Even-Power Weighted Distribution

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Abstract
Since the widely using of the weighted distribution in many fields of real life such various areas including medicine, ecology, reliability, and so on , then we try to shed light and record our contribution in this field thru the research. Derivation Even-power weighted distribution (EPWD) with some of statistical properties is discussed in this paper.
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1. Introduction

The Concept of weighted distributions can be traced to the work of Fisher (1934), in connection with his studies on how methods of ascertainment can influence the form of distribution of recorded observations. Later it was introduced and formulated in general terms by Rao (1965), in connection with modeling statistical data where the usual practice of using standard distributions for the purpose was not found to be appropriate. In Rao' s paper, he identified various situations that can be modeled by weighted distributions. These situations refer to instances where the recorded observations cannot be considered as a random sample from the original distributions. This may occur due to non-observability of some events or damage caused to the original observation resulting in a reduced value, or adoption of a sampling procedure which gives unequal chances to the units in the original.
The usefulness and applications of weighted distributions to biased samples invarious areas including medicine, ecology, reliability, and branching processes can be seen in Patil and Rao (1978), Gupta and Kirmani(1990), Gupta and Keating(1985), Oluyede (1999) and in references there in. Within the context of cell kinetics and the early detection of disease, Zelen (1974) introduced weighted distributions to represent what he broadly perceived as length-biased sampling (introduced earlier in Cox, D.R. (1962)). For additional and important results on weighted distributions, see Rao (1997), Patil and Ord(1997), Zelen and Feinleib (1969), Application examples for weighted distribution see El-Shaarawi and Walter (2002), and there are many researches for weighted distribution as, Priyadarshani (2011) introduced a new class of weighted generalized gamma distribution and related distribution, theoretical properties of the generalized gamma model, Jing (2010) introduced the weighted inverse Weibull distribution and beta-inverse Weibull distribution, theoretical properties of them, Castillo and Perez-Casany (1998) introduced new exponential families, that come from the concept of weighted distribution, that include and generalize the poisson distribution, Shaban and Boudrissa (2000) have shown that the length-biased version of the Weibull distribution known as Weibull Length-biased (WLB) distribution is unimodal throughout examining its shape, with other properties, Das and Roy (2011) discussed the length-biased Weighted Generalized Rayleigh distribution with its properties, also they are develop the length-biased from of the weighted Weibull distribution see Das and Roy (2011). *On Some Length-Biased Weighted Weibull Distribution*, Patil and Ord (1976), introduced the concept of size-biased sampling and weighted distributions by identifying some of the situations where the underlying models retain their form. For more important results of weighted distribution you can see also (Oluyede and George (2000), Ghitany and Al-Mutairi (2008), Ahmed ,Reshi and Mir (2013*)*, Broderick X. S., Oluyede and Pararai (2012), Oluyede and Terbeche M (2007*)*).

A mathematical definition of the weighted distribution is as follows. Let $(\Omega; \Upsilon, P)$ be aprobability space, $X: \Omega \rightarrow H$ be a random variable (rv) where $H = (a, b)$be an intervalon real line with $a > 0$ and $b(> a)$can be finite or infinite. When the distribution function (df) $F(x)$of $X$is absolutely continuous with probability density function (pdf) $f(x)$and$w(x)$be a non-negative weight function satisfying $\mu_w = E(w(X)) < \infty$, then the (rv) X_w having pdf

$$f_w(x) = \frac{w(x)f(x)}{\mu_w}, \qquad a < x < b \tag{1}$$

is said to have weighted distribution, corresponding to the distribution of X. The definition in the discrete case is analogous. One of the basic problems when one use weighted distributions as a tool in the selection of suitable models for observed data is the choice of the weight function that fits the data. Depending upon the choice of weight function(x), we have different weighted models. For example, when the weight function depends on the lengths of units of interest, i.e. $w(x) = x$, the resulting distribution is called length-biased. In this case, the pdf of a length-biased (rv) $X_L$is defined as

$$f_L(x) = \frac{xf(x)}{\mu}, \qquad a < x < b \tag{2}$$

Where $\mu = E(X) < \infty$ .More generally, when the sampling mechanism selects units with probability proportional to some measure of the unit size, i.e., when $w(x) = x^c$; $c > 0$, then the resulting distribution is called size-biased. This type of sampling is a generalization of length-biased sampling and majority of the literature is centered on this weight function. Denoting$\mu_c = E(x^c) < \infty$, distribution of the size-biased (rv) X_s of order c is specified by the pdf

$$f_s(x) = \frac{x^c f(x)}{\mu_c}, \qquad a < x < b \tag{3}$$

Clearly, when $c = 1$, (3) reduces to the pdf of a length-biased (rv).

