

## Exponential Pareto Distribution

Kareema Abed Al-Kadim<sup>\*(1)</sup>      Mohammad Abdalhussain Boshi<sup>(2)</sup>

College of Education of Pure Sciences/ University of Babylon/ Hilla

<sup>(1)\*</sup>[kareema.kadim@yahoo.com](mailto:kareema.kadim@yahoo.com)    <sup>(2)</sup>[Mohammad\\_abd31@yahoo.com](mailto:Mohammad_abd31@yahoo.com)

### Abstract

In this paper we introduce a new distribution that is dependent on the Exponential and Pareto distribution and present some properties such that the moment generated function, mean, mode, median, variance, the r-th moment about the mean, the r-th moment about the origin, reliability, hazard functions, coefficients of variation, of skewness and of kurtosis. Finally, we estimate the parameter.

**Keyword:** Exponential distribution, Pearson distribution, moment estimation

### Introduction

In the last years many researches based on the beta distribution and described as a new distribution like the distributions: Beta-Pareto which is introduced by Akinsete, Famoye and Lee (2008), Barreto-Souza, Santos, and Cordeiro (2009) constructed Beta generalized exponential, Beta-half-Cauchy is presented by Cordeiro, and Lemonte (2011), Gastellares, Montenegro, and Gauss derived Beta Log-normal, while Morais, Cordeiro, and Audrey (2011) introduced Beta Generalized Logistic, Beta Burr III model for life time data is introduced by Gomes, da-Silva, Cordeiro, and Ortega, the Beta Burr III Model for Lifetime Data, Beta-hyperbolic Secant(BHS) by Mattheas, David(2007), Beta Frechet by Nadarajah, and Gupta(2004) by [8], Beta normal distribution and its application Eugene, N., Lee, C. and Famoye, F. (2002) and Beta exponential by Nadarajah, S. and Kotz, S., 2004. Exponential Pearson coming from the question that is, why the above distribution used only Beta distribution? we propose another kind of this distribution as the form

$$F(x) = \int_a^b f(x)dx, \text{ where } b = \infty \text{ such that } \infty = \frac{1}{1-F^\#(x)}$$

, where  $F^\#(x)$  is any distribution like Normal, Exponential, or Gamma ..... atc. While the previous definition of such distribution is as  $G(x) = \int_0^{F(x)} f(x)dx$ , where  $f(x)$  is the p.d.f. of Beta distribution, and  $F(x)$  is the c.d.f. of any distribution.

### 1.The Probability Density Function of the EPD

The c.d.f. of the EPD is given by the form as

$$F_{e.p}(x) = \int_0^{\frac{1}{1-F^\#(x)}} f^\exists(x) dx$$

where  $F^\#$  is the pareto distribution,  $F(X) = 1 - \left(\frac{p}{x}\right)^\theta$ , and  $f^\exists$  is the exponential distribution,

$$F(X) = e^{-\lambda x}.$$

so that

$$F_{e.p}(X; p, \lambda, \theta) = \int_0^{\frac{1}{1-\left(\frac{p}{x}\right)^\theta}} \lambda e^{-\lambda x} dx, \text{ where } p \text{ is a constant of the pareto distribution}$$

$$F(X; p, \lambda, \theta) = \int_0^{\left(\frac{x}{p}\right)^\theta} \lambda e^{-\lambda x} dx = 1 - e^{-\lambda \left(\frac{x}{p}\right)^\theta}$$

So the c.d.f. of the exponential pareto distribution (E.P.D) is given by

$$\therefore F_{e,p}(X; p, \lambda, \theta) = 1 - e^{-\lambda\left(\frac{x}{p}\right)^\theta} \quad (1)$$

Also the p.d.f. of this distribution is given by :

$$f_{e,p}(X; p, \lambda, \theta) = \frac{dF_{e,p}(X; p, \lambda, \theta)}{dx} = \frac{\lambda\theta}{p} \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} I_{(0,\infty)}(x) \quad (2)$$

Which is similar to Weibull distribution that is given as

$$f(x, \theta, \beta) = s\beta x^{\beta-1} e^{-sx^\beta} I_{(0,\infty)}(x) \quad (3) \quad \text{That is equal where } \theta = \beta, p = 1, s = \lambda. \text{ The plot of the p.d.f. and c.d.f. for the EPD is given in Figure 1, 2, \dots, 6.}$$

## 2. Limit of the Probability Density and Distribution Functions

The limit of this distribution is given by the form as

$$\lim_{x \rightarrow 0} f_{e,p}(X; p, \lambda, \theta) = 0 \quad (4)$$

Because

$$\lim_{x \rightarrow 0} \frac{\lambda\theta}{p} \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} = \frac{\lambda\theta}{p} \lim_{x \rightarrow 0} \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} = \frac{\lambda\theta}{p} \underbrace{\left(\frac{0}{p}\right)^{\theta-1}}_0 \underbrace{e^{-\lambda\left(\frac{0}{p}\right)^\theta}}_1 = 0$$

