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Fixed Point Theorems for Random Variables in Complete Metric Spaces

Deepak Singh Kaushal

Abstract
In this paper we prove some fixed point theorem for random variables with one and two self maps satisfying rational inequality in complete metric spaces
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1.INTRODUCTION

Many theorems of fixed point have been proved using rational inequality in ordinary metric space. Some of the noteworthy contributions are by Bhardwaj, Rajput and Yadava [3], Jaggi and Das[9], Das and Gupta[5] Fisher [7,8] who obtained some fixed point theorems using rational inequality in complete metric spaces.

Random fixed point theory has received much attention in recent years.Some of the recent results in random fixed points have been proved by Beg and Shahzad [1, 2], Choudhary and Ray [4], Dhagat, Sharma and Bhardwaj [6]. In particular random iteration schemes leading to random fixed point of random operator.Throughout this paper $(\Omega, \sum)$ denotes a measurable space, (X,d) be a complete metric space and C is non empty subset of X.

2.PRELIMINARIES

Definition 2.1:A function $f:\Omega\times C\to C$ is said to be random operator, if $f(.,X):\Omega\to C$ is measurable for every $X\in C$.

Definition 2.2: A function $f:\Omega\to C$ is said to be measurable if $f'(B\cap C)\in\sum$ for every Borel subset B of X.

Definition 2.3:A measurable function $g:\Omega\to C$ is said to be random fixed point of the random operator $f:\Omega\times C\to C$, if $f(t,g(t))=g(t),\forall t\in\Omega.$

Definition 2.4:A random operator $f:\Omega\times C\to C$ is said to be continuous if for fixed $t\in\Omega, f(t,.):C\times C$ is continuous.

3.MAIN RESULTS

Theorem3.1: Let E be a self mapping on a complete metric space (X,d) satisfying the condition

$$\begin{aligned} d\left(E(\xi,g(\xi)),E(\xi,h(\xi))\right) \geq\ & \alpha\frac{d\left(g(\xi),E(\xi,g(\xi))\right).d\left(h(\xi),E(\xi,h(\xi))\right)}{d\left(g(\xi),h(\xi)\right)} \\ & +\beta\frac{d\left(g(\xi),E(\xi,h(\xi))\right).d\left(h(\xi),E(\xi,g(\xi))\right)}{d\left(g(\xi),h(\xi)\right)} \\ & +\gamma d\left(g(\xi),E(\xi,g(\xi))\right)+\delta d\left(h(\xi),E(\xi,h(\xi))\right) \\ & +\eta\, d\left(g(\xi),h(\xi)\right) \end{aligned}$$

For all $g(\xi),h(\xi)\in X$ with $g(\xi)\neq h(\xi)$,where $\alpha,\beta,\gamma,\delta,\eta>0$ and $\alpha+\beta+\gamma+\delta+\eta>1$ and $\alpha+\gamma<1$ and E is onto.Then E has a fixed point in X.

Proof:- Let $g_0(\xi)\in X$.Since E is onto there is an element $g_1(\xi)$ satisfying $g_1(\xi)\in E^{-1}(\xi,g_0(\xi))$ By the same way we can choose $g_n(\xi)\in E^{-1}(\xi,g_{n-1}(\xi))$ where $n=2,3,4,...$

If $g_{m-1}(\xi)=g_m(\xi)$ for some m then $g_m(\xi)$ is a fixed point of E.

Without loss of generality we can suppose that $g_{n-1}(\xi)=g_n(\xi)$,for every n.

So

$$\begin{aligned}
d\left(g_{n-1}(\xi),g_n(\xi)\right) &= d\left(E(\xi,g_n(\xi)),E(\xi,g_{n+1}(\xi))\right)\\
&\geq \alpha\frac{d\left(g_n(\xi),E(\xi,g_n(\xi))\right).d\left(g_{n+1}(\xi),E(\xi,g_{n+1}(\xi))\right)}{d\left(g_n(\xi),g_{n+1}(\xi)\right)}\\
&\quad+\beta\frac{d\left(g_n(\xi),E(\xi,g_{n+1}(\xi))\right).d\left(g_{n+1}(\xi),E(\xi,g_n(\xi))\right)}{d\left(g_n(\xi),g_{n+1}(\xi)\right)}\\
&\quad+\gamma d\left(g_n(\xi),E(\xi,g_n(\xi))\right)+\delta d\left(g_{n+1}(\xi),E(\xi,g_{n+1}(\xi))\right)\\
&\quad+\eta\, d\left(g_n(\xi),g_{n+1}(\xi)\right)
\end{aligned}$$

