

Some cases of reducible Generalized Hypergeometric Functions

Awadhesh Kumar Pandey, Ramakant Bhardwaj*, Kamal Wadhwa** and Nitesh Singh Thakur***

Department of Mathematics, Patel Institute of Technology, Bhopal, M.P.

*Truba Institute of Technology, Bhopal, M.P.

**Govt. Narmada P.G. College, Hoshangabad, M.P.

***Patel College of Science & Technology, Bhopal, M.P.

Abstract

In this paper we consider the integrals of Generalized hypergeometric function of three variables given by Saran Shanti [4] and obtained functions of two variables of Horn's list given in ErdelyiA.[1]. Our results are also motivated by Singh Pooja & Singh Prof. (Dr.) Harish [3].

Corresponding author E.mail:- pandey1172@gmail.com, pandey.awadhesh1972@gmail.com

Introduction

Horn investigated in particular hypergeometric series of order two and found that , apart from certain series which are either expressible in terms of product of two hypergeometric series in one variable, there are 34 distinct convergent hypergeometric series Erdelyi A [1].

Saran Shanti [4], gave some integral associated with hypergeometric function of three variables. We are using some of them for our investigation such as F_E and F_K . Recently the method adopting here has been used by Singh Pooja & Singh Prof. (Dr.) Harish [3]. We are giving here same treatment with some modifications.

Saran Shanti [4], gave the following summations

$$F_E(a_1, a_1, a_1, b_1, b_2, b_2; c_1, c_2, c_3; x, y, z) = \sum_{m,n,p=0}^{\infty} \frac{(a_1)_m+n+p (b_1)_m (b_2)_n+p}{m! n! p! (c_1)_m (c_2)_n (c_3)_p} x^m y^n z^p \quad \dots \quad (1.1)$$

$$F_K(a_1, a_2, a_2, b_1, b_2, b_1; c_1, c_2, c_3; x, y, z) = \sum_{m,n,p=0}^{\infty} \frac{(a_1)_m (a_2)_n+p (b_1)_m+p (b_2)_n}{m! n! p! (c_1)_m (c_2)_n (c_3)_p} x^m y^n z^p \quad \dots \quad (1.2)$$

and their integral results are also given by Saran Shanti [4], as

$$F_E(a_1, a_1, a_1, b_1, b_2, b_2; c_1, c_2, c_3; x, y, z) = \frac{\Gamma c_2 \Gamma c_3 \Gamma(2-c_2-c_3)}{(2\pi i)^2} \int_c (-t)^{-c_2} (t-1)^{-c_3} \times F_2(a_1, b_1, b_2, c_1, c_2+c_3-1; x, \frac{y}{t} + \frac{z}{1-t}) dt \quad \dots \quad (1.3)$$

where $|x| + \left| \frac{y}{t} + \frac{z}{1-t} \right| < 1$ along the contour.

$$F_K(a_1, a_2, a_2, b_1, b_2, b_1; c_1, c_2, c_3; x, y, z) = \frac{\Gamma r \Gamma r_1 \Gamma(2-r-r_1)}{(2\pi i)^2} \int_c (-t)^{-r} (t-1)^{-r_1} \times F_2(r, a_1; c_1 + \frac{1}{2}, \frac{x}{t}) \times F_2(a_2, b_2, r_1, c_2, c_3; y, \frac{z}{1-t}) dt \quad \dots \quad (1.4)$$

Where $b_1 = r + r_1 - 1$, $|t| > |x|$, $\left| \frac{z}{1-t} \right| < 1 - |y|$ along the integral.