Also

$$\lim_{x \rightarrow \infty} f(x; p, \lambda, \theta) = \lim_{x \rightarrow \infty} \frac{\lambda\theta}{p} \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} = 0 \times \infty = 0 \quad (5)$$

Also since the c.d.f. of this distribution is:

$$F_{e,p}(X; p, \lambda, \theta) = 1 - e^{-\lambda\left(\frac{x}{p}\right)^\theta}$$

So

$$\lim_{x \rightarrow 0} F_{e,p}(X; p, \lambda, \theta) = 0 \quad (6)$$

$$\text{Because } \lim_{x \rightarrow 0} \left[ 1 - e^{-\lambda\left(\frac{x}{p}\right)^\theta} \right] = 1 - 1 = 0$$

$$\text{Where } \lim_{x \rightarrow 0} e^{-\lambda\left(\frac{x}{p}\right)^\theta} = e^{-\lambda\left(\frac{0}{p}\right)^\theta} = 1$$

Also

$$\lim_{x \rightarrow \infty} F_{e,p}(X; p, \lambda, \theta) = 1 \quad (7)$$

Since

$$\lim_{x \rightarrow \infty} F_{e,p}(X; p, \lambda, \theta) = \lim_{x \rightarrow \infty} \left[ 1 - e^{-\lambda\left(\frac{x}{p}\right)^\theta} \right] = 1 - e^{-\lambda\left(\frac{\infty}{p}\right)^\theta} = 1 - 0 = 1$$

### Proposition 1:

The rth moment about the mean of this EPD is as follows :

$$\therefore E(X - \mu)^r = \sum_{j=0}^r C_j^r \left(\frac{p}{\theta\sqrt{\lambda}}\right)^j (-\mu)^{r-j} \Gamma\left(\frac{j}{\theta} + 1\right), r = 1, 2, 3, \dots \quad (8)$$

And the r-moment about the origin is

$$EX^r = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^r \Gamma\left(\frac{r}{\theta} + 1\right) \text{ with } r = 1,2,3, \dots \quad (9)$$

Proof :-

The r-th moment about the mean is given by

$$E(X - \mu)^r = \int_0^\infty (x - \mu)^r f_{e,p}(X; p, \lambda, \theta) dx$$

So

$$E(X - \mu)^r = \int_0^\infty (x - \mu)^r \frac{\lambda\theta}{p} \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} dx$$

Let

$$u = \lambda \left(\frac{x}{p}\right)^\theta \Rightarrow du = \frac{\lambda\theta}{p} \left(\frac{x}{p}\right)^{\theta-1} dx, u(0) = 0 \text{ and } u(\infty) = \infty$$

$$\text{Also } x = p \left(\sqrt{\frac{u}{\lambda}}\right)$$

So we put the above formulas in the integration to have

$$\begin{aligned} E(X - \mu)^r &= \int_0^\infty \left[ p \left(\sqrt{\frac{u}{\lambda}}\right) - \mu \right]^r e^{-u} du \\ &= \int_0^\infty \left[ \frac{p}{\theta\sqrt{\lambda}} u^{\frac{1}{\theta}} - \mu \right]^r e^{-u} du \\ &= \int_0^\infty \sum_{j=0}^r C_j^r \left(\frac{p}{\theta\sqrt{\lambda}} u^{\frac{1}{\theta}}\right)^j (-\mu)^{r-j} e^{-u} du \\ &= \sum_{j=0}^r C_j^r \left(\frac{p}{\theta\sqrt{\lambda}}\right)^j (-\mu)^{r-j} \int_0^\infty u^{\frac{j}{\theta}} e^{-u} du \\ &= \sum_{j=0}^r C_j^r \left(\frac{p}{\theta\sqrt{\lambda}}\right)^j (-\mu)^{r-j} \Gamma\left(\frac{j}{\theta} + 1\right) \end{aligned}$$

Thus the r-th moment about the mean is

$$\therefore E(X - \mu)^r = \sum_{j=0}^r C_j^r \left(\frac{p}{\theta\sqrt{\lambda}}\right)^j (-\mu)^{r-j} \Gamma\left(\frac{j}{\theta} + 1\right), r = 1,2,3, \dots$$

Where  $\mu = EX$ , then if  $\mu = 0$ , thus the r-th moment about the origin is

$$EX^r = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^r \Gamma\left(\frac{r}{\theta} + 1\right) \quad r = 1,2,3, \dots \quad \blacksquare$$