$$\left[1-(\alpha+\gamma)\right]d\left(g_{n-1}(\xi),g_n(\xi)\right)\geq(\delta+\eta)d\left(g_n(\xi),g_{n+1}(\xi)\right)$$

$$d\left(g_n(\xi),g_{n+1}(\xi)\right)\leq\frac{\left[1-(\alpha+\gamma)\right]}{(\delta+\eta)}d\left(g_{n-1}(\xi),g_n(\xi)\right)$$

It follows that

$$d\left(g_n(\xi),g_{n+1}(\xi)\right)\leq k\,d\left(g_{n-1}(\xi),g_n(\xi)\right)\text{where } k=\frac{\left[1-(\alpha+\gamma)\right]}{(\delta+\eta)}<1.$$

By routine calculation the following inequality holds for $p>n$.

$$\begin{aligned}
d\left(g_n(\xi),g_{n+p}(\xi)\right) &\leq\sum d\left(g_{n+i-1}(\xi),g_{n+i}(\xi)\right)\\
&\leq k^{n+i-1}d\left(g_0(\xi),g_1(\xi)\right)\\
&\leq\frac{k^n}{1-k}d\left(g_0(\xi),g_1(\xi)\right)
\end{aligned}$$

Now making $n\to\infty$ we obtain

$$d\left(g_n(\xi),g_{n+p}(\xi)\right)\to 0$$

Hence $\{g_n(\xi)\}$ is a Cauchy sequence. Since X is complete $\{g_n(\xi)\}$ converges to $g(\xi)$,for some $g(\xi)\in X$.

Since E is onto then there exists $h(\xi)\in X$ such that $h(\xi)\in E^{-1}(\xi,g(\xi))$ and for infinitely many $n, g_n(\xi)\neq g(\xi)$ for such n.

$$\begin{aligned}
d\left(g_n(\xi),g(\xi)\right) &= d\left(E(\xi,g_{n+1}(\xi)),E(\xi,h(\xi))\right)\\
&\geq\alpha\frac{d\left(g_{n+1}(\xi),E(\xi,g_{n+1}(\xi))\right).d\left(h(\xi),E(\xi,h(\xi))\right)}{d\left(g_{n+1}(\xi),h(\xi)\right)}\\
&\quad+\beta\frac{d\left(g_{n+1}(\xi),E(\xi,h(\xi))\right).d\left(h(\xi),E(\xi,g_{n+1}(\xi))\right)}{d\left(g_{n+1}(\xi),h(\xi)\right)}\\
&\quad+\gamma d\left(g_{n+1}(\xi),E(\xi,g_{n+1}(\xi))\right)+\delta d\left(h(\xi),E(\xi,h(\xi))\right)\\
&\quad+\eta\, d\left(g_{n+1}(\xi),h(\xi)\right)
\end{aligned}$$

$$\geq \alpha \frac{d\left(g_{n+1}(\xi), g_n(\xi)\right).d\left(h(\xi), g(\xi)\right)}{d\left(g_{n+1}(\xi), h(\xi)\right)}$$
$$+\beta \frac{d\left(g_{n+1}(\xi), g(\xi)\right).d\left(h(\xi), g_n(\xi)\right)}{d\left(g_{n+1}(\xi), h(\xi)\right)}$$
$$+\gamma d\left(g_{n+1}(\xi), g_n(\xi)\right)+\delta d\left(h(\xi), g(\xi)\right)$$
$$+\eta d\left(g_{n+1}(\xi), h(\xi)\right)$$

On taking limit as $n \to \infty$ we have

$$0 \geq \delta d\left(h(\xi), g(\xi)\right)+\eta \lim_{n\to\infty} d\left(g_{n+1}(\xi), h(\xi)\right)$$

Since $\delta, \eta > 0$, So $d\left(g(\xi), h(\xi)\right)=0$

and $\lim_{n\to\infty} d\left(g_{n+1}(\xi), h(\xi)\right)=0$

So in both cases we get $g(\xi)=h(\xi)$ .Thus E has a fixed point in X.

