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A New Method of Constructing SRGD & Resolvable SRGD Designs

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Abstract
In this paper some methods of construction of certain SRGD designs are described. These designs have been constructed by making use of known self complementary balanced incomplete block (BIB) designs.A series of resolvable SRGD designs is also obtained here. Designs constructed in this paper are not listed in the existing lists of the available tables of these designs.
Keywords: Block design , incidence matrix , BIBD , self complementary BIBD design , semi-regular group divisible design , resolvable SRGD design .

Introduction
The largest, simplest and perhaps most important class of two-associate partially balanced incomplete block (PBIB) designs is the class of group divisible (GD) designs.

Let $v = mn$ objects (= treatments) be partitioned into m disjoint sets $\mathbf{S}_i\,(i = 1, 2, \ldots\ldots., m)$ each containing $n\,(n \geq 2)$ objects. Two objects belonging to the same set are first associates whereas two objects belonging to different sets are second associates.

Suppose now that we have a GD scheme on $v = mn$ treatments as above. A G.D. design D $(v, b, r, k, m, n, \lambda_1, \lambda_2)$ is an arrangement of these v treatments $t_1, t_2, \ldots\ldots.., t_v$ into b distinct subsets $\mathbf{B}_1, \mathbf{B}_2, \ldots., \mathbf{B}_b$ (called blocks) satisfying the following conditions :

(i) $|\mathbf{B}_i| = k \;(i = 1, 2, \ldots\ldots, b)$,

(ii) each treatment occurs in exactly r blocks,

(iii) two treatments from the same set $\mathbf{S}_i$ appear together in exactly λ_1 blocks and two treatments from distinct sets $\mathbf{S}_i$ and $\mathbf{S}_j$ occur together in exactly λ_2 blocks.

The parameters of a GD design are denoted by $v,\, b,\; r,\, k,\, m,\, n,\, \lambda_1,\, \lambda_2$ [1]

Bose and Connor (1952) have classified G.D. designs into three subtypes :

(1) Singular, if $r = \lambda_1$.

(2) Semi-regular, if $r > \lambda_1$ and $rk = v\lambda_2 = 0$.

(3) Regular, if $r > \lambda_1$ and $rk - v\lambda_2 > 0$.

For complete description of balanced incomplete block (BIB) designs, G.D. designs ,we refer to Raghavarao (1971).

For a BIB design with parameters $v, b, r, k, \lambda,$ if the blocks can be separated into t classes $S_1, S_2, \ldots\ldots., S_t$ of β blocks each such that each class contains every treatment exactly α times, then the design is said to be α-resolvable[3]. The necessary conditions for the existence of a resolvable BIB design are that

i) v is divisible by k ,

ii) an inequality $b \geq v + r - 1$ holds, in particular,

iii) when $b = v + r - 1,\; k^2$ must be divisible by v .

For further properties of resolvable designs we refer Raghava rao (1971).

The concept of resolvable GD design with $\lambda_1 = 0$ is of special interest[8].

When s is prime or a prime power, a symmetric BIB design with parameters

$v^* = b = s^2 + s + 1, , r = k = s + 1,\ \lambda = 1.$ …(1.1)

can be obtained from a projective plane of order s [See Raghavarao (1971)]. The residual of this symmetric BIB design is also a BIB design with parameters

$$v = s^2, b = s^2 + s, r = s + 1, k = s, \lambda = 1 \quad \text{…(1.2)}$$

which is resolvable as the blocks form a resolution having the treatments common with the deleted block of a design with parameters (1.1). Hence there are $s+1$ resolutions each containing $S = v/k$ blocks[5].

Furthermore, in the design with parameters (1.2) by deleting a treatment along with the $r = s+1$ blocks containing it, we can obtain a Regular GD design with parameters .

$v^* = s^2 - 1 = b^*, r^* = k^* = s,\ \lambda_1^* = 0, \lambda_2^* = 1,\ m = s+1,\ n = s-1,$ …(1.3)

which is known (see Raghavarao, 1971, p.139)[9].

When we delete a treatment and the blocks containing it, a block containing the treatment is deleted from each resolution. Then the remaining blocks of the resolutions form partial parallel classes [cf. Furino, Miao and Yin (1996)][6]. Thus we get $s+1$ partial parallel classes corresponding to $s+1$ resolutions of the design with parameters (1.2) each having $s-1$ blocks.