Singh Pooja & Singh Prof. (Dr.) Harish[3], used argument as hyperbolic function in (1.3) & (1.4) as

$$F_E(a_1, a_1, a_1, b_1, b_2, b_2; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) = \frac{\Gamma c_2 \Gamma c_3 \Gamma(2-c_2-c_3)}{(2\pi i)^2} \int_c (-t)^{-c_2} (t-1)^{-c_3} \times F_2(a_1, b_1, b_2, c_1, c_2+c_3-1; \cosh x, \frac{\cosh y}{t} + \frac{\cosh z}{1-t}) dt \quad \dots \quad (1.5)$$

$$F_K(a_1, a_2, a_2, b_1, b_2, b_1; c_1, c_2, c_3; \cosh x, \cosh y, z) = \frac{\Gamma r \Gamma r_1 \Gamma(2-r-r_1)}{(2\pi i)^2} \int_c (-t)^{-r} (t-1)^{-r_1} \times F_2(r, a_1; c_1 + \frac{1}{2}, \cosh y, \frac{\cosh x}{t}) \times F_2(a_2; b_2, r_1, c_2, c_3; \cosh y, \frac{\cosh z}{1-t}) dt \quad \dots \quad (1.6)$$

The following results are due to Singh Pooja & Singh Prof. (Dr.) Harish[3],

$$F_E(a_1, a_1, a_1, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) = (1 - \cosh x - \cosh y)^{-a_1} \times H_1(1-c_3, a_1, c_2 + c_3 - 1, c_2; \frac{\cosh y}{\cosh x + \cosh y - 1}, \frac{\cosh z}{\cosh x + \cosh z - 1}) \quad \dots \quad (1.7)$$

$$F_E(a_1, a_1, a_1, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) = (1 - \cosh x - \cosh y - \cosh z)^{-a_1}$$

$$\times G_1(a_1, 1 - c_2, 1 - c_3; \frac{\cosh y}{1 - \cosh x - \cosh y - \cosh z}, \frac{\cosh z}{1 - \cosh x - \cosh y - \cosh z}) \quad \dots (1.8)$$

$$\begin{aligned} F_K(a_1, a_2, a_2, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) \\ = (1 - \cosh y - \cosh z)^{-a_2} \\ \times H_2(1 - c_1, a_2, a_1, c_1 + c_3 - 1, c_3; \frac{\cosh x}{\cosh y + \cosh z - 1}, -\cosh x) \end{aligned} \quad \dots (1.9)$$

$$\begin{aligned} F_K(a_1, a_2, a_2, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) \\ = (1 - \cosh x)^{-a_1} (1 - \cosh y)^{-a_2} \\ \times H_2(1 - c_3, a_1, a_2, c_1 + c_3 - 1, c_1; \frac{\cosh z}{\cosh y - 1}, \frac{\cosh z}{\cosh y - 1}) \end{aligned} \quad \dots (1.10)$$

$$\begin{aligned} F_K(a_1, a_2, a_2, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; \cosh x, \cosh y, \cosh z) \\ = (1 - \cosh x)^{-a_1} (1 - \cosh y)^{-a_2} \\ \times G_2(a_1, a_2, 1 - c_1, 1 - c_3; \frac{\cosh z}{1 - \cosh y}, \frac{\cosh z}{1 - \cosh y - \cosh z}) \end{aligned} \quad \dots (1.11)$$

Where H_1, H_2 & G_1, G_2 are Horn's functions defined by Erdelyi A. [1], as

$$H_1(a, b, c, d; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m-n}(b)_{m+n}(c)_n x^m y^n}{m! n! (d)_m} \quad \dots (1.12)$$

$$H_2(a, b, c, d, e; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m-n}(b)_m (c)_n (d)_n}{m! n! (e)_m} x^m y^n \quad \dots (1.13)$$

$$G_1(a, b, b'; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_{m-n}(b')_{m-n} x^m y^n}{m! n!} \quad \dots (1.14)$$

$$G_2(a, a', b, b'; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_m (a')_n (b')_{m-n} (b)_{n-m} x^m y^n}{m! n!} x^m y^n \quad \dots (1.15)$$