Theorem 3.2: Let E be a self mapping on a complete metric sapce (X, d) satisfying the condition

$$d\left(E(\xi, g(\xi)), E(\xi, h(\xi))\right)$$
$$\geq \alpha \min\left\{\begin{array}{l}\dfrac{d\left(g(\xi), E(\xi, g(\xi))\right).d\left(h(\xi), E(\xi, h(\xi))\right)+d\left(g(\xi), E(\xi, h(\xi))\right).d\left(h(\xi), E(\xi, g(\xi))\right)}{d\left(g(\xi), h(\xi)\right)}, \\ \dfrac{d\left(g(\xi), E(\xi, h(\xi))\right).d\left(h(\xi), E(\xi, g(\xi))\right)+d\left(h(\xi), E(\xi, h(\xi))\right).d\left(h(\xi), E(\xi, h(\xi))\right)}{d\left(g(\xi), h(\xi)\right)}, \\ d\left(h(\xi), E(\xi, h(\xi))\right), d\left(g(\xi), E(\xi, g(\xi))\right), d\left(g(\xi), h(\xi)\right)\end{array}\right\}$$

For all $g(\xi), h(\xi) \in X$ with $g(\xi) \neq h(\xi)$, where $\alpha > 1$ and E is onto. Then E has a fixed point in X.

Proof:- Let $g_0(\xi) \in X$. Since E is onto there is an element $g_1(\xi)$ satisfying $g_1(\xi) \in E^{-1}(\xi, g_0(\xi))$ By the same way we can choose $g_n(\xi) \in E^{-1}(\xi, g_{n-1}(\xi))$ where $n = 2, 3, 4, ...$

If $g_{m-1}(\xi) = g_m(\xi)$ for some m then $g_m(\xi)$ is a fixed point of E.

Without loss of generality we can suppose that $g_{n-1}(\xi) = g_n(\xi)$ for every n.

So $d\left(g_{n-1}(\xi), g_n(\xi)\right) = d\left(E(\xi, g_n(\xi)), E(\xi, g_{n+1}(\xi))\right)$

$$\geq \alpha \min\left\{\begin{array}{l}\dfrac{d\left(g_n(\xi), E(\xi, g_n(\xi))\right).d\left(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))\right)+d\left(g_n(\xi), E(\xi, g_{n+1}(\xi))\right).d\left(g_{n+1}(\xi), E(\xi, g_n(\xi))\right)}{d\left(g(\xi), g_{n+1}(\xi)\right)}, \\ \dfrac{d\left(g_n(\xi), E(\xi, g_{n+1}(\xi))\right).d\left(g_{n+1}(\xi), E(\xi, g_n(\xi))\right)+\left[d\left(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))\right)\right]^2}{d\left(g_n(\xi), g_{n+1}(\xi)\right)}, \\ d\left(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))\right), d\left(g_n(\xi), E(\xi, g_n(\xi))\right), d\left(g_n(\xi), g_{n+1}(\xi)\right)\end{array}\right\}$$

$$= \alpha \min \left\{ \begin{array}{l} \dfrac{d(g_n(\xi), g_{n-1}(\xi)).d(g_{n+1}(\xi), g_n(\xi)) + d(g_n(\xi), g_n(\xi))).d(g_{n+1}(\xi), g_{n-1}(\xi)))}{d(g_n(\xi), g_{n+1}(\xi))}, \\ \dfrac{d(g_n(\xi), g_n(\xi)).d(g_{n+1}(\xi), g_{n-1}(\xi)) + \left[d(g_{n+1}(\xi), g_n(\xi))\right]^2}{d(g_n(\xi), g_{n+1}(\xi))}, \\ d(g_{n+1}(\xi), g_n(\xi)), d(g_n(\xi), g_{n-1}(\xi))), d(g_n(\xi), g_{n+1}(\xi)) \end{array} \right\}$$