We present the Even-Power Weighted distribution (EPWD), take two types of weighted functions,$w_1(x) = x$ and $w_2(x) = e^x$, we derive the pdf, cdf, and some other useful distributional properties.

2. Even-Power Weighted Distribution

2.1. Definition: The Even-Power Weighted Distribution (EPWD) is given by

$$f_{w^{2r}}(x) = \frac{(w(x))^{2r} f(x)}{W}, \; -\infty < x < \infty, \; r = 1,2,3,\ldots \tag{4}$$

where

$$W = E\left[(w(x))^{2r}\right] = \int_{-\infty}^{\infty} (w(x))^{2r} f(x)dx$$

And $X \in R$, the weight function $w(x)$ raised to the power of $2r$, where $r \in Z^+$. Therefore we can use any other distribution such as the normal distribution or any other distributions.

2.2.1. Even-Power Weighted Normal Distribution

Consider the weight function $w_1(x) = x$, $-\infty < x < \infty$ and Normal distribution given by

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < \mu < \infty \;\; and \;\; \sigma^2 > 0$$

Now let $r = 1$ and

$$W = E\left[(w_1(x))^{2r}\right] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \sigma^2 + \mu^2$$

And according to equation (1) the pdf of EPWND is as follows

$$f_{w_1{}^2}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{5}$$

And the cdf of EPWND is:

$$F_{w_1{}^2}(x; \mu, \sigma^2) = \frac{\int_{-\infty}^{x} t^2 e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} = \frac{1}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} \times \Phi(x) \tag{6}$$

Where $\Phi(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{x} t^2\, e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$

Now let $w_2(x) = e^x$, $-\infty < x < \infty$ with Normal pdf, $f(x; \mu, \sigma^2)$

And $$W = E\left[(w_2(x))^{2r}\right] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} e^{2rx} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx, \; r = 1,2,3,\ldots$$

Now let $x - \mu = u \Longrightarrow x = u + \mu, dx = du$

$$W = \frac{e^{2r\mu+2r^2\sigma^2}}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{u-2r\sigma^2}{\sigma}\right)} du = e^{2r\mu+2r^2\sigma^2}$$

Then the pdf of (EPWND) is

$$f_{w_2{}^{2r}}(x;\mu,\sigma^2,r) = \frac{1}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}}\, e^{2rx-\frac{(x-\mu)^2}{2\sigma^2}} \tag{7}$$

And the cdf of (EPWND) is

$$F_{w_2{}^{2r}}(x;\mu,\sigma^2,r) = \frac{\int_{-\infty}^{x} e^{2rt} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}} = \frac{\int_{-\infty}^{x-\mu} e^{2ru} e^{-\frac{u^2}{2\sigma^2}} du}{\sqrt{2\pi}\,\sigma e^{2r^2\sigma^2}}$$

$$= \frac{\int_{-\infty}^{\frac{x-2r\sigma^2}{\sigma}} e^{-\frac{y^2}{2}} dy}{\sqrt{2\pi}} = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu-2r\sigma^2}{\sqrt{2}\sigma}\right)\right] \tag{8}$$

Where 1) Let $t-\mu = u \implies t = u+\mu,\ dt = du$

2) $\frac{u-2r\sigma^2}{\sigma} = y \implies u = \sigma y + 2r\sigma^2,\ du = \sigma dy$

2.2.1.1. The shape

The shapes of the density functions given in (5) and (7) can be clarified by studying those functions defined over the real line (-∞, ∞) and the behavior of its derivative as follows:

2.2.1.1. 1. Limit and Mode of the function

Note that the limits of the Density functions given in (5) and (7) are as follow:-

$$\lim_{x\to 0} f_{w_1{}^2}(x;\mu,\sigma^2) = \frac{\lim_{x\to 0} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} = \frac{1}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} \times 0 \times e^{-\frac{\mu^2}{2\sigma^2}} = 0 \tag{9}$$

Also

$$\lim_{x\to 0} f_{w_2{}^{2r}}(x;\mu,\sigma^2,r) = \frac{\lim_{x\to 0} e^{2rx-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}} = \frac{1}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}}\, e^{-\frac{\mu^2}{2\sigma^2}} \tag{10}$$

Therefore $$\lim_{x\to\infty} f_{w_1{}^2}(x;\mu,\sigma^2) = \frac{\lim_{x\to\infty} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} = 0 \tag{11}$$

$$\lim_{x\to\infty} f_{w_2{}^{2r}}(x;\mu,\sigma^2,r) = \frac{\lim_{x\to\infty} e^{2rx-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}} = 0 \tag{12}$$

And $$\lim_{x\to-\infty} f_{w_1{}^2}(x;\mu,\sigma^2) = \frac{\lim_{x\to-\infty} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma(\sigma^2+\mu^2)} = 0 \tag{13}$$

$$\lim_{x\to-\infty} f_{w_2{}^{2r}}(x;\mu,\sigma^2,r) = \frac{1}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}} \lim_{x\to-\infty} e^{2rx-\frac{(x-\mu)^2}{2\sigma^2}} = 0 \tag{14}$$

Since $\lim_{x\to\pm\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0$ and $\lim_{x\to-\infty} e^{2rx} = 0$ .From these limits, we conclude that

1) the pdf of EPWND when $w_1(x) = x$, $r = 1$ have two maximum values, modes say x_1 and x_2,
2) the pdf of EPWND when $w_2(x) = e^x$ has one mode say x.