Due to Singh Pooja & Singh Prof. (Dr.) Harish[3],

$$\begin{aligned} (1 - \cosh x - \frac{\cosh y}{t} - \frac{\cosh z}{1-t})^{-a_1} \\ = (1 - \cosh x - \cosh y)^{-a_1} \\ \times \sum_{m,n=0}^{\infty} \frac{(a_1)_{m+n}}{m! n!} \left(\frac{\cosh y}{(1 - \cosh x - \cosh y)} \cdot \frac{(1-t)}{t} \right)^m \left(\frac{\cosh z}{(1 - \cosh x - \cosh y)} \cdot \frac{1}{(1-t)} \right)^m \end{aligned} \quad \dots (1.16)$$

where $\left| \frac{\cosh y}{(1 - \cosh x - \cosh y)} \cdot \frac{(1-t)}{t} \right| < 1$, $\left| \frac{\cosh z}{(1 - \cosh x - \cosh y)} \cdot \frac{1}{(1-t)} \right| < 1$, along the contour.

$$\begin{aligned} (1 - \cosh x - \frac{\cosh y}{t} - \frac{\cosh z}{1-t})^{-a_1} = (1 - \cosh x - \cosh y - \cosh z)^{-a_1} \\ \times \sum_{m,n=0}^{\infty} \frac{(a_1)_{m+n}}{m! n!} \left(\frac{\cosh y}{(1 - \cosh x - \cosh y - \cosh z)} \cdot \frac{(1-t)}{t} \right)^m \\ \times \left(\frac{\cosh z}{(1 - \cosh x - \cosh y - \cosh z)} \cdot \frac{1}{(1-t)} \right)^m \end{aligned} \quad \dots (1.17)$$

where $\left| \frac{\cosh y}{(1 - \cosh x - \cosh y - \cosh z)} \cdot \frac{(1-t)}{t} \right| < 1$, $\left| \frac{\cosh z}{(1 - \cosh x - \cosh y - \cosh z)} \cdot \frac{1}{(1-t)} \right| < 1$, along the contour.

Due to Erdelyi A.[1], we have

$$\begin{aligned} F_2(a_2, b_2, r_1, r_2, r_1; y, \frac{z}{1-t}) = (1 - y - \frac{z}{1-t})^{-a_2} \\ = (1 - y - z)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} \left(\frac{z}{(1-y-z)} \cdot \frac{t}{(1-t)} \right)^m \end{aligned} \quad \dots (1.18)$$

$$\begin{aligned} F_2(a_2, b_2, r_1, r_2, r_1; y, \frac{z}{1-t}) = (1 - y - \frac{z}{1-t})^{-a_2} \\ = (1 - y)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} \left(\frac{z}{(1-y-z)} \cdot \frac{t}{(1-t)} \right)^m \end{aligned} \quad \dots (1.19)$$

$$\begin{aligned} {}_2F_1(r, a_1, r; y, \frac{x}{t}) = (1 - \frac{x}{t})^{-a_2} \\ = (1 - x)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} \left(\frac{x}{(1-x)} \cdot \frac{1}{(1-t)} \right)^m \end{aligned} \quad \dots (1.20)$$

2. Main results

Reduction of F_E & F_K into Horn's function (Modification)

$$\begin{aligned} F_E(c_2 + c_3, c_2 + c_3, c_2 + c_3, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1 - x - y)^{-c_2 - c_3} \\ \times H_1(1 - c_3, c_2 + c_3, c_2 + c_3 - 1, c_2; \frac{y}{x+y-1}, \frac{z}{x+y-1}) \end{aligned} \quad \dots (2.1)$$

$$\begin{aligned} F_E(c_2 + c_3, c_2 + c_3, c_2 + c_3, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1 - x - y - z)^{-c_2 - c_3} \\ \times G_1(c_2 + c_3, 1 - c_2, 1 - c_3; \frac{y}{1 - x - y - z}, \frac{z}{1 - x - y - z}) \end{aligned} \quad \dots (2.2)$$