$$= \alpha \min \{ d(g_n(\xi), g_{n-1}(\xi)), d(g_n(\xi), g_{n+1}(\xi)) \}$$

$$\frac{1}{\alpha} d(g_{n-1}(\xi), g_n(\xi)) \geq d(g_n(\xi), g_{n+1}(\xi))$$

$$d(g_n(\xi), g_{n+1}(\xi)) \leq \frac{1}{\alpha} d(g_{n-1}(\xi), g_n(\xi))$$

Where $\frac{1}{\alpha} < 1$,therefore by well known way $\{g_n(\xi)\}$ is a Cauchy sequence in X. Since X is complete $\{g_n(\xi)\}$ converges to $g(\xi)$,for some $g(\xi) \in X$. So by continuity of E,we have

$E(\xi, g_n(\xi)) = g_{n-1}(\xi) \to E(\xi, g(\xi)), \quad \text{as} \quad n \to \infty$

Hence $E(\xi, g(\xi)) = g(\xi)$

Thus E has a fixed point in X.

Theorem3.3: Let E be a self mapping on a complete metric sapce (X,d) satisfying the condition

$$(E(\xi, g(\xi)), E(\xi, h(\xi))) \geq \alpha \begin{bmatrix} d(g(\xi), E(\xi, g(\xi))).d(h(\xi), E(\xi, h(\xi))) \\ +d(g(\xi), E(\xi, h(\xi))).d(h(\xi), E(\xi, g(\xi))) \\ +d(g(\xi), E(\xi, g(\xi)))d(g(\xi), h(\xi)) \\ +d(g(\xi), E(\xi, h(\xi)))d(g(\xi), h(\xi)) \end{bmatrix}^{\frac{1}{2}}$$

For each $g(\xi), h(\xi) \in X$ with $g(\xi) \neq h(\xi)$, where $\alpha > 1$ and E is onto.Then E has a fixed point in X.

Proof:- Let $g_0(\xi) \in X$.Since E is onto there is an element $g_1(\xi)$ satisfying $g_1(\xi) \in E^{-1}(\xi, g_0(\xi))$ By the same way we can choose $g_n(\xi) \in E^{-1}(\xi, g_{n-1}(\xi))$ where $n = 2,3,4,...$

If $g_{m-1}(\xi) = g_m(\xi)$ for some m then $g_m(\xi)$ is a fixed point of E.

Without loss of generality we can suppose that $g_{n-1}(\xi) = g_n(\xi)$ for every n.

So

$$d(g_{n-1}(\xi), g_n(\xi))) = d(E(\xi, g_n(\xi)), E(\xi, g_{n+1}(\xi)))$$

$$\geq \alpha \begin{bmatrix} d(g_n(\xi), E(\xi, g_n(\xi)))d(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))) \\ +d(g_n(\xi), E(\xi, g_{n+1}(\xi)))d(g_{n+1}(\xi), E(\xi, g_n(\xi))) \\ +d(g_n(\xi), E(\xi, g_n(\xi)))d(g_n(\xi), g_{n+1}(\xi)) \\ +d(g_n(\xi), E(\xi, g_{n+1}(\xi)))d(g_n(\xi), g_{n+1}(\xi)) \end{bmatrix}^{\frac{1}{2}}$$

$$\geq \alpha \begin{bmatrix} d\left(g_n(\xi), g_{n-1}(\xi)\right) d\left(g_{n+1}(\xi), g_n(\xi)\right) \\ +d\left(g_n(\xi), g_n(\xi)\right) d\left(g_{n+1}(\xi), g_{n-1}(\xi)\right) \\ +d\left(g_n(\xi), g_{n-1}(\xi)\right) d\left(g_n(\xi), g_{n+1}(\xi)\right) \\ +d\left(g_n(\xi), g_n(\xi)\right) d\left(g_n(\xi), g_{n+1}(\xi)\right) \end{bmatrix}^{\frac{1}{2}}$$

$$\geq \alpha\left[2d\left(g_n(\xi), g_{n-1}(\xi)\right) d\left(g_{n+1}(\xi), g_n(\xi)\right)\right]^{\frac{1}{2}}$$

$$d\left(g_n(\xi), g_{n+1}(\xi)\right) \leq \frac{1}{2\alpha^2} d\left(g_{n-1}(\xi), g_n(\xi)\right)$$