**Proposition 2:**

The coefficients of variance, of skeweness and of kurtosis are given, respectively, as

$$CV = \frac{\sqrt{\left[\Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right]}{\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta}+1\right)} \quad (10)$$

$$CS. = \frac{\left\{ \Gamma\left(\frac{3}{\theta}+1\right) - 3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right) + 2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3 \right\}}{\left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}^{3/2}} \quad (11)$$

$$CK. = \frac{\left\{ -3\left(\Gamma\left(\frac{1}{\theta}+1\right)\right)^4 + 6\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\Gamma\left(\frac{2}{\theta}+1\right) - 4\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{3}{\theta}+1\right) + \Gamma\left(\frac{4}{\theta}+1\right) \right\}}{\left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}^2} - 3 \quad (12)$$

**Proof**

For  $r = 2$  in equation (8), we have

$$E(X - \mu)^2 = \sigma^2 = Var(X) = E(X^2) - (EX)^2$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta} + 1\right) - \left[\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right)\right]^2 = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \left\{ \Gamma\left(\frac{2}{\theta} + 1\right) - \left[\Gamma\left(\frac{1}{\theta} + 1\right)\right]^2 \right\}$$

$$CV. = \frac{\sigma}{\mu} = \frac{\sqrt{E(X-\mu)^2}}{E(X)} = \frac{\sqrt{\left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}}}{\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)} = \frac{\sqrt{\left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}}}{\Gamma\left(\frac{1}{\theta}+1\right)}$$

Now for  $r = 3$ ,

$$E(X - \mu)^3 = E[(x^2 - 2x\mu + \mu^2)(x - \mu)]$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \left\{ \Gamma\left(\frac{3}{\theta} + 1\right) - 3\Gamma\left(\frac{1}{\theta} + 1\right)\Gamma\left(\frac{2}{\theta} + 1\right) + 2\left[\Gamma\left(\frac{1}{\theta} + 1\right)\right]^3 \right\}$$

$$\therefore E(X - \mu)^3 = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \left\{ \Gamma\left(\frac{3}{\theta} + 1\right) - 3\Gamma\left(\frac{1}{\theta} + 1\right)\Gamma\left(\frac{2}{\theta} + 1\right) + 2\left[\Gamma\left(\frac{1}{\theta} + 1\right)\right]^3 \right\}$$

$$CS. = \frac{E(X-\mu)^3}{\sigma^3} = \frac{\left\{ \Gamma\left(\frac{3}{\theta}+1\right) - 3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right) + 2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3 \right\}}{\left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}^{3/2}}$$

And if  $r = 4$

$$E(X - \mu)^4 = E\left[\sum_{j=0}^4 C_j^4 x^j (-\mu)^{4-j}\right] = \sum_{j=0}^4 C_j^4 (-\mu)^{4-j} E(x^j)$$

$$= (-\mu)^4 + 4(-\mu)^3 \frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right) + 6(-\mu)^2 \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta} + 1\right) - 4\mu \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \Gamma\left(\frac{3}{\theta} + 1\right) + \left(\frac{p}{\theta\sqrt{\lambda}}\right)^4 \Gamma\left(\frac{4}{\theta} + 1\right)$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right)\right)^4 - 4\left(\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right)\right)^3 \frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right) + 6\left[\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right)\right]^2 \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta} + 1\right) - 4\frac{p}{\theta\sqrt{\lambda}} \Gamma\left(\frac{1}{\theta} + 1\right) \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \Gamma\left(\frac{3}{\theta} + 1\right) + \left(\frac{p}{\theta\sqrt{\lambda}}\right)^4 \Gamma\left(\frac{4}{\theta} + 1\right)$$

$$\therefore E(X - \mu)^4 = \left[\frac{p}{\theta\sqrt{\lambda}}\right]^4 \left\{ -3\left(\Gamma\left(\frac{1}{\theta} + 1\right)\right)^4 + 6\left[\Gamma\left(\frac{1}{\theta} + 1\right)\right]^2 \Gamma\left(\frac{2}{\theta} + 1\right) - 4\Gamma\left(\frac{1}{\theta} + 1\right)\Gamma\left(\frac{3}{\theta} + 1\right) + \Gamma\left(\frac{4}{\theta} + 1\right) \right\}$$

$$CK. = \frac{E(X-\mu)^4}{\sigma^4} - 3$$

Where  $\sigma$  is the standard deviation

$$= \frac{\left\{ -3\left(\Gamma\left(\frac{1}{\theta}+1\right)\right)^4 + 6\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\Gamma\left(\frac{2}{\theta}+1\right) - 4\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{3}{\theta}+1\right) + \Gamma\left(\frac{4}{\theta}+1\right) \right\}}{\left\{ \Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \right\}^2} - 3$$

The plot of the CS., CS. And CK. are given as follows in figure 7,8 and 9 respectively.