Then we must verify for that by finding the mode (modes) of them as:

2.2.1.1. 2. The mode (modes) of EPWND when

1) $w_1(x) = x$, $r = 1$ is given by $\frac{\partial \log f_{w_1{}^2}(x)}{\partial x} = 0$

Now $\log f_{w_1{}^2}(x) = \log\left(\frac{1}{\sqrt{2\pi}\sigma(\sigma^2+\mu^2)}\right) + 2\log x - \frac{1}{2\sigma^2}(x-\mu)^2$

$$\frac{d\log f_{w_1{}^2}(x)}{dx} = \frac{2}{x} - \frac{1}{\sigma^2}(x-\mu) = 0 \Rightarrow \frac{2\sigma^2 - x(x-\mu)}{x\sigma^2} = 0$$

That is the two modes are as $x_1 = \frac{\mu}{2} + \frac{\sqrt{\mu^2+8\sigma^2}}{2}$ and $x_2 = \frac{\mu}{2} - \frac{\sqrt{\mu^2+8\sigma^2}}{2}$ (15)

2) $w_2(x) = e^x$ we get $\log f_{w_2{}^{2r}}(x) = \log\left(\frac{1}{\sqrt{2\pi}\,\sigma e^{2r\mu+2r^2\sigma^2}}\right) + 2rx - \frac{(x-\mu)^2}{2\sigma^2}$

$$\frac{d\log f_{w_2{}^{2r}}(x)}{dx} = 2r - \frac{x-\mu}{\sigma^2} = 0$$

Then the mode of EPWND is $x = 2r\sigma^2 + \mu$ (16)

The Figure 1 (part -1- and -2-) and Figure 2 shows the shapes of pdf and cdf of EPWND where $w_1(x) = x$, $r = 1$ depending on different values of the parameters, and Figure 3(part -1-, -2- and -3-) shows the shapes of pdf of EPWND where $w_2(x) = e^x$ depending on different values of the parameters.

2.2.1.2. Reliability function

The reliability functions of EPWND are given by

$$R_{f_{w_1{}^2}(x;\mu,\sigma^2)} = 1 - F_{w_1{}^2}(x) = 1 - \frac{1}{\sqrt{2\pi}\sigma(\sigma^2+\mu^2)} \times \Phi(x) \quad (17)$$

$$R_{f_{w_2{}^{2r}}(x;\mu,\sigma^2)} = 1 - F_{w_2{}^{(2r)}}(x) = \frac{1}{2}\left[1 - erf\left(\frac{x-\mu-2r\sigma^2}{\sqrt{2}\sigma}\right)\right] \quad (18)$$

2.2.1.3. Hazard function

The hazard functions of EPWND are given by

$$h_{f_{w_1{}^2}(x;\mu,\sigma^2)} = \frac{x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma(\sigma^2+\mu^2) - \Phi(x)} \quad (19)$$

$$h_{f_{w_2{}^{2r}}(x;\mu,\sigma^2)} = \frac{2e^{2rx}e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma e^{2r\mu+2r^2\sigma^2}\left[1-erf\left(\frac{x-\mu-2r\sigma^2}{\sqrt{2}\sigma}\right)\right]} \quad (20)$$

Figure4 shows the hazard functions of EPWND where $w_1(x) = x$, $r = 1$ and $w_2(x) = e^x$.

2.2.1.4. Moment Generating Function

Theorem (1): If X is distributed EPWND, then its moment generating function is:

1) $M_{X(f_{w_1{}^2})}(t) = \left(1 + \frac{\sigma^2(t^2\sigma^2+2t\mu)}{\sigma^2+\mu^2}\right)e^{t\mu+\frac{t^2\sigma^2}{2}}$, if $w_1(x) = x,\ r = 1$ (21)

2) $M_{X(f_{w_2{}^{2r}})}(t) = \frac{e^{t\mu+\frac{\sigma^2(t+2r)}{2}}}{e^{2r^2\sigma^2}}$, if $w_2(x) = e^x$ (22)

Proof:

1) We have $M_{X(f_{w_1{}^2})}(t) = E(e^{tx}) = \frac{1}{\sqrt{2\pi}\sigma(\sigma^2+\mu^2)}\int_{-\infty}^{\infty} e^{tx}x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$

Let $x - \mu = u \Rightarrow x = u + \mu$, $dx = du$, so

$$M_{X(f_{w_1{}^2})}(t) = \frac{e^{t\mu}\int_{-\infty}^{\infty}(u^2+2\mu u+\mu^2)e^{-\frac{(u^2-2\sigma^2 tu)}{2\sigma^2}}du}{\sqrt{2\pi}\sigma(\sigma^2+\mu^2)} = \left(1 + \frac{\sigma^2 t(t\sigma^2+2\mu)}{\sigma^2+\mu^2}\right)e^{t\mu+\frac{t^2\sigma^2}{2}}$$

2) We have $M_{X(f_{w_2{}^{2r}})}(t) = \frac{\int_{-\infty}^{\infty} e^{(t+2r)x}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx}{\sqrt{2\pi}\sigma e^{2r\mu+2r^2\sigma^2}} = \frac{e^{t\mu+\frac{\sigma^2(t+2r)^2}{2}}}{e^{2r^2\sigma^2}}$ ■

Result (1): The mean of EPWND is given by:

1) $\boldsymbol{\mu}_{f_{w_1{}^2}}(x) = \frac{\mu(3\sigma^2+\mu^2)}{\sigma^2+\mu^2}$, where $w_1(x) = x$, $r = 1$ (23)

2) $\boldsymbol{\mu}_{f_{w_2{}^{2r}}}(x) = \mu + 2r\sigma^2$, where $w_2(x) = e^x$ (24)

Proof: We know that $M_X^{(r)}(t)\big|_{t=0} = E(X)^r,\ r = 1,2,\ldots.$ (25)

So that

1) $M'_{X(f_{w1^2})}(t) = \{\left(1 + \frac{t^2\sigma^4 + 2\mu t\sigma^2}{\sigma^2+\mu^2}\right)(\mu + t\sigma^2) + \left(\frac{2t\sigma^4+2\mu\sigma^2}{\sigma^2+\mu^2}\right)\}e^{t\mu+\frac{t^2\sigma^2}{2}}$

$$M'_{X\left(f_{w1^2}\right)}(0) = \frac{\mu(3\sigma^2+\mu^2)}{\sigma^2+\mu^2}$$

2) $M'_{X(f_{w_2 2r})}(t) = \frac{[\mu+\sigma^2(t+2r)]}{e^{2r^2\sigma^2}} e^{t\mu+\frac{\sigma^2(t+2r)^2}{2}}$

$$M'_X(0) = \mu + 2r\sigma^2$$ ■

Result (2): *The variance of EPWND is given by:*

1) $var_{f_{w_1{}^2}}(x) = \frac{3\sigma^4+6\sigma^2\mu^2+\mu^4}{\sigma^2+\mu^2}$, if $w_1(x) = x\,,\ r = 1$ (26)
2) $var_{f_{w_2{}^{2r}}}(x) = \sigma^2$, if $w_2(x) = e^x$ (27)

Proof:

1) $M''_{X(f_{w1^2})}(t) = \left\{\left(1 + \frac{t^2\sigma^4+2\mu t\sigma^2}{\sigma^2+\mu^2}\right)[(\mu + t\sigma^2)^2 + \sigma^2] + (\mu + t\sigma^2)\left(\frac{2t\sigma^4+2\mu\sigma^2}{\sigma^2+\mu^2}\right) + \left(\frac{2\sigma^4}{\sigma^2+\mu^2}\right) + \left(\frac{2t\sigma^4+2\mu\sigma^2}{\sigma^2+\mu^2}\right)(\mu + t\sigma^2)\right\} e^{t\mu+\frac{t^2\sigma^2}{2}}$

$$M''_{X(f_{w1^2})}(0) = \frac{3\sigma^4+6\sigma^2\mu^2+\mu^4}{\sigma^2+\mu^2} = E_{f_{w_1{}^2}}(X^2)$$

Then the variance of EPWND, if $w_1(x) = x\,,\ r = 1$, is given by

$$var_{f_{w_1{}^2}}(x) = E_{f_{w_1{}^2}}(X^2) - \left[\mu_{f_{w_1{}^2}}(x)\right]^2 = \frac{\sigma^2[3\sigma^4+\mu^2(9\sigma^2+\mu^2-9)]}{(\sigma^2+\mu^2)^2}$$

