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On Common Fixed point Theorem in Fuzzy Metric space

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ABSTRACT
In this research article we are proving common fixed point theorem using Occasionally Weakly Compatible Mapping in fuzzy metric space.
KEYWORDS: Common Fixed point, Fuzzy Metric space, Occasionally Weakly Compatible Mapping, Continuous t-norm.

1. INTRODUCTION

It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [24] which laid the foundation of fuzzy mathematics. Kramosil and Michalek [11] introduced the notion of a fuzzy metric space by generalizing the concept of the probabilistic metric space to the fuzzy situation. George and Veeramani [7] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [11]. There are many view points of the notion of the metric space in fuzzy topology for instance one can refer to Kaleva and Seikkala [10], Kramosil and Michalek [11], George and Veeramani [7].

2. PRELIMINARIES:

Definition 2.1. [24] Let X be any non empty set. A fuzzy set M in X is a function with domain X and values in [0, 1].

Definition 2.2. [19] A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-norm if it satisfy the following condition:
(i) $*$ is associative and commutative .
(ii) $*$ is continous function.
(iii) $a*1=a$ for all $a\in [0,1]$
(iv) $a*b \le c*d$ whenever $a \le c$ and $b \le d$ and $a, b,c,d \in[0,1]$

Definition 2.3. [11] The 3 − tuple $(X, M,*)$ is called a fuzzy metric space in the sense of Kramosil and Michalek if X is an arbitrary set, is a continuous t − norm and M is a fuzzy set in $X^2\times [0,\infty)$ satisfying the following conditions:
(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t)\ M(y, z, s) \le M(x, z, t + s)$,
(e) $M(x, y, .) : [0,\infty) \rightarrow [0, 1]$ is a continuous function, for all $x, y, z \in X$ and $t, s > 0$.

Definition 2.4 [11] Let $(X, M, *)$ be a fuzzy metric space . Then
(i) A sequence $\{x_n\}$ in X converges to x if and only if for each $t >0$ there exists $n_0 \in N$, such that, $\lim n\rightarrow\infty\ M (x_n, x, t) = 1$, for all $n \ge n_0$.
(ii) The sequence $(x_n)\ n\in N$ is called Cauchy sequence if $\lim_{n\rightarrow\infty} M (x_n,x_{n+p}, t) = 1$, for all $t > 0$ and $p \in N$.
(iii) A fuzzy metric space X is called complete if every Cauchy sequence is convergent in X.

Definition 2.5. [23] Two self-mappings *f* and *g* of a fuzzy metric space $(X, M, *)$ are said to be weakly commuting if $M(fgx, gfx, t) \ge M(fx, gx, t)$, for each $x \in X$ and for each $t > 0$.

Definition 2.6 [5] Two self mappings f and g of a fuzzy metric space$(X, M,*)$ are called compatible if $\lim_{n\rightarrow\infty} M (fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that$\lim_{n\rightarrow\infty} fx_n = \lim_{n\rightarrow\infty} gx_n = x$ for some x in X.
Definition 2.7.[2] A pair of mappings f and g from a fuzzy metric space $(X,M,*)$ into itself are weakly compatible if they commute at their coincidence points,i.e., $fx = gx$ implies that $fgx = gfx$.

Definition 2.8 Let X be a set, f, g selfmaps of X. A point x in X is called a coincidence point of f and g iff $fx = gx$. We shall call $w = fx = gx$ a point of coincidence of f and g.

Definition 2.9 [2] A pair of maps S and T is called weakly compatible pair if they commute at coincidence points.

Definition 2.10.[4] Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

A. Al-Thagafi and Naseer Shahzad [4] shown that occasionally weakly compatible is weakly compatible but converse is not true.

Lemma 2.11 [4] Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

3. IMPLICIT RELATIONS:

(a) Let **(Φ)** be the set of all real continuous functions $\emptyset : (R^+)^5 \to R^+$ satisfying the condition ∅: (u, u, v, v, u,) ≥0 imply u ≥ v, for all u, v ∈ [0,1].
(b) Let **(Φ)** be the set of all real continuous functions $\emptyset : (R^+)^4 \to R^+$ satisfying the condition ∅: (u, v, u, u,) ≥0 imply u≥ v, for all u, v ∈ [0,1].