**Proposition 3**

The mode and median of the **EPD** is given as :-

$$mode = p \left( \frac{\theta-1}{\lambda\theta} \right)^{\frac{1}{\theta}} \tag{13}$$

$$median = p \left( \frac{\ln 2}{\lambda} \right)^{\frac{1}{\theta}} \tag{14}$$

**Proof**

The mode of the **EPD** is given as :

$$Mode = \arg \max (f(x))$$

So that

$$\begin{aligned} \ln f_{e,p}(X; p, \lambda, \theta) &= \ln \left\{ \frac{\lambda\theta}{p} \left( \frac{x}{p} \right)^{\theta-1} e^{-\lambda \left( \frac{x}{p} \right)^\theta} \right\} \\ &= \ln \left( \frac{\lambda\theta}{p} \right) + (\theta - 1) \ln \left( \frac{x}{p} \right) - \lambda \left( \frac{x}{p} \right)^\theta \\ \frac{df_{e,p}(X;p,\lambda,\theta)}{dx} &= (\theta - 1) \left( \frac{1}{p} \right) \left( \frac{x}{p} \right)^{\theta-2} - \frac{\lambda\theta}{p} \left( \frac{x}{p} \right)^{\theta-1} \\ \frac{df_{e,p}(X;p,\lambda,\theta)}{dx} = 0 &\Rightarrow (\theta - 1) \left( \frac{1}{p} \right) \left( \frac{x}{p} \right)^{\theta-2} - \frac{\lambda\theta}{p} \left( \frac{x}{p} \right)^{\theta-1} = 0 \\ \frac{\theta-1}{x} &= \frac{\lambda\theta x^{\theta-1}}{p^\theta} \Rightarrow \frac{p^\theta(\theta-1)}{\lambda\theta} = x^\theta \\ \therefore x &= p \left( \frac{\theta-1}{\lambda\theta} \right)^{\frac{1}{\theta}} \end{aligned}$$

The median of this **EPD** is given by

$$\begin{aligned} 0.5 &= \int_0^m f_{e,p}(x; p, \lambda, \theta) dx \\ 0.5 &= 1 - e^{-\lambda \left( \frac{m}{p} \right)^\theta} \Rightarrow 0.5 = e^{-\lambda \left( \frac{m}{p} \right)^\theta} \Rightarrow \ln 2 = \lambda \left( \frac{m}{p} \right)^\theta \\ \therefore m &= p \left( \frac{\ln 2}{\lambda} \right)^{\frac{1}{\theta}} \quad \blacksquare \end{aligned}$$

**Reliability and Hazard Functions**

The reliability function and hazard function is given as:

$$R(x) = e^{-\lambda \left( \frac{x}{p} \right)^\theta} \tag{15}$$

$$h(x) = \frac{\lambda\theta}{p} \left( \frac{x}{p} \right)^{\theta-1} \tag{16}$$

Proof :-

The reliability function of the **EPD** is given by the form as :

$$R(x) = 1 - F(x) = 1 - \left[ 1 - e^{-\lambda \left(\frac{x}{p}\right)^\theta} \right] = e^{-\lambda \left(\frac{x}{p}\right)^\theta}$$

The hazard function is given by

$$h(x) = \frac{f(x)}{F(x)} = \frac{\frac{\lambda \theta \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda \left(\frac{x}{p}\right)^\theta}}{1 - e^{-\lambda \left(\frac{x}{p}\right)^\theta}}}{e^{-\lambda \left(\frac{x}{p}\right)^\theta}} = \frac{\lambda \theta \left(\frac{x}{p}\right)^{\theta-1}}{1 - e^{-\lambda \left(\frac{x}{p}\right)^\theta}}$$

The plot of the hazard function is given in **Figures 12,13 and 14**.

### 3. Estimate of the parameter

Using the MLE, we estimate the parameter of this distribution .