Therefore by well known way $\{g_n(\xi)\}$ is a Cauchy sequence in X. Since X is complete $\{g_n(\xi)\}$ converges to $g(\xi)$, for some $g(\xi) \in X$. Since E is onto there exists $h(\xi) \in X$. such that $h(\xi) \in E^{-1}(\xi, g(\xi))$ and for infinitely many n, $g_n(\xi) \neq g(\xi)$ for all n.

$$d\left(g_n(\xi), g(\xi)\right) = d\left(E(\xi, g_{n+1}(\xi)), E(\xi, h(\xi))\right)$$

$$\geq \alpha \begin{bmatrix} d\left(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))\right) d\left(h(\xi), E(\xi, h(\xi))\right) \\ +d\left(g_{n+1}(\xi), E(\xi, h(\xi))\right) d\left(h(\xi), E(\xi, g_{n+1}(\xi))\right) \\ +d\left(g_{n+1}(\xi), E(\xi, g_{n+1}(\xi))\right) d\left(g_{n+1}(\xi), h(\xi)\right) \\ +d\left(g_{n+11}(\xi), E(\xi, h(\xi))\right) d\left(g_{n+1}(\xi), h(\xi)\right) \end{bmatrix}^{\frac{1}{2}}$$

On taking limit as $n \to \infty$,we obtain

$$d\left(g_n(\xi), g(\xi)\right) \to 0$$

So we have

$$d\left(g(\xi), h(\xi)\right) = 0 \Rightarrow g(\xi) = h(\xi)$$

Hence E has a fixed point in X.

Theorem3.4: Let E and F be two self mappings on a complete metric space (X, d) satisfying the condition

$$\begin{aligned} d\left(E(\xi, g(\xi)), F(\xi, h(\xi))\right) \geq\ & \alpha \frac{d\left(g(\xi), E(\xi, g(\xi))\right).d\left(h(\xi), F(\xi, h(\xi))\right)}{d\left(g(\xi), h(\xi)\right)} \\ & + \beta \frac{d\left(g(\xi), F(\xi, h(\xi))\right).d\left(h(\xi), E(\xi, g(\xi))\right)}{d\left(g(\xi), h(\xi)\right)} \\ & + \gamma d\left(g(\xi), E(\xi, g(\xi))\right) + \delta d\left(h(\xi), F(\xi, h(\xi))\right) \\ & + \eta\, d\left(g(\xi), h(\xi)\right) \end{aligned}$$

For all $g(\xi), h(\xi) \in X$ with $g(\xi) \neq h(\xi)$,where $\alpha, \beta, \gamma, \delta, \eta > 0$ and $\alpha + \beta + \gamma + \delta + \eta > 1$ and $\alpha + \gamma < 1$ .Then E and F have a common fixed point.

Proof: Let $g_0(\xi)$ be any point of X, we define a sequence $\{g_n(\xi)\}$ as follows

$$g_0(\xi)=E(\xi,g_1(\xi)),$$
$$g_1(\xi)=E(\xi,g_2(\xi)),$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$g_{2n}(\xi)=E(\xi,g_{2n+1}(\xi)),$$
$$g_{2n+1}(\xi)=E(\xi,g_{2n+2}(\xi))$$