4. MAIN RESULTS

Theorem 4.1.: Let (X, M,∗) be a fuzzy metric space with ∗ continuous t-norm. Let A, B, S, T be self mappings of X satisfying

(i) The pair (A, S) and (B, T) be owc.

(ii) For some ∅ ∈ **Φ** and for all x, y ∈ X and every t > 0,
∅{M (Ax, By, t), M (Sx, Ty, t), M (Sx, Ax, t), M (Ty, By, t), M (Sx, By, t)}≥ 0

then there exists a unique point w ∈X such that Aw = Sw = w and a unique point z ∈X such that Bz = T z = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points x, y ∈X such that Ax = Sx and By = T y. We claim that Ax = By. If not, by inequality (ii)

∅{M (Ax, By, t), M (Ax, By, t), M (Ax, Ax, t), M (By, By, t), M (Ax, By, t)}≥ 0

∅{M (Ax, By, t), M (Ax, By, t), 1, 1, M (Ax, By, t)}≥ 0

∅{M (Ax, By, t), M (Ax, By, t), 1, 1, M (Ax, By, t)}≥ 0

In view of Φ we get Ax = By i.e. Ax = Sx = By = T y

Suppose that there is a another point z such that Az = Sz then by (i) we have Az = Sz = By = T y, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 2.11 w is the only common fixed point of A and S. Similarly there is a unique point z ∈ X such that z = Bz = T z.
Assume that w ≠ z. We have

∅{M(Aw, Bz, t), M(Sw,Tz, t), M(Sw, Aw, t), M(Tz,Bz,t),M(Sw,Bz, t)}≥ 0

∅{M (w, z, t), M (w, z, t), M (w, w, t), M (z, z, t), M (w, z, t)}≥ 0

∅{M (w, z, t), M (w, z, t), 1, 1, M (w, z, t)}≥ 0

∅{M (w, z, t), M (w, z, t), 1, 1, M (w, z, t)}≥ 0

In view of Φ we get **w** = z. by Lemma 2.11 and z is a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (ii)

Theorem 4.2.: Let (X, M, ∗) be a fuzzy metric space with ∗ continuous t-norm. Let A, B, S, T be self mappings of X satisfying

(i) The pair (A, S) and (B, T) be owc.

(ii) For some $\emptyset \in \mathbf{\Phi}$ and for all x, y ∈ X and every t > 0,

$\emptyset\{M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t)\} \geq 0$
then there exists a unique point w ∈ X such that Aw = Sw = w and a unique point z ∈X such that Bz = T z = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points x, y ∈ X such that Ax = Sx and By = T y. We claim that Ax = By. If not, by inequality (ii).

$\emptyset\{, M(Ax, By, t), M(Ax, Ax, t), M(Ax, By, t), M(By, Ax, t)\} \geq 0$
$\emptyset\{M(Ax, By, t), M(Ax, Ax, t), M(Ax, By, t), M(Ax, By, t)\} \geq 0$
$\emptyset\{\{M(Ax, By, t), 1, M(Ax, By, t), M(Ax, By, t)\}\} \geq 0$

In view of Φ we get **Ax** = By i.e. Ax = Sx = By = T y

Suppose that there is a another point z such that Az = Sz then by (i) we have Az = Sz = By = T y, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 2.12 w is the only common fixed point of A and S. Similarly there is a unique point z ∈ X such that z = Bz = T z.

$\emptyset\{ M(Sw, Tz, t), M(Sw, Aw, t), M(Sw, Bz, t), M(Tz, Aw, t)\} \geq 0$
$\emptyset\{ M(w, z, t), M(w, w, t), M(w, z, t), M(z, w, t)\} \geq 0$
$\emptyset\{M(w, z, t), 1, M(w, z, t), M(w, z, t)\} \geq 0$
In view of Φ we get **w** = z. by Lemma 2.11 and z is a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (ii)

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