2) $M''_{X(f_{w_2 2r})}(t) = \frac{[\sigma^2]}{e^{2r^2\sigma^2}} e^{t\mu+\frac{\sigma^2(t+2r)^2}{2}} + \frac{[\mu+\sigma^2(t+2r)]^2}{e^{2r^2\sigma^2}} e^{t\mu+\frac{\sigma^2(t+2r)^2}{2}}$

$$M''_{X(f_{w_2 2r})}(0) = (\mu + 2r\sigma^2)^2 + \sigma^2 = E_{f_{w_2{}^{2r}}}(X^2)$$

Then the variance of EPWND if $w_2(x) = e^x$, is $var_{f_{w_2(2r)}}(x) = E_{f_{w_2{}^{2r}}}(X^2) - \mu^2_{f_{w_2(2r)}}(x) = \sigma^2$ ■

Result (3): The coefficient of variation, skewness and kurtosis of EPWND are, respectively as follows:

1) When $w_1(x) = x\,,\ r = 1$

$$CV_{f_{w1^2}} = \frac{var_{f_{w_1{}^2}}(x)}{E_{f_{w_1{}^2}}(X)} = \frac{\sigma^2[3\sigma^4+\mu^2(9\sigma^2+\mu^2-9)]}{\mu(\sigma^2+\mu^2)(3\sigma^2+\mu^2)} \quad (28)$$

$$CS_{f_{w1^2}} = \frac{(\sigma^2+\mu^2)^2(15\sigma^4\mu+9\sigma^2\mu^2+\mu^5)-3\mu(\sigma^2+\mu^2)(3\sigma^2+\mu^2)(3\sigma^4+6\sigma^2\mu^2+\mu^4)+2\mu^3(3\sigma^2+\mu^2)^3}{\sigma^3[3\sigma^4+\mu^2(9\sigma^2+\mu^2-9)]^{3/2}} \quad (29)$$

$$CK_{f_{w1^2}} = [(\sigma^2 + \mu^2)^3(15\sigma^6 + 21\sigma^4\mu^4 + 30\sigma^5\mu + 15\sigma^2\mu^4 + 6\sigma^3\mu^3 + \mu^6) - 4(\sigma^2 + \mu^2)^2\mu(3\sigma^2 + \mu^2)(15\sigma^4\mu + 9\sigma^2\mu^2 + \mu^5) + 6(\sigma^2 + \mu^2)\mu^2(3\sigma^2 + \mu^2)^2(3\sigma^4 + 6\sigma^2\mu^2 + \mu^4) - 3\mu^4(3\sigma^2 + \mu^2)^4]/\sigma^4(3\sigma^4 + \mu^2(9\sigma^2 + \mu^2 - 9))^2 \quad (30)$$

Proof Using (25), (26) we can prove this result. ■

Figure 5 shows the plots of $CV_{f_{w1^2}}$, $CS_{f_{w1^2}}$ and $CK_{f_{w1^2}}$ of EPWND where $w_1(x) = x\,,\ r = 1$.

2) When $w_2(x) = e^x$

$$CV_{f_{w1^{2r}}} = \frac{\sigma^2}{\tau} \quad \text{where} \quad \tau = \mu + 2r\sigma^2 \quad (31)$$

$$Cs_{f_{w1^{2r}}} = \frac{\tau^3+3\sigma^2\tau-3\tau(\tau^2+\sigma^2)+2\tau^3}{\sigma^2} = 0 \quad (32)$$

$$CK_{f_{w1^{2r}}} = \frac{\tau^4+6\sigma^2\tau^2+3\sigma^4-4\tau(\tau^3+3\sigma^2\tau)+6\tau^2(\tau^2+\sigma^2)-3\tau^4}{\sigma^4} = 3 \quad (33)$$

Proof Using (25), then

$$CV_{f_{w1^{2r}}} = \frac{\sigma^2}{\mu+2r\sigma^2} = \frac{\sigma^2}{\tau}$$

$$CS_{f_{w1^{2r}}} = \frac{E_{f_{w_2{}^{2r}}}(X^3)-3E_{f_{w_2{}^{2r}}}(X)E_{f_{w_2{}^{2r}}}(X^2)+2\left(E_{f_{w_1{}^{2}}}(X)\right)^3}{\left(var_{f_{w_2{}^{2r}}}(X)\right)^{3/2}} = 0$$

$$CK_{f_{w1^{2r}}} = \frac{E_{f_{w_2(2r)}}(X)^4-4E_{f_{w_2(2r)}}(X)E_{f_{w_2{}^{2}}}(X^3)+6\left(E_{f_{w_1(2r)}}(X)\right)^2E_{f_{w_2{}^{2}}}(X^2)-3\left(E_{f_{w_2(2r)}}(X)\right)^4}{\left(var_{f_{w_2(2r)}}(X)\right)^2}$$

where $E_{f_{w_2{}^{2r}}}(X^3) = \tau^3 + 3\sigma^2\tau$ (34)

and $E_{f_{w_2(2r)}}(X^4) = \tau^4 + 6\sigma^2\tau^2 + 3\sigma^4$ (35)

■

Since $Cs_{f_{w1^{2r}}} = 0$ then the shape of the pdf of EPWND where $w_2(x) = e^x$ is symmetric and bell-shaped. From equation (34) we get $CK_{f_{w1^{2r}}} = 3$, then the shape of the pdf of EPWND where $w_2(x) = e^x$ is peaked like the normal distribution.