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1 - y - z)^{-c_1 - c_3} H_2(1 - c_1, c_1 + c_3, a_1, c_1 + c_3 - 1, c_3; \frac{x}{y+z-1}, -x) \end{aligned} \quad \dots (2.3)$$

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1 - x)^{-a_1} (1 - y)^{-c_1 - c_3} H_2(1 - c_3, a_1, c_1 + c_3, c_1 + c_3 - 1, c_1; \frac{z}{y-1}, \frac{z}{y-1}) \end{aligned} \quad \dots (2.4)$$

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-x)^{-a_1} (1-y-z)^{-c_1-c_3} G_2(a_1, c_1 + c_3, 1-c_1, 1-c_3; \frac{z}{1-y}, \frac{z}{1-y-z}) \end{aligned} \quad \dots \dots \dots (2.5)$$

Proof of (2.a) & (2.b) :-

From equation (1.3), we have

$$\begin{aligned} F_E(a_1, a_1, a_1, b_1, b_2, b_2; c_1, c_2, c_3; x, y, z) &= \frac{\Gamma(c_2)\Gamma(c_3)\Gamma(2-c_2-c_3)}{(2\pi i)^2} \int_c (-t)^{-c_2} (t-1)^{-c_3} \\ &\quad \times F_2(a_1, b_1, b_2, c_1, c_2+c_3-1; x, \frac{y}{t} + \frac{z}{1-t}) dt \end{aligned} \quad \dots \dots \dots (2.6)$$

where $|x| + \left| \frac{y}{t} + \frac{z}{1-t} \right| < 1$ along the contour.

Putting $a_1 = c_2 + c_3$, $b_1 = c_1$ and $b_2 = c_2 + c_3 - 1$, in equation (2.6), we have

$$\begin{aligned} F_E(c_2 + c_3, c_2 + c_3, c_2 + c_3, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = \frac{\Gamma(c_2)\Gamma(c_3)\Gamma(2-c_2-c_3)}{(2\pi i)^2} \int_c (-t)^{-c_2} (t-1)^{-c_3} (1-x - \frac{y}{t} - \frac{z}{1-t})^{-c_2-c_3} dt \end{aligned} \quad \dots \dots \dots (2.7)$$

on expansion, we get

$$\begin{aligned} (1-x - \frac{y}{t} - \frac{z}{1-t})^{-c_2-c_3} &= (1-x-y)^{-c_2-c_3} \\ &\quad \times \sum_{m,n=0}^{\infty} \frac{(c_2+c_3)_{m+n}}{m!n!} \left(\frac{y}{(1-x-y)} \cdot \frac{(1-t)}{t} \right)^m \left(\frac{z}{(1-x-y)} \cdot \frac{1}{(1-t)} \right)^n \end{aligned} \quad \dots \dots \dots (2.8)$$

where $\left| \frac{y}{(1-x-y)} \cdot \frac{(1-t)}{t} \right| < 1$, $\left| \frac{z}{(1-x-y)} \cdot \frac{1}{(1-t)} \right| < 1$ along the contour.

and

$$\begin{aligned} (1-x - \frac{y}{t} - \frac{z}{1-t})^{-c_2-c_3} &= (1-x-y-z)^{-c_2-c_3} \\ &\quad \times \sum_{m,n=0}^{\infty} \frac{(c_2+c_3)_{m+n}}{m!n!} \left(\frac{y}{(1-x-y-z)} \cdot \frac{(1-t)}{t} \right)^m \left(\frac{z}{(1-x-y-z)} \cdot \frac{1}{(1-t)} \right)^n \end{aligned} \quad \dots \dots \dots (2.9)$$

where $\left| \frac{y}{(1-x-y-z)} \cdot \frac{(1-t)}{t} \right| < 1$, $\left| \frac{z}{(1-x-y-z)} \cdot \frac{1}{(1-t)} \right| < 1$ along the contour.