Since

$$f_{e,p}(x; p, \lambda, \theta) = \frac{\lambda \theta \left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda \left(\frac{x}{p}\right)^\theta}}{1 - e^{-\lambda \left(\frac{x}{p}\right)^\theta}}$$

The likelihood function is given by :

$$L(x; p, \lambda, \theta) = \frac{\lambda^n \theta^n \left(\prod_{i=1}^n x_i\right)^{\theta-1} e^{-\lambda \frac{\sum_{i=1}^n x_i^\theta}{p^\theta}}}{p^n}$$

Then the natural logarithm of likelihood is

$$\ln L(x; p, \lambda, \theta) = n \ln \lambda + n \ln \theta - n \ln p + (\theta - 1) \ln \prod_{i=1}^n x_i - n(\theta - 1) \ln p - \lambda \frac{\sum_{i=1}^n x_i^\theta}{p^\theta}$$

$$\frac{\partial \ln L(x; p, \lambda, \theta)}{\partial \theta} = \frac{n}{\theta} + \ln \prod_{i=1}^n x_i - n \ln p - \lambda \sum_{i=1}^n \left(\frac{x_i}{p}\right)^\theta \ln \left(\frac{x_i}{p}\right)$$

$$\frac{\partial \ln L(x; p, \lambda, \theta)}{\partial \lambda} = \frac{n}{\lambda} - \frac{\sum_{i=1}^n x_i^\theta}{p^\theta}$$

$$\frac{\partial \ln L(x; p, \lambda, \theta)}{\partial p} = -\frac{n}{p} - \frac{n(\theta-1)}{p} + \frac{\lambda \theta \sum_{i=1}^n x_i^\theta}{p^{\theta+1}}$$

$$\text{For } \frac{\partial \ln L(x; p, \lambda, \theta)}{\partial \theta} = \frac{\partial \ln L(x; p, \lambda, \theta)}{\partial \lambda} = \frac{\partial \ln L(x; p, \lambda, \theta)}{\partial p} = 0$$

$$\frac{n}{\theta} + \ln \prod_{i=1}^n x_i - n \ln p - \lambda \sum_{i=1}^n \left(\frac{x_i}{p}\right)^\theta \ln \left(\frac{x_i}{p}\right) \dots \dots \dots 1$$

To find the parameter theta using numerical method for solve above equation .

$$\frac{n}{\lambda} - \frac{\sum_{i=1}^n x_i^\theta}{p^\theta} = 0$$

Estimate the parameter lambda when

$$\hat{\lambda} = \frac{np^\theta}{\sum_{i=1}^n x_i^\theta} = \frac{n}{\sum_{i=1}^n \left(\frac{x_i}{p}\right)^\theta} \dots \dots \dots 2$$

$$-\frac{n}{p} - \frac{n(\theta-1)}{p} + \frac{\lambda \theta \sum_{i=1}^n x_i^\theta}{p^{\theta+1}} = 0 \Rightarrow -n - n\theta + n + \frac{\lambda \theta \sum_{i=1}^n x_i^\theta}{p^\theta} = 0 \Rightarrow n\theta = \frac{\lambda \theta \sum_{i=1}^n x_i^\theta}{p^\theta}$$

Estimate of parameter  $\hat{\theta}$  when theta and lambda is know:

$$\hat{\theta} = \sqrt{\frac{\lambda \sum_{i=1}^n x_i^\theta}{n}} \dots\dots\dots 3$$

#### 4. Conclusion

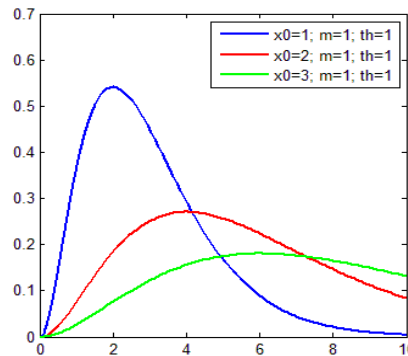
We can construct new distribution based on other the beta distribution like Exponential Pareto with discussion some of its properties.