Figure 6 shows the plots of $CV_{f_{w1^{2r}}}$ of EPWND where $w_2(x) = e^x$.

3. Conclusions

We can develop the weighted distribution into Derivation Even-power weighted distribution (EPWD), like Even-Power Weighted distribution, Even-power Weighted Normal distribution. Therefore, we can discuss some of statistical properties on them.

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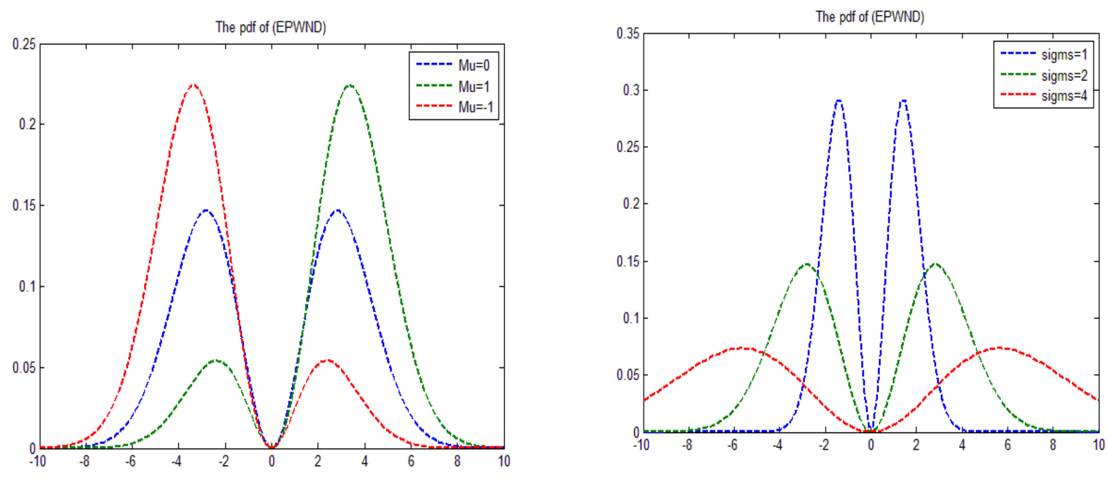
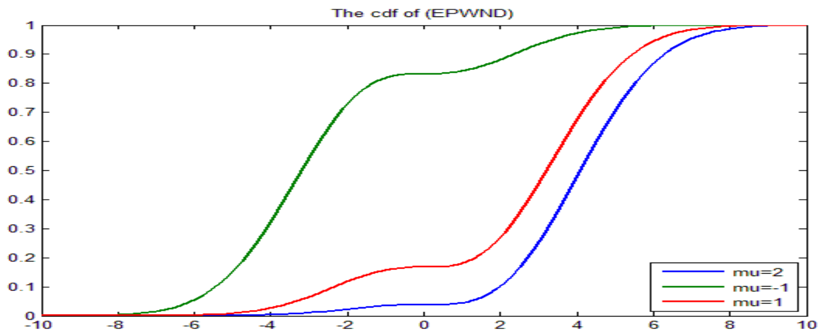


-1- -2-

Figure 1 the pdf of EPWND where $w_1(x) = x$, $r = 1$. (-1-) the parameter μ take the values (0,1,-1) with $\sigma = 2$. (-2-) the parameter σ take the values (1,2,4) with $\mu = 0$.

From Figure 1(-1-), (-2-), we note that μ behaves as a shape parameter, while it is location parameter in the original (Normal) distribution. Also we note if $\mu = 0$ then there are two equal peaks in height, while if $\mu = 1$ then the peak at the right of zero to is more high than other, and if $\mu = -1$ then the peak at the left of zero is more high peak than other. From this we conclude that the more the value of the parameter μ increased peak height on the positive side of zero and decreased peak height on the negative side of zero. Conversely, the smaller the value of the parameter μ increased peak height in the negative direction of the zero and reduced peak height in the positive direction of the zero. In Figure 1(-2-) we can see that the parameter σ behaves as scale and location parameter, the impact parameter σ on the spread of shape the bigger σ, the more spread out the graph is. While in Figure 2 below, we see the impact parameter μ in simple bow in shape of the cdf , the less value of μ, increased curvature simple bow, therefore stay behaves μ as a shape parameter.