Using equation (2.8), (2.9) and then evaluating the integral after changing the order of integration and summation, keeping

$$\frac{(2\pi i)^2}{\Gamma(1-a)\Gamma(1-b)\Gamma(a+b)} = \int_c (-t)^{a-1} (t-1)^{b-1} dt \quad \dots \dots \dots (2.10)$$

$$\begin{aligned} F_E(c_2 + c_3, c_2 + c_3, c_2 + c_3, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-x-y)^{-c_2-c_3} \\ \times H_1(1-c_3, c_2 + c_3, c_2 + c_3 - 1, c_2; \frac{y}{x+y-1}, \frac{z}{x+y-1}) \end{aligned} \quad \dots \dots \dots (2.11)$$

Which is new reduction result in F_E and

$$\begin{aligned} F_E(c_2 + c_3, c_2 + c_3, c_2 + c_3, c_1, c_2 + c_3 - 1, c_2 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-x-y-z)^{-c_2-c_3} \\ \times G_1(c_2 + c_3, 1-c_2, 1-c_3; \frac{y}{1-x-y-z}, \frac{z}{1-x-y-z}) \end{aligned} \quad \dots \dots \dots (2.12)$$

Which is also new reduction result in F_E .

In this way equations (2.1) & (2.2) are proved.

Proof of (2.3), (2.4) & (2.5) :-

From equation (1.2), we have

$$\begin{aligned} F_K(a_1, a_2, a_2, b_1, b_2, b_1; c_1, c_2, c_3; x, y, z) \\ = \sum_{m,n,p=0}^{\infty} \frac{(a_1)_m (a_2)_{n+p} (b_1)_{m+p} (b_2)_n}{m!n!p!(c_1)_m (c_2)_n (c_3)_p} x^m y^n z^p \end{aligned} \quad \dots \dots \dots (2.13)$$

absolutely convergent if $p=(1-m)(1-n)$, where $|x|<1$, $|y|<1$ and $|z|<1$.

From equation (1.4), we have

$$\begin{aligned} F_K(a_1, a_2, a_2, b_1, b_2, b_1; c_1, c_2, c_3; x, y, z) \\ = \frac{\Gamma r \Gamma r_1 \Gamma(2-r-r_1)}{(2\pi i)^2} \int_c (-t)^{-r} (t-1)^{-r_1} \times {}_2F_1(r, a_1; c_1 + \frac{1}{2}; \frac{x}{t}) \\ \times F_2(a_2, b_2, r_1, c_2, c_3; y, \frac{z}{1-t}) dt \end{aligned} \quad \dots \dots \dots (2.14)$$

$|x|<1$ and $\left| \frac{z}{1-t} \right| < 1 - |y|$ along the contour.

Using equation (1.18) and (1.19) & by replacing $c_2=r_2$, $c_3=r_1$, we have

$$\begin{aligned} F_2(a_2, b_2, r_1, c_2, c_3; y, \frac{z}{1-t}) &= F_2(a_2, b_2, r_1, r_2, r_1; y, \frac{z}{1-t}) \\ &= (1-y - \frac{z}{1-t})^{-a_2} \\ &= (1-y-z)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} \left(\frac{z}{(1-y-z)} \cdot \frac{t}{(1-t)} \right)^m \end{aligned} \quad \dots \dots \dots (2.15)$$

and

$$\begin{aligned} F_2(a_2, b_2, r_1, r_2, r_1; y, \frac{z}{1-t}) &= (1-y - \frac{z}{1-t})^{-a_2} \\ &= (1-y)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} (\frac{z}{(1-y-z)} \cdot \frac{t}{(1-t)})^m \end{aligned} \quad \dots \dots (2.16)$$

putting $c_1 + \frac{1}{2} = r$ and using equation (1.20), we get

$$\begin{aligned} {}_2F_1(r, a_1; c_1 + \frac{1}{2}, \frac{x}{t}) &= {}_2F_1(r, a_1; r, \frac{x}{t}) \\ &= (1 - \frac{x}{t})^{-a_2} \\ &= (1-x)^{-a_2} \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} (\frac{x}{(1-x)} \cdot \frac{1}{(1-t)})^m \end{aligned} \quad \dots \dots (2.17)$$

putting $a_2 = c_1 + c_3$, $b_1 = c_1 + c_3 - 1$ and $b_2 = c_2$ in equation (2.14) & also using equation (2.15) and equation (2.17), we get

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = \frac{\Gamma(c_1)\Gamma(c_3)\Gamma(2-c_1-c_3)}{(2\pi i)^2} \int_C (-t)^{-c_1} (t-1)^{-c_3} (1-\frac{x}{t})^{-a_2} (1-y - \frac{z}{1-t})^{-c_1-c_3} dt \end{aligned} \quad \dots \dots (2.18)$$

Now using (2.15) and $(1 - \frac{x}{t})^{-a_2} = \sum_{m=0}^{\infty} \frac{(a_2)_m}{m!} (\frac{x}{t})^m$ and integrating term by term we will have

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-y-z)^{-a_2} H_2(1-c_1, c_1 + c_3, a_1, c_1 + c_3 - 1, c_3; \frac{x}{y+z-1}, -x). \end{aligned} \quad \dots \dots (2.19)$$

Which is new reduction **result** in F_K .

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-x)^{-a_2} (1-y)^{-c_1-c_3} H_2(1-c_1, a_1, c_1 + c_3, c_1 + c_3 - 1, c_1; \frac{z}{y-1}, \frac{z}{y-1}) \end{aligned} \quad \dots \dots (2.20)$$

Which is also new reduction **result** in F_K .

$$\begin{aligned} F_K(a_1, c_1 + c_3, c_1 + c_3, c_1 + c_3 - 1, c_2, c_1 + c_3 - 1; c_1, c_2, c_3; x, y, z) \\ = (1-x)^{-a_2} (1-y-z)^{-c_1-c_3} G_2(a_1, c_1 + c_3, 1-c_1, 1-c_3; \frac{z}{1-y}, \frac{z}{1-y-z}) \end{aligned} \quad \dots \dots (2.21)$$

Which is also new reduction **result** in F_K .

In this way equations (2.3), (2.4) & (2.5) are proved.

Acknowledgement

Corresponding Author (A.P.) is thankful to all reviewers and Authors whose references are taken to prepare this article.

References :

- [1] Erdelyi A., "Higher transcendental functions", New York Toronto London, McGraw-Hill Book Company, INC, (1953).
- [2] Exton Harold, "On certain hypergeometric differential system", Funkcialaj Ekvacioj, 14(1971), pp. 79-87.
- [3] Singh Pooja & Singh Prof. (Dr.) Harish, "Certain case of reducible hypergeometric functions of hyperbolic functions as argument", International Journal of Scientific and Research Publications, Volume 2, Issue 10, October (2012), pp.1-18.
- [4] Saran Shanti, "Integral associated with hypergeometric function of three variables", (1955).

This academic article was published by The International Institute for Science, Technology and Education (IISTE). The IISTE is a pioneer in the Open Access Publishing service based in the U.S. and Europe. The aim of the institute is Accelerating Global Knowledge Sharing.

More information about the publisher can be found in the IISTE's homepage:
<http://www.iiste.org>

CALL FOR PAPERS

The IISTE is currently hosting more than 30 peer-reviewed academic journals and collaborating with academic institutions around the world. There's no deadline for submission. **Prospective authors of IISTE journals can find the submission instruction on the following page:** <http://www.iiste.org/Journals/>

The IISTE editorial team promises to the review and publish all the qualified submissions in a **fast** manner. All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Printed version of the journals is also available upon request of readers and authors.

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digital Library , NewJour, Google Scholar

