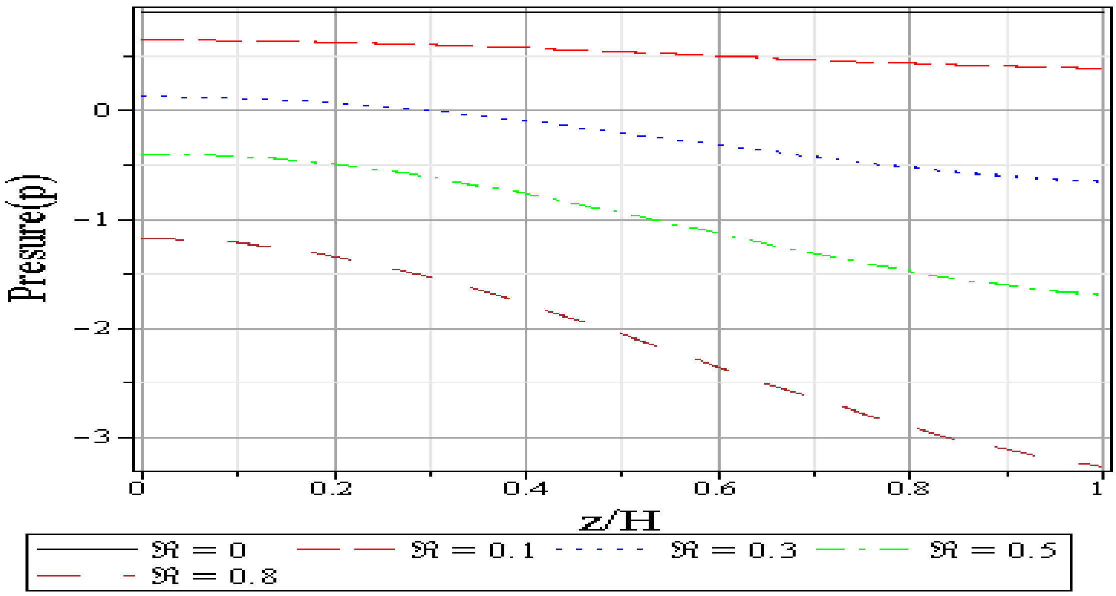
The influence of fluid inertia is taken into account when studying axisymmetric viscous flow formed by two big parallel plates slowly approaching each other. Recursive analysis is used to solve the problem. The resulting findings (perturbation method) are compared, indicating that the recursive approach appears to be more effective and simple to apply. The data is examined based on the figures.



The impact of radial velocities components at various positions of the radial axis is shown in figure(a,b) and observed that as usually increasing the value of r, getting in results increasing velocity. As well it is observed that as usually increasing the value of Re, and getting in results decreasing velocity.



In figure, it is observed that curves of axial velocities w in the domain (0,1) becomes negative on positive values at slightly viscoelastic parameter values components at various positions of thep radial axis is shown in figure(a,b) and observed that as usually increasing the value of r, getting in results increasing velocity.



In figure, high pressure is noticed for the small aspect ratio of film thickness and in case of slightly viscoelastic parameter same behavior of pressure is also observed.

6. Conclusions

The analytical solution of a non – linear system of partial differential equations with non–homogeneous boundary conditions by the recursive approach is obtained and getting the exact expressions of the velocity profile and pressure between the disks. It is critically observed that the axial and radial velocities increase at the higher values of slightly viscoelastic parameter (), this signifies the behavior of fluid. If the viscoelastic parameter () is decreased to zero (δ=0), the behavior of this fluid becomes Newtonian.

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