Figure 2 the cdf of EPWND where $w_1(x) = x$, $r = 1$ and the parameter μ take the values (2, 1,-1) with fixed σ.



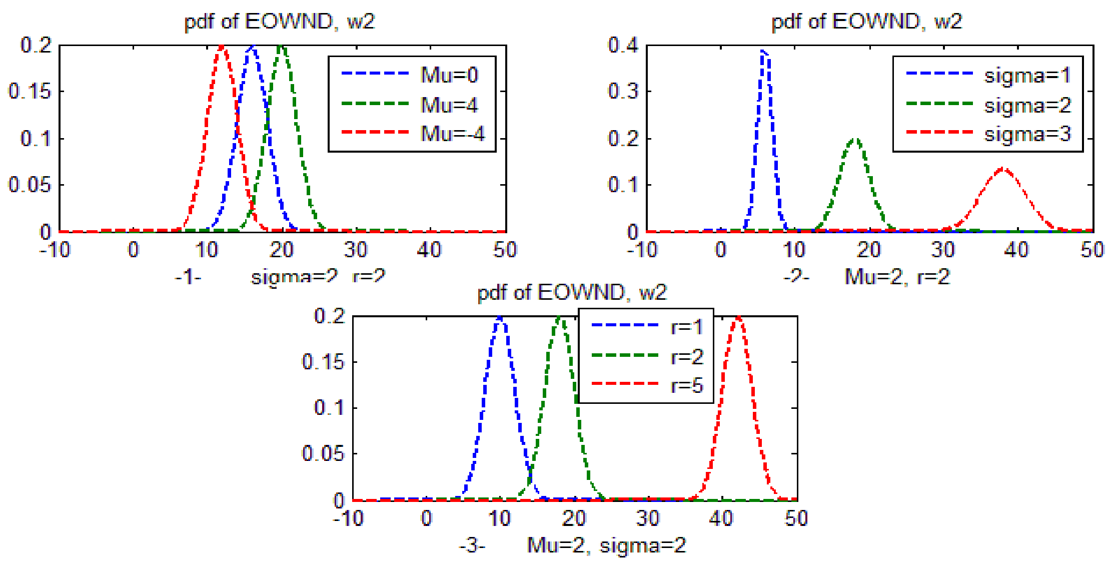


Figure 3 the pdf of EPWND where $w_2(x) = e^x$, the parameters μ, σ and r take the values $(0,4,-4)$, $(1,2,3)$ and $(1,2,5)$, with $\sigma = 2$, $r = 2$, $\mu = 2, r = 2$ and $\mu = 2, \sigma = 2$ respectively.

From Figure 3 we note that the parameter μ behaves as location parameter as in the Normal distribution. In Figure 2 -2- the parameter σ behaves as location and scale parameter, while in the original distribution it is behaves as scale parameter. Also in Figure 2 -3- the parameter r behaves as location parameter

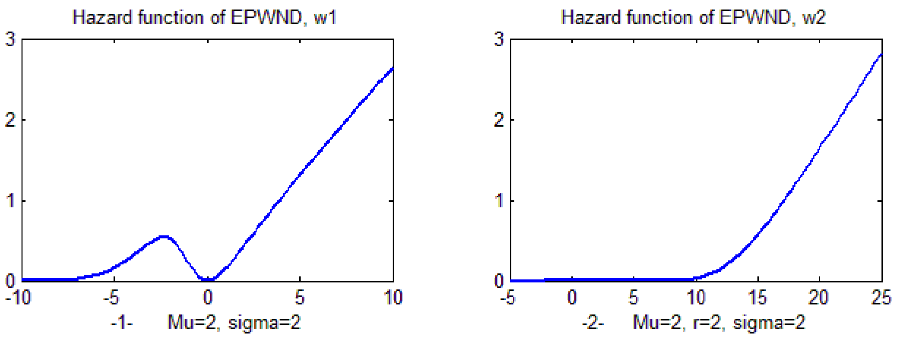


Figure 4: The hazard function of EPWND where $w_1(x) = x$, $r = 1, \mu = -2, \sigma = 2$ and where $w_2(x) = e^x$, $\mu = -2, r = 2, \sigma = 2$ respectively.

Figure 4 -1- shows us that the curve of hazard function is changed from increasing to decreasing into the zero, and then it is strictly increasing, that because the effect of the variable x^2 in the hazard function, while Figure 3 -2- shows us that the hazard function is strictly increasing.