Now if $g_{2n+1}(\xi)\neq g_{2n+2}(\xi)$ then $d\left(g_{2n}(\xi),g_{2n+1}(\xi)\right)=d\left(E(\xi,g_{2n+1}(\xi)),F(\xi,g_{2n+2}(\xi))\right)$

$$\geq\alpha\frac{d\left(g_{2n+1}(\xi),E(\xi,g_{2n+1}(\xi))\right).d\left(g_{2n+2}(\xi),F(\xi,g_{2n+2}(\xi))\right)}{d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)}$$
$$+\beta\frac{d\left(g_{2n+1}(\xi),F(\xi,g_{2n+2}(\xi))\right).d\left(g_{2n+2}(\xi),E(\xi,g_{2n+1}(\xi))\right)}{d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)}$$
$$+\gamma d\left(g_{2n+11}(\xi),E(\xi,g_{2n+1}(\xi))\right)+\delta d\left(g_{2n+2}(\xi),F(\xi,g_{2n+2}(\xi))\right)$$
$$+\eta\, d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)$$
$$\geq\alpha\frac{d\left(g_{2n+1}(\xi),g_{2n}(\xi)\right).d\left(g_{2n+2}(\xi),g_{2n+1}(\xi)\right)}{d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)}$$
$$+\beta\frac{d\left(g_{2n+1}(\xi),g_{2n+1}(\xi)\right).d\left(g_{2n+2}(\xi),g_{2n}(\xi)\right)}{d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)}$$
$$+\gamma d\left(g_{2n+1}(\xi),g_{2n}(\xi)\right)+\delta d\left(g_{2n+2}(\xi),g_{2n+1}(\xi)\right)$$
$$+\eta\, d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)$$
$$d\left(g_{2n}(\xi),g_{2n+1}(\xi)\right)\geq(\alpha+\gamma)d\left(g_{2n+1}(\xi),g_{2n}(\xi)\right)+(\delta+\eta)d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)$$
$$\left[1-(\alpha+\gamma)\right]d\left(g_{2n}(\xi),g_{2n+1}(\xi)\right)\geq(\delta+\eta)d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)$$
$$d\left(g_{2n+1}(\xi),g_{2n+2}(\xi)\right)\leq\left[\frac{1-(\alpha+\gamma)}{\delta+\eta}\right]d\left(g_{2n}(\xi),g_{2n+1}(\xi)\right),\text{Since }\ 0<\left[\frac{1-(\alpha+\gamma)}{\delta+\eta}\right]<1$$

It follow that $\{g_n(\xi)\}$ is a Cauchy sequence .By completeness of X there is some point $z(\xi)$ in X and $\{g_n(\xi)\}$ converge to $z(\xi)$ .By the condition there is another point $u(\xi)$ in X such that $E(\xi,u(\xi))=z(\xi)$ Since we can suppose $u(\xi)\neq g_{2n+2}(\xi)$ for many infinitely n we can write

$$d\left(z(\xi),g_{2n+1}(\xi)\right)=d\left(E(\xi,u(\xi)),F(\xi,g_{2n+2}(\xi))\right)$$
$$\geq\alpha\frac{d\left(u(\xi),E(\xi,u(\xi))\right)d\left(g_{2n+2}(\xi),F(\xi,g_{2n+2}(\xi))\right)}{d\left(u(\xi),g_{2n+2}(\xi)\right)}$$
$$+\beta\frac{d\left(u(\xi),F(\xi,g_{2n+2}(\xi))\right)d\left(g_{2n+2}(\xi),E(\xi,u(\xi))\right)}{d\left(u(\xi),g_{2n+2}(\xi)\right)}$$
$$+\gamma d\left(u(\xi),E(\xi,u(\xi))\right)+\delta d\left(g_{2n+2}(\xi),F(\xi,g_{2n+2}(\xi))\right)$$
$$+\eta\, d\left(u(\xi),g_{2n+2}(\xi)\right)$$

On taking limit as $n\to\infty$ we obtain

$$0 \geq \gamma d\left(u(\xi), E(\xi, u(\xi))\right) + \eta\, d\left(u(\xi), z(\xi)\right)$$

$$0 \geq \left(\gamma + \eta\right) d\left(u(\xi), z(\xi)\right)$$

Which implies that $u(\xi) = z(\xi) = E(\xi, u(\xi))$

i.e. $z(\xi) = E(\xi, u(\xi))$

Similarly $z(\xi) = F(\xi, u(\xi))$

Hence E and F have a common fixed point.

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Deepak Singh Kaushal
Sagar Institute of Science,Technology & Research
Bhopal (M.P.)
email:deepaksinghkaushal@yahoo.com