#### Reference

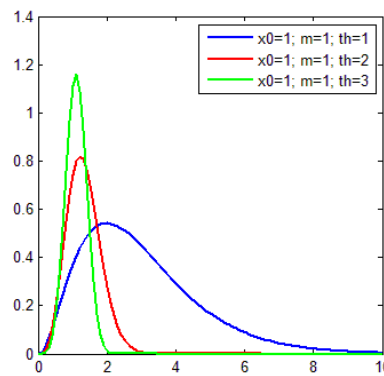
- [1] Akinsete, A., Famoye F. and Lee, C. (2008). **The Beta-Pareto Distribution**. Statistics, 42:6, pp. 547–563.
- [2] Barreto-Souza, W., Santos, A.H.S. and Cordeiro, G.M. (2009). **The beta generalized exponential distribution**. Journal of Statistical Computation and Simulation, 80:2, pp.159–172.
- [3] Cordeiro, Gauss M.; Lemonte, Artur J. (2011). **The Beta-half-Cauchy Distribution**. Journal of probability and statistic, vol 2011 Article ID904705.
- [4] Gastellares, F., Montenegro, L.C. and Gauss, M. C. **The Beta Log-normal Distribution**. [www.ime.unicamp.br/.../Beta\\_Lognormal\\_Distribution](http://www.ime.unicamp.br/.../Beta_Lognormal_Distribution)
- [5] Morais, A., Cordeiro, G and Audrey, H. (2011). **The beta Generalized Logistic Distribution**. Journal of Probability and statistic. [www.imstat.org/bjps/papers/BJPS166.pdf](http://www.imstat.org/bjps/papers/BJPS166.pdf)
- [6] Gomes, A. E., da-Silva, C. Q., Cordeiro, G. M., and Ortega, E. M. M.. **The Beta Burr III Model for Lifetime Data**, [imstat.org/bjps/papers/BJPS179.pdf](http://imstat.org/bjps/papers/BJPS179.pdf)
- [7] Mattheas, J., David, V. (2007). **The Beta-Hyperbolic Secant (BHS) Distribution**. journal of statistic [www.iiste.org/Journals/index.php/MTM](http://www.iiste.org/Journals/index.php/MTM)
- [8] Nadarajah, S., and Gupta, A. K. (2004). **The Beta Frechet Distribution**. Far East Journal of Theoretical Statistics .14, 15–24.
- [9] Eugene, N., Lee, C. and Famoye, F. (2002). **Beta-normal distribution and its applications**. Commun. Statist. - Theory and Methods, 31, 497-512.
- [10] Nadarajah, S. and Kotz, S., 2004. **The beta Gumbel distribution**. Math. Probability. Eng., 10, 323-332.

Dr. Kareema A. k., assistant professor, Babylon University for Pure Mathematics Dept.. Her general specialization is statistics and specific specialization is Mathematics Statistics. Her interesting researches are : Mathematics Statistics, Time Series, and Sampling that are published in international and local Journals. Date of attainability of :

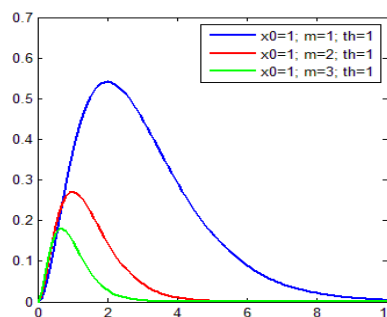
- 1) the Doctor degree in Mathematical Statistics: 30/12/2000
- 2) the Master degree in Statistics: 1989/1990
- 3) the Bachelor's degree in Statistics: 1980/1981
- 4) the Bachelor's degree



**Figure 1:** plot the p.d.f. of the **EPD** with the parameters  $x_0 = 1, 2, 3 ; m = 2 ; th = \theta = 1$



**Figure 2:** plot of the p.d.f. of the exponential pareto distribution with the parameter  $x_0 = 1 ; m = 1, th = \theta = 1, 2, 3$

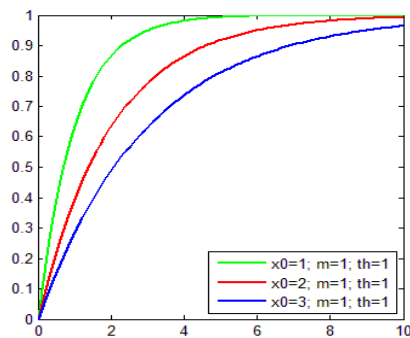


**Figure 3:** plot the p.d.f. of the **EPD** with the parameter  $x_0 = 1, m = 1, 2, 3 ; th = \theta = 1$ .

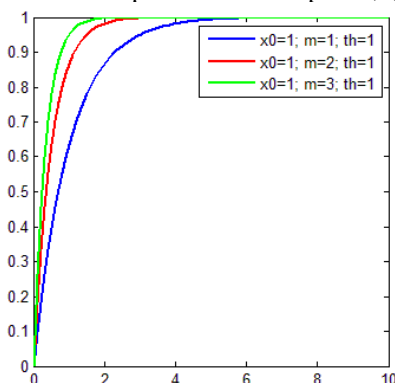
**Figure 1,2,3** show us that the shape of the p.d.f. of **EPD** based on fixed two parameters with change the other parameter, which is similar to Weibull distribution.