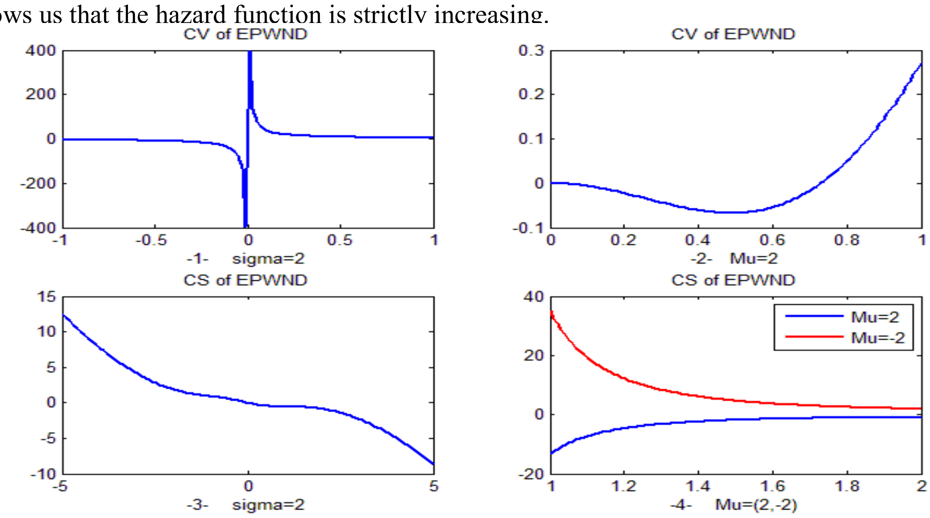


Figure 5 The coefficient of (Variation, Skewness and Kurtosis) of EPWND when $w_1(x) = x$, $r = 1$

Figure 5 -1-, shows us the relationship between $CV_{f_{w1^2}}$ and μ at fixed σ such that the $CV_{f_{w1^2}}$ behaves a constantly decreasing when σ increases and then $CV_{f_{w1^2}}$ monotonically decreases when μ approaches to zero, at which $CV_{f_{w1^2}}$ increases to maximum value. After that $CV_{f_{w1^2}}$ decreases and then constantly increases with increasing μ. Figure 5 -2-, shows us the relationship between $CV_{f_{w1^2}}$ and σ at fixed μ such that the $CV_{f_{w1^2}}$ monotonically decreases and then increases when σ increases. The relationship between $CS_{f_{w1^2}}$ and μ at fixed σ is shown in Figure 5 -3- such that $CS_{f_{w1^2}}$ monotonically decreases with increasing μ . While in Figure 5 -4- $CS_{f_{w1^2}}$ behaves as an exponential distribution with increasing σ at $\mu = -2$, and behaves as natural logarithm function at $\mu = 2$.

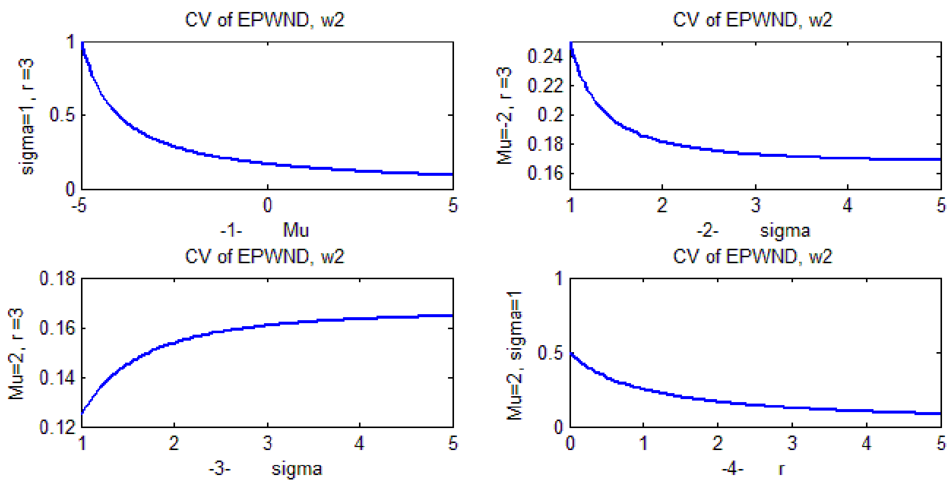


Figure 6 The coefficient of variation of EPWND where $w_2(x) = e^x$ with fixed $(\sigma, r), (\mu, r), (\mu, r)$ and (μ, σ) in parts -1-, -2-, -3-, -4- respectively.

Figures 6, the relationship between μ, σ, r and $CV_{f_{w1^2r}}$ in which $CV_{f_{w1^2r}}$ monotonically decreasing at increasing μ (from -5 to 5 in part -1-), σ (from 1 to 5 in part -2-), r (from 0 to 5 in part -4-) respectively. While in part -3- , the relationship between σ and $CV_{f_{w1^2r}}$ shows that $CV_{f_{w1^2r}}$ monotonically increasing at increasing σ (from 1 to 5 in).