Throughout this paper, N denotes incidence matrix of BIB design of appropriate size, $\overline{\mathbf{N}} = \mathbf{J} - \mathbf{N}$, $\mathbf{J}_{\mathbf{r}\times\mathbf{s}}$ denotes all-one flat matrix of order $r \times s$, $\mathbf{I}_{\mathbf{n}}$ the identity matrix of order n and $\mathbf{A} \otimes \mathbf{B}$ denotes Kronecker product of two matrices A and B[2].

MAIN RESULTS

Theorem 1 : Let N be the incidence matrix of order $v \times b$ of a self complementary BIBD (v, b, r, k, λ) and $\mathbf{J} - \mathbf{N}$ be the complement of N, then the incidence structure

$$S = \begin{pmatrix} \mathbf{N} & \mathbf{J}-\mathbf{N} & \mathbf{N} & \mathbf{J}-\mathbf{N} \\ \mathbf{J}-\mathbf{N} & \mathbf{N} & \mathbf{N} & \mathbf{J}-\mathbf{N} \end{pmatrix}$$

is the incidence matrix of semiregular GD Design with the parameters

$$v^* = 2v, b^* = 4b,\ r^* = 4r,\ k^* = 2k,\ \lambda_1^* = 4\lambda,\ \lambda_2^* = 2r\ ;\ m,\ n.$$

Proof: The proof of the theorem is straight forward.

As an illustration, consider N, the incidence matrix of BIBD (6,10,5,3,2) in Theorem 1, we obtain SRGD design with parameters.

$v^* = 12, b^* = 40,\ r^* = 20,\ k^* = 6,\ \lambda_1^* = 8,\ \lambda_2^* = 10\ ;\ m = 2\ \ n = 6.$

This design are not available in Parihar (1981), Parihar and Shrivastava (1988), Thakur (1990) Freeman (1967) Singh (1994), Kageyama and Mukerjee (1987).

Theorem 2 : A series of resolvable SRGD design with parameters

$$\begin{aligned} &v^* = s^2 - 1,\ b^* = (s+1)(s-1)^2,\ \ r^* = s^2 - 1,\ k^* = s+1, \\ &\lambda_1^* = 0,\ \ \lambda_2^* = s+1,\ \ m = s+1,\ \ n = s-1, \end{aligned} \quad \text{…(1.4)}$$

always exists when s is a prime or a prime power.

Proof : Since $\lambda_1^* = 0$ and $k^* = s = m-1$ in (1.3) implies that each block of the design with parameters (1.3) has no treatments from one of the m groups. By repeating each partial parallel class of the design with parameters given in (1.3) $n = s-1$ times and adding one distinct treatment from the unrepresented group in each of the partial parallel classes in n cyclic permutations of $(x_1, x_2, \ldots\ldots, x_n)$ where $x_i^{'}s$ are members of the unrepresented group. We can get the resultant resolvable SRGD design [4]with parameters (1.4).

Note 2.1: When $s = 3$, we get a new resolvable solution, which is non-isomorphic to SR39 in Clatworthy (1973)[7] as [(1,2,3,4), (5,6,7,8)], [(1,2,4,7), (3,5,6,8)] developed cyclically for four steps only,having parameters

$$v^* = 8,\ b^* = 16,\ r^* = 8,\ k^* = 4,\ \lambda_1^* = 0,\ \lambda_2^* = 4\ ;\ m = 4, n = 2.$$

in fact, in the solution presented by Clatworthy (1973) each block is duplicated whereas in our solution all the blocks are distinct.

The blocks of the above design is presented in the form of columns :

1	2	3	4	5	1	1	1	1	2	1	2	3	1	2	1
2	3	4	5	6	6	2	2	2	3	3	4	5	4	5	3
3	4	5	6	7	7	7	3	4	5	4	5	6	6	7	6
4	5	6	7	8	8	8	8	7	8	6	7	8	7	8	8

The GD association scheme is represented as follows

$\mathbf{G}_1$	$\mathbf{G}_2$	$\mathbf{G}_3$	$\mathbf{G}_4$
1	2	3	4
5	6	7	8

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