Also the c.d.f. of the **E.P.D** is given by :

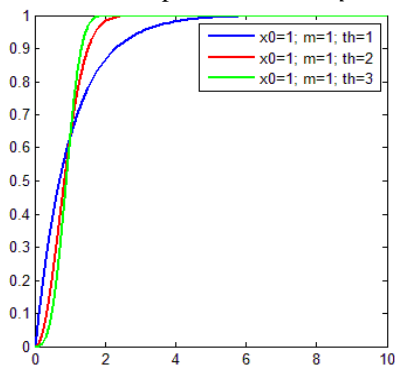




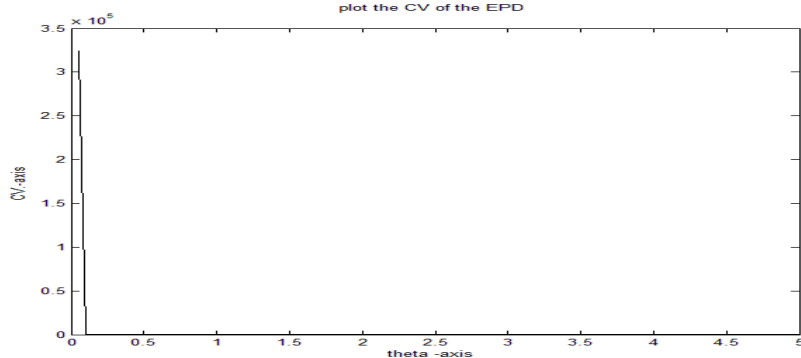
**Figure 4:** plot of c.d.f of the EPD with the parameter  $x_0 = p = 1, 2, 3$  and  $m = \lambda = 1, th = \theta = 1$



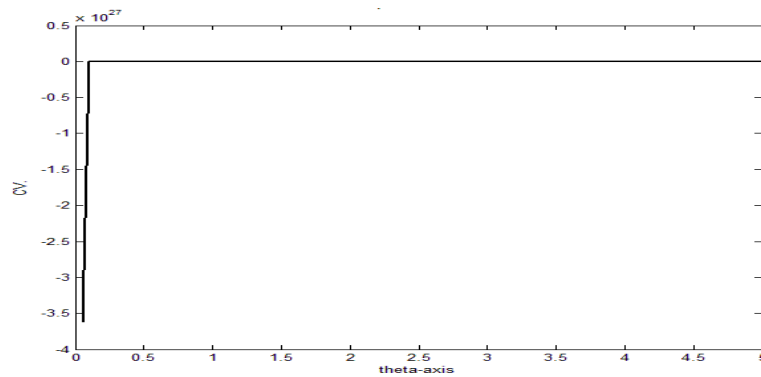
**Figure 5:** plot of c.d.f of the EPD with the parameter  $x_0 = p = 1, m = \lambda = 1$  and  $th = \theta = 1$



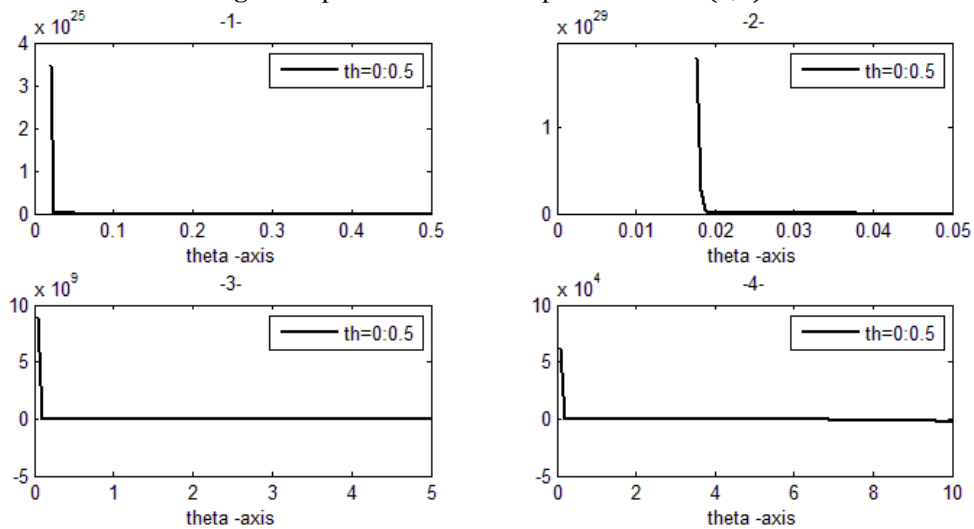
**Figure 6:** plot of c.d.f of the EPD with the parameter  $x_0 = p = 1, m = \lambda = 1, th = \theta = 1, 2, 3$ .



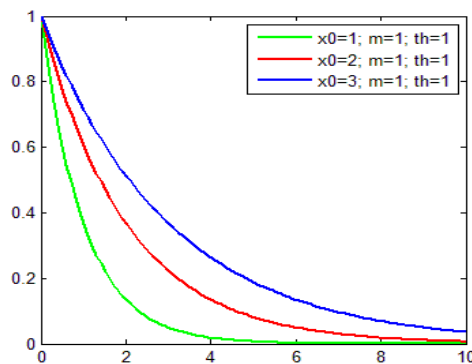
**Figure 7:** plot of CV. Of the EPD .



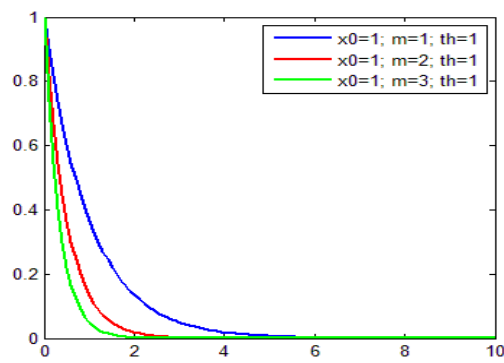
**Figure 8:** plot of CK with the parameter  $\theta = (0,5)$



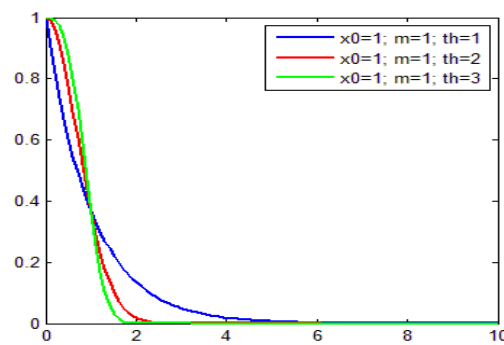
**Figure 3.9:** plot of CS of the E.P.D with the parameter just theta because the CS donete dependent on another distribution in figure 1,.....,4 the CS is dicresing but with different bigining in figure -1- bigining with  $3.4581 \times 10^{25}$  in subfigure 2 bigining with infinity ,in subfigure 3 bigine  $8.8743 \times 10^9$  and figure -4- bigining with  $6.2048 \times 10^4$ .



**Figure 3.12:** plot of  $R(x)$  with the change parameter  $x_0 = p$ .

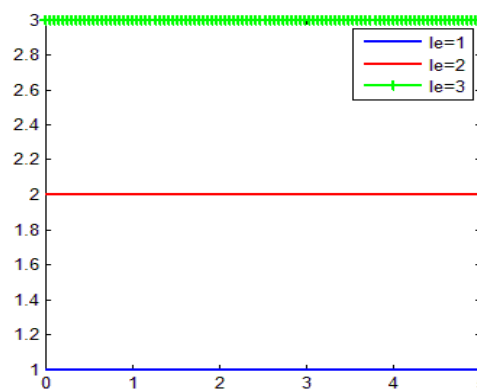


**Figure 3.13:** plot of  $R(x)$  with change parameter  $m = \lambda$  .

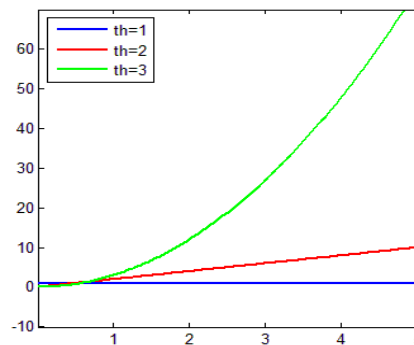


**Figure 3.14:** plot of  $R(x)$  with change parameter  $th = \theta$

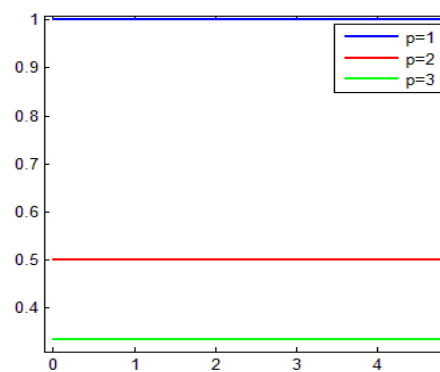
The Reliability function is decreasing with increasing of  $X$  that is take the same curve in the change of parameter  $\lambda$  and  $p$  but in the other parameter that is different of the shape .



**Figure 12:** plot of hazard function with fixed point  $\theta = p = 1$  and change the parameter  $le = \lambda = 1,2,3$



**Figure 13 :** plot of hazard function of the EPD with fixed lambda and p that is equal to one and changing the parameter theta.



**Figure 14:** plot of hazard function with change parameter p and  $\theta = \lambda = 1$ .

As we show that the shape of the hazard function is not similar to the plot of the p.d.f with some of represent by  $\frac{1}{R(x)}$  as weighted function, that it is taken the straight line in the **Figures 12 and 14** with the parameter  $\theta = 1$  but when the parameter  $\theta > 1$  the shape of the  $h(x)$  is increasing in each figure but when the parameter  $0 < \theta < 1$  the shape of the hazard function is decreasing with increasing of  